

# BASIC OF SETS Theory

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**Abstract-** This research paper will examine the set theory. This paper provides an overview of the set and their types like finite set, infinite set, disjoint set, null set, power set etc. A set is defined as a collection of distinct objects of same type or class of objects. A set is usually denoted by capital letters. eg: A,B,C etc. A set can be formed by two way .Tabular and Builder form of a set. Set theory is seen as the foundation from which virtually all of mathematics can be derived. For example, structures in abstract algebra, such as groups, fields and rings, are sets closed under one or more operations. One of the main applications of naive set theory is constructing relations. Set operators exist and act on two sets to give us new sets. The basic operations of sets are Union ( $\cup$ ), Intersection ( $\cap$ ), Difference ( $-$ ), Complement (“—”), Symmetric Difference ( $\oplus$ ).

## I. Introduction

A set is defined as a collection of distinct objects of same types or class of objects. The objects of a set are called elements or members of the set. Objects can be number alphabets, names etc.

## II. SET FORMATION

Tabular form of set => if a set is defined by actually listing its member, e.g., if set p contains elements a, b, c, d then its is expressed as  $p = \{a, b, c, d\}$ .

Builder Form of a set=> if a set is defined by properties which its elements must satisfy.

$P = \{x: x \text{ is a real number and } -2 < x < 5\}$

**FINITE SET:** A set that is consist of specific number of different elements is said to be finite set. e.g.,

$R = \{\text{Months of year}\}$ .

**INFINITE SET:** if a set consists of infinite number of different elements or if the counting of different elements of the set does not come to an end, the set is called infinite set. E.g,

$I = \{\text{The set of all integer}\}$

**NULL or EMPTY SET:** A set that has no elements is called the empty set or null set and is denoted by  $\emptyset$ .

**SINGLETON SET:** A set that has one element is called a singleton set.

For example:  $\{a\}$ , with brackets, is a singleton set a, without brackets, is an element of the set  $\{a\}$ .

**DISJOINT SET:** Two sets A and B are said to be disjoint if no element of A is in B and no element of B is in A. e.g.

$$A = \{a, b, c, d\}$$

$$B = \{k, l, m, n\}$$

R and S are disjoint sets.

**MULTI SET:** A multi-set is a set where you specify the number of occurrences of each element:  $\{m_1.a_1, m_2.a_2, \dots, m_r.a_r\}$  is a set where

$m_1$  occurs  $a_1$  times

$m_2$  occurs  $a_2$  times

⋮

⋮

⋮

$m_r$  occurs  $a_r$  times.

A multi set is an unordered collection of elements, in which the multiplicity of an element may be one or more than one.

The multiplicity of an element is the number of times the element repeated in the multiset.

e.g.  $A = \{1, 1, m, m, m, n, n\}$ ;  $S = \{b, b, b, b, c\}$

**POWER SET:** The power set of a set S, denoted  $P(S)$ , is the set of all subsets of S.

Ex: Let  $A = \{a, b, c\}$ ,  
 $P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

Let  $A = \{\{a, b\}, c\}$ ,  $P(A) = \{\emptyset, \{\{a, b\}\}, \{c\}, \{\{a, b\}, c\}\}$

The power set is a fundamental combinatorial object useful when considering all possible combinations of elements of a set

**Fact:** Let S be a set such that  $|S|=n$ , then

$$|P(S)| = 2^n.$$

**TUPLES:** The ordered n- tuple  $(a_1, a_2, \dots, a_n)$  is the ordered collection with the element  $a_i$  being the i-th element for  $i=1, 2, \dots, n$

Two ordered n-tuples  $(a_1, a_2, \dots, a_n)$  and  $(b_1, b_2, \dots, b_n)$  are equal if and only if for every  $i=1, 2, \dots, n$  we have  $a_i = b_i$  ( $a_1, a_2, \dots, a_n$ )

A 2-tuple ( $n=2$ ) is called an ordered pair.

**UNIVERSEL SET:** If all the sets under investigation are sub sets of a fixed set  $U$ , then the set  $U$  is called universal set.

e.g. In plane geometry, the universal set consists of all the points in the plane.

**SUBSET OF SET:**

if every element of a set  $A$  is also an element of a set  $B$  then  $A$  is called subset of  $B$  and is written as

$$A \subseteq B, B \text{ is called superset of } A.$$

**PROPER SUBSET:** Given two sets  $A$  and  $B$ , we say  $A$  is a proper subset of  $B$ , denoted by  $A \subset B$ , if every element of  $A$  is an element of  $B$ , But there is an element in  $B$  that is not contained in  $A$ .

$$(b) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

**De Morgan's Laws**

$$(a) (A \cup B)^c = A^c \cap B^c$$

$$(b) (A \cap B)^c = A^c \cup B^c$$

**Identity laws**

$$(a) A \cup \Phi = A$$

$$(b) A \cap U = A$$

$$(c) A \cup U = U$$

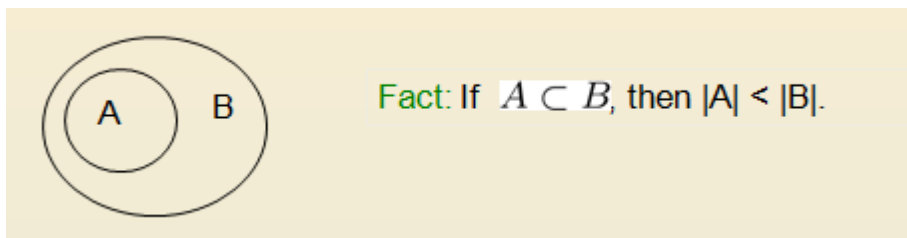
$$(d) A \cap \Phi = \Phi$$

**Complement Laws**

$$(a) A \cup A^c = U$$

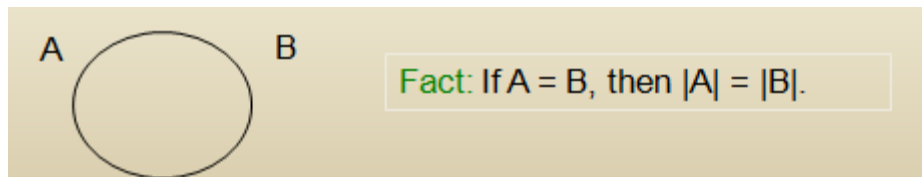
$$(b) A \cap A^c = \Phi$$

$$(c) U^c = \Phi$$



$$(d) \Phi^c = U$$

**Involution Law**



**IMPROPER SUBSET:** if  $A$  is subset of  $B$  and  $A=B$ , then  $A$  is said to be an improper subset of  $B$ .

**CARDINALITY OF A SET:** The total numbers of unique elements in the set if is called the cardinality of the set. The cardanility of the caountably infinite set is countably infinite.

$$\text{Let } P = \{k, l, m, n\}$$

The cardanility of the set  $P$  is 4.

$$(a) (A^c)^c = A$$

IV. Application of Set Theory

Set theory is seen as the foundation from which virtually all of mathematics can be derived. For example, structures in abstract algebra, such as groups, fields and rings, are sets closed under one or more operations. One of the main applications of naive set theory is constructing relations. A relation from a domain  $A$  to a codomain  $B$  is a subset of the Cartesian product  $A \times B$ . Given this concept, we are quick to see that the set  $F$  of all ordered pairs  $(x, x^2)$ , where  $x$  is real, is quite familiar. It has a domain set  $R$  and a codomain set that is also  $R$ , because the set of all squares is subset of the set of all reals. If placed in functional notation, this relation becomes  $f(x) = x^2$ . The reason these two are equivalent is for any given value,  $y$  that the function is defined for, its corresponding ordered pair,  $(y, y^2)$  is a member of the set  $F$ .

III. SET IDENTITES

**Idempotent Laws**

$$(a) A \cup A = A$$

$$(b) A \cap A = A$$

**Associative Laws**

$$(a) (A \cup B) \cup C = A \cup (B \cup C)$$

$$(b) (A \cap B) \cap C = A \cap (B \cap C)$$

**Commutative Laws**

$$(a) A \cup B = B \cup A$$

$$(b) A \cap B = B \cap A$$

**Distributive Laws**

$$(a) A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

## V. CONCLUSION

we in this, studied the different types of sets and also describe their properties and their laws. It is part of mathematics. It is important to know about the basic of sets and their theory.

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