

Brief study of two port network and its parameters

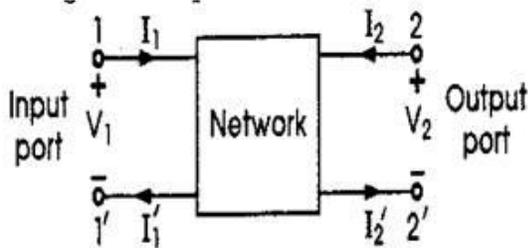
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Abstract- this paper proposes the study of the various types of parameters of two port network and different type of interconnections of two port networks. This paper explains the parameters that are Z-, Y-, T-, T', h- and g-parameters and different types of interconnections of two port networks. We will also discuss about their applications.

Index Terms- two port network, parameters, interconnections.

I. INTRODUCTION

A two-port network (a kind of four-terminal network or quadripole) is an electrical network (circuit) or device with two pairs of terminals to connect to external circuits. Two terminals constitute a port if the currents applied to them satisfy the essential requirement known as the port condition: the electric current entering one terminal must equal the current emerging from the other terminal on the same port. The ports constitute interfaces where the network connects to other networks, the points where signals are applied or outputs are taken. In a two-port network, often port 1 is considered the input port and port 2 is considered the output port.



II. VARIOUS TYPES OF PARAMETERS

2.1 Impedance parameters

Z-parameters are also known as *open-circuit impedance parameters* as they are calculated under open circuit conditions. i.e., $I_x=0$, where $x=1,2$ refer to input and output currents flowing through the

ports (of a two port network. in this case) respectively.

The Z-parameter matrix for the two-port network is probably the most common. In this case the relationship between the port currents, port voltages and the Z-parameter matrix is given by:

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

where

$$\begin{aligned} Z_{11} &= \left. \frac{V_1}{I_1} \right|_{I_2=0} & Z_{12} &= \left. \frac{V_1}{I_2} \right|_{I_1=0} \\ Z_{21} &= \left. \frac{V_2}{I_1} \right|_{I_2=0} & Z_{22} &= \left. \frac{V_2}{I_2} \right|_{I_1=0} \end{aligned}$$

For the general case of an N -port network,

$$Z_{nm} = \left. \frac{V_n}{I_m} \right|_{I_k=0 \text{ for } k \neq m}$$

The input impedance of a two-port network is given by:

$$Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + Z_L}$$

where Z_L is the impedance of the load connected to port two. Similarly, the output impedance is given by:

$$Z_{out} = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{11} + Z_S}$$

Where Z_S is the impedance of the source connected to port one.

2.2 Admittance parameters

A Y-parameter matrix describes the behavior of any linear electrical network that can be regarded as a black box with a number of ports. A *port* in this context is a pair of electrical terminals carrying equal and opposite currents into and out-of the network, and having a particular voltage between them. The Y-matrix gives no information about the behavior of the network when the currents at any port are not balanced in this way (should this be possible), nor does it give any information about the voltage

between terminals not belonging to the same port. Typically, it is intended that each external connection to the network is between the terminals of just one port, so that these limitations are appropriate. The Y-parameter matrix for the two-port network is probably the most common. In this case the relationship between the port voltages, port currents and the Y-parameter matrix is given by:

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Where

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} \quad Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

The input admittance of a two-port network is given by:

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L}$$

where Y_L is the admittance of the load connected to port two. Similarly, the output admittance is given by:

$$Y_{out} = y_{22} - \frac{y_{12}y_{21}}{y_{11} + Y_S}$$

Where Y_S is the admittance of the source connected to port one.

2.3 Transfer parameters

The ABCD-parameters are known variously as chain, cascade, or transmission line parameters. There are a number of definitions given for ABCD parameters, the most common is,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

For reciprocal networks $AD-BC=1$. For symmetrical networks $A=D$. For networks which are reciprocal and lossless, A and D are purely real while B and C are purely imaginary. This representation is preferred because when the parameters are used to represent a cascade of two-ports, the matrices are written in the same order that a network diagram would be drawn, that is, left to right. However, the examples given below are based on a variant definition;

$$\begin{bmatrix} V_2 \\ I'_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

Where

$$A' \stackrel{\text{def}}{=} \left. \frac{V_2}{V_1} \right|_{I_1=0} \quad B' \stackrel{\text{def}}{=} \left. \frac{V_2}{I_1} \right|_{V_1=0}$$

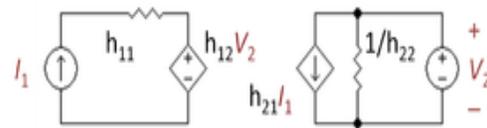
$$C' \stackrel{\text{def}}{=} \left. -\frac{I_2}{V_1} \right|_{I_1=0} \quad D' \stackrel{\text{def}}{=} \left. -\frac{I_2}{I_1} \right|_{V_1=0}$$

The negative signs in the definitions of parameters C' and D' arise because I'_2 is defined with the opposite sense to I_2 , that is, $I'_2 = -I_2$. The reason for adopting this convention is so that the output current of one cascaded stage is equal to the input current of the next. Consequently, the input voltage/current matrix vector can be directly replaced with the matrix equation of the preceding cascaded stage to form a combined matrix. The terminology of representing the parameters as a matrix of elements designated a_{11} etc. as adopted by some authors and the inverse parameters as a matrix of elements designated b_{11} etc. is used here for both brevity and to avoid confusion with circuit elements.

$$[\mathbf{a}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$[\mathbf{b}] = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$$

2.4 Hybrid parameters



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

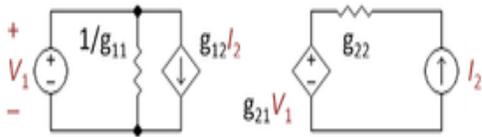
where

$$h_{11} \stackrel{\text{def}}{=} \left. \frac{V_1}{I_1} \right|_{V_2=0} \quad h_{12} \stackrel{\text{def}}{=} \left. \frac{V_1}{V_2} \right|_{I_1=0}$$

$$h_{21} \stackrel{\text{def}}{=} \left. \frac{I_2}{I_1} \right|_{V_2=0} \quad h_{22} \stackrel{\text{def}}{=} \left. \frac{I_2}{V_2} \right|_{I_1=0}$$

This circuit is often selected when a current amplifier is wanted at the output. The resistors shown in the diagram can be general impedances instead.

2.5 Inverse hybrid parameters



$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

Where

$$g_{11} \stackrel{\text{def}}{=} \left. \frac{I_1}{V_1} \right|_{I_2=0} \quad g_{12} \stackrel{\text{def}}{=} \left. \frac{I_1}{I_2} \right|_{V_1=0}$$

$$g_{21} \stackrel{\text{def}}{=} \left. \frac{V_2}{V_1} \right|_{I_2=0} \quad g_{22} \stackrel{\text{def}}{=} \left. \frac{V_2}{I_2} \right|_{V_1=0}$$

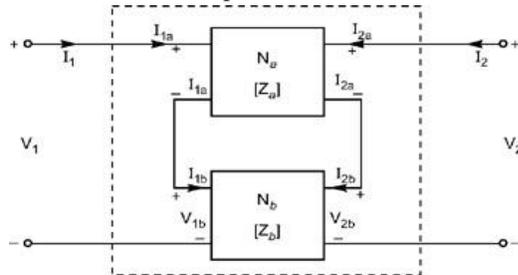
Often this circuit is selected when a voltage amplifier is wanted at the output. Notice that off-diagonal g-parameters are dimensionless, while diagonal members have dimensions the reciprocal of one another. The resistors shown in the diagram can be general impedances instead.

III. COMBINATION OF TWO PORT NETWORKS

When two or more two-port networks are connected, the two-port parameters of the combined network can be found by performing matrix algebra on the matrices of parameters for the component two-ports. The matrix operation can be made particularly simple with an appropriate choice of two-port parameters to match the form of connection of the two-ports. For instance, the z-parameters are best for series connected ports.

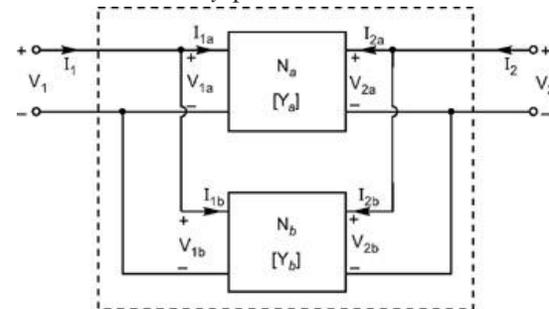
3.1 Series connections

When two-ports are connected in a series-series configuration, the best choice of two-port parameter is the z-parameters. The z-parameters of the combined network are found by matrix addition of the two individual z-parameter matrices.



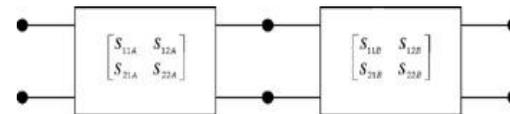
3.2 Parallel Connections

When two-ports are connected in a parallel-parallel configuration, the best choice of two-port parameter is the y-parameters. The y-parameters of the combined network are found by matrix addition of the two individual y-parameter matrices.



3.3 Cascade Connection

When two-ports are connected with the output port of the first connected to the input port of the second (a cascade connection), the best choice of two-port parameter is the ABCD-parameters. The a-parameters of the combined network are found by matrix multiplication of the two individual a-parameter matrices.



IV. CONCLUSION

This paper concludes about a brief theory of two port networks, its different parameters and their different interconnections.

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