LAPLACE TRANSFORMS AND ITS APPLICATIONS

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Abstract- Laplace transform is a very powerful mathematical tool applied in various areas of engineering and science. With the increasing complexity of engineering problems, Laplace transforms help in solving complex problems with a very simple approach just like the applications of transfer functions to solve ordinary differential equations. This paper will discuss the applications of Laplace transforms in the area of physics followed by the application to electric circuit analysis. A more complex application on Load frequency control in the area of power systems engineering is also discussed.

I. INTRODUCTION

Laplace transform is an integral transform method which is particularly useful in solving linear ordinary differential equations. It finds very wide applications in various areas of physics, electrical engineering, control engineering, optics, mathematics and signal processing. The Laplace transform can be interpreted as a transformation from the time domain where inputs and outputs are functions of time to the frequency domain where inputs and outputs are functions of complex angular frequency. In order for any function of time f(t) to be Laplace transformable, it must satisfy the following Dirichlet conditions [1]:

- f(t) must be piecewise continuous which means that it must be single valued but can have a finite number of finite isolated discontinuities for $t > 0$.

- f(t) must be exponential order which means that f(t) must remain less than $Se^{-a_o t}$ as t approaches ∞ where S is a positive constant and ao is a real positive number

A. Some Important Properties of Laplace Transforms

The Laplace transforms of different functions can be found in most of the mathematics and engineering books and hence, is not included in this paper. Some of the very important properties of Laplace transforms which will be used in its applications to be discussed later on are described as follows:[1][2]

• Linearity

The Laplace transform of the linear sum of two Laplace transformable functions f(t) + g(t) is given by

$L(f(t) + g(t)) = F(s) + G(s)$

• Differentiation

If the function f(t) is piecewise continuous so that it has a continuous derivative $f^{n-1}(t)$ of order n-1 and a sectionally continuous derivative $f^n(t)$ in every finite interval $0 \le t \le T$, then let, f(t) and all its derivatives through $f^{n-1}(t)$ be of exponential order e^{ct} as $t \to \infty$.

Then, the transform of $f^n(t)$ exists when Re(s) > c and has the following form:

$Lf^n(t) = s^nF(s) - s^{n-1}f(0+) - s^{n-2}f^{(1)}(0+) - \ldots\ldots - s^{n-1}f^{(n-1)}(0+)$

• Time delay

The substitution of $t - \lambda$ for the variable t in the transform Lf(t) corresponds to the multiplication of the function F(s) by $e^{-\lambda s}$, that is

$L(f(t - \lambda)) = e^{-s\lambda}F(s)$

II. APPLICATIONS OF LAPLACE TRANSFORMS

This section describes the applications of Laplace transforms in the areas of science and engineering. At first, simple application in the area of Physics and Electric Circuit theory is presented which will be followed by a more complex application to power system which includes the description of Load Frequency Control (LFC) for transient stability studies.

A. Application in Electric Circuit Theory

The Laplace transform can be applied to solve the switching transient phenomenon in the series or parallel RL,RC or RLC circuits [4]. A simple example of showing this application follows next.

Let us consider a series RLC circuit as shown in Fig 1. to which a d.c. voltage Vo is suddenly applied. Now, applying Kirchhoff's Voltage Law (KVL) to the circuit, we have,

Ri + Ldi/dt + 1/C Ridt = Vo (3)

Differentiating both sides,

Ld2i/di2 + 1/Ci + Rdi/dt = 0;

or, d2i/dt2 + (R/L)di/dt + (1/LC)i = 0 (4)

Now, applying laplace transform to this equation, let us assume that the solution of this equation is i(t) Kest where K and s are constants which may be real, imaginary or complex. Now, from eqn (4),

LKs2est + RKest + 1/CKest = 0 which on simplification gives,

or, s2 + (R/L)s + 1/LC = 0

p

The roots of this equation would be s1, s2 = R/2L ±(R2/4L2) − (1/LC)$^{1/2}$

The general solution of the differential equation is thus,

i(t) = K1es1t + K2es2t where K1 and K2 are determined from the intial conditions.

Now, if we define, α = Damping Coefficient = R/2L and Natural Frequency, ωn = 1/√LC which is also

known as undamped natural frequency or resonant frequency.

Thus, roots are : s1, s2 = −α ±(α2 − ω2)$^{1/2}$

The final form of solution depends on whether (R2/4L2) > 1/LC; (R2/4L2) = 1/LC and (R2/4L2) <1/LC

The three cases can be analysed based on the initial conditions of the circuit which are : overdamped case if α > ωn Critically damped case if α = ωn and underdamped case if α < ωn.

B. Application in Power Systems Load Frequency control

Power systems are comprised of generation, transmission and distribution systems. A generating system consists of a turbogenerator set in which a turbine drives the electrical generator and the generator serves the loads through transmission and distribution lines. It is required that the system voltage and frequency has to be maintained at some pre-specified standards eg. Frequency have to be maintained at 50 or 60 Hz and voltage magnitude should be 0.95-1.05 per unit. In an interconnected power system, Load Frequency Control (LFC) and Automatic Voltage Regulator (AVR) equipment are installed for each generator. The controllers are set for a particular operating condition and take care of small changes in load demand to maintain the frequency and voltage within specified limits. Changes in real power is dependent on the rotor angle, δ and thus system frequency and the reactive power is dependent on the voltage magnitude that is, the generator excitation In order to design the control system, the initial step is the modeling of generator, load, prime mover (turbine) and governer [5].

a. Generator Model

The modeling of a generator by applying the swing equation of a synchronous machine [5]. When small per-3 turbation is applied to the swing equation, the equation modifies as follows:

(2H/ωs)(d2∆δ/dt2) = ∆Pm − ∆Pe (5)

This can be written for a small deviation in speed with speed expressed in per unit as

d∆ω/dt = 1/2H(∆Pm − ∆Pe) (6)

Now, applying Laplace transform to Eq(6), we obtain

$\Delta\Omega(s) = 1/2Hs[\Delta Pm(s) - \Delta Pe(s)]$ (7)

b. Load model

The loads in the power system comprise of different kinds of electrical devices. Some loads are frequency dependent such as motor loads and other loads like lighting and heating loads are independent of frequency. The frequency sensitivity of the loads depend on the speed load characteristics of all the driven devices. The speed load characteristic of a composite load is approximated by $\Delta Pe = \Delta PL + D\Delta\omega$ (8) where ΔPL is the non frequency sensitive load change and $D\ \Delta\omega$ is the frequency sensitive load change. D is expressed as a percentage change in load divided by percent change in frequency. The combined block diagram representation of generator and load.

c. Prime mover model

Prime mover is the source of mechanical power which can be hydraulic turbines or steam turbines. The modeling of the turbine is related to the change in mechanical power output ΔPm to the change in steam valve position ΔPv. The simplest prime mover model for a steam turbine can be developed by a single time constant, τT and hence, the resulting transfer function is as follows:

$GT(s) = \Delta Pm(s)/\Delta PV(s) = 1/1+\tau T(s)$ (9)

d. Governer Model

During the cases when the generator load is suddenly increased, the electrical power exceeds the mechanical power input and this deficiency of power is supplied by the kinetic energy stored in the rotating system. Due to this reduction in kinetic energy, the turbine speed and hence, the generator frequency gets reduced. The turbine governer senses this reduction in speed and acts to adjust the turbine input valve to change the mechanical power output to bring the speed to a new steady state.

The governers are designed to permit the speed to drop as the load is increased. The steady state characteristics of a governer is as shown in Fig 5. The slope of this curve represents the speed regulation R. The speed governer mechanism acts as a comparator whose output $\Delta Pg$is the difference between the reference set power $\Delta Pref$ and the power $1/R\Delta\omega$ given by the governer speed characteristic. which can be expressed as follows:

$\Delta Pg = \Delta Pref - 1/R\Delta\omega$ (10)

Again, applying Laplace transform, in s-domain,

$\Delta Pg(s) = \Delta Pref(S) - 1/R\Delta\Omega(s)$ (11)

The command ΔPg is transformed to the steam valve position, ΔPV , assuming the linear relationship and considering a time constant τg, so that we have following4 s-domain relationship:

$\Delta PV(s) = 1/1+\tau g(s)\Delta Pg(s)$ (12)

The combination of eqn (11) and (12) can be represented by a block diagram of a governer model.

Now, combining the block diagrams of generator, load, turbine and governer systems we obtain the overall block diagram of the load frequency control of an isolated power system. power system

The closed loop transfer function relating the load change ΔPL to the frequency deviation $\Delta\Omega$ is

given by:

$\Delta\Omega(s)/-\Delta PL(s) = (1+\tau g(s))(1+\tau T(s))/(2Hs+D)(1+\tau g(s))(1+\tau T(s))+1/R$

That is: $\Delta\Omega(s) = -\Delta PL(s)T(s)$

The load change is a step input i.e. $\Delta PL(s) = \Delta PL/s$. Thus again applying the final value theorem we, obtain the steady state value of $\Delta\omega$ as $\omega ss = \lim(s \to 0)s\Delta\Omega(s) = (-\Delta PL)1/D+1/R$

A simple simulation is carried out in MATLAB simulink with the load frequency . The values of the different parameters are: turbine time constant

$\tau T = 0.5$ sec;governer time constant $\tau g = 0.2$ sec; Generator inertia constant, H = 5 sec; speed regulation, $1/R = 0.05$ pu and $\Delta PL = 0.2$ pu.

The plot in Fig 8 shows that there is a steady error in frequency of around -0.0096 pu. with this load frequency control mechanism. This application can be extended to a more complex Automatic Generation Control (AGC) in which the system frequency is automatically adjusted to the nominal value as the system load change continuously with zero steady state frequency error.

III. CONCLUSION

The paper presented the application of Laplace transform in different areas of physics and electrical power engineering. Besides these, Laplace transform is a very effective mathematical tool to simplify very complex problems in the area of stability and control. With the ease of application of Laplace transforms in myriad of scientific applications, many research softwares have made it possible to simulate the Laplace transformable equations directly which has made a good advancement in the research field.

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