Dynamic Modeling, Design and PID Control for an Autonomous Quadrotor

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Abstract- **This paper presents a rapid prototyping approach to attitude control algorithms of a quad rotor aerial vehicle. The control of Unmanned Aerial Vehicle (UAV) is a very challenging field of research especially for Vertical Take-Off and Landing (VTOL) vehicles or aircrafts for their numerous advantages over the traditional airplanes and due to the rapid advances that were made in this field with the development of light weight Micro-Electromechanical System (MEMS) sensors and actuators it has become possible to build an autonomous model for a light weight quad rotor and to develop various controls for it. This paper focuses on the development and implementation of an obstacle avoidance controller for quad rotor using four ultra-sonic (US) sensors for detecting obstacles and the triple-axis MEMS gyroscope/Accelerometer sensor for controlling its altitude. This paper focuses on the mathematical model of a quad rotor vehicle and performance of various control loops evaluated experimentally.**

***Index Terms*—Quadrotor, proportional-integral-derivative (PID) controller, vertical take-off and landing (VTOL), unmanned aerial vehicles (UAV), Quadrotor modeling and control, quadrotor modeling simulation in MATLAB ,autonomous aerial robot**

I. INTRODUCTION

A quadrotor helicopter or quadrotor for short is an aircraft lifted and propelled by four motors and propellers. They are extremely maneuverable and are capable of hovering and vertical takeoff and landing. The first type of quadrotors was flown in the 1920's with the aim to be used as manned vehicles, but the first successful manned flight was in the1960's when the Curtiss-Wright X-19A was developed [16]. These early quadrotors lacked systems that provided stability during flight and controlling four motors individually made them difficult for pilots to control and the manned development programs were scrapped. However, recent advances in technology mean that onboard computers can correct instabilities and control is much easier. Due to their design they are much simpler than helicopters to fly. They are now becoming much more common and the most recent types of quadrotors are designed to be unmanned aerial vehicles (UAV). Quadrotors work using a symmetrical design with the motors located in each of the four corners. The rotors are fixed in their pitch with two of the rotors move clockwise and two of them move anti-clockwise. This design naturally cancels out torque and prevents quadrotor from rotating in the air. Control over the aircraft is produced by changing the speed of the motors. The simplicity of the design has a number of advantages over traditional helicopters. Quadrotors are free from complex mechanical control linkages which mean that they require less maintenance and are more efficient at smaller sizes. In addition, by surrounding the rotors within a frame, the rotors can be protected from damage during collisions. This allows flights indoors and in cluttered surroundings, with minimal risk of damaging the aircraft or its environment. These safety benefits speed-up the design and test flight process as flights can be carried out indoors, by inexperienced pilots, with a quick turnaround time after an incident.

II. MODELING AND CONTROL

The basic concept of flight mechanism is based on the propeller's speed and direction of rotation. Generally Quadrotor (UAV) is equipped with four fixed pitch rotors; each one includes a Brush Less DC (BLDC) motor, a one-stage gearbox and a propeller. The rotors are directed upwards and they are placed in a square formation with equal distance from the center of mass of the quadrotor. By changing the control command to these motors Ωi, their speed will vary and the quadrotor direction is updated, accordingly the quadrotor can navigate in different directions. For instance, the quadrotor can move in the vertical Z direction by varying the speed of all propellers at the same time and by the same amount. To command the quadrotor to move in the X direction, the speed of the front and rear propellers should be changed by the same amount and in opposite directions . Moving the quadrotor in the Y direction can be done by changing the speed of the right and left propellers by the same amount and in opposite directions. To control quadrotor heading, the speed of all propellers is commanded by the same amount but in different directions, front and rear propellers with the same direction and right and left the propellers' with opposite direction.

A. Quadrotor Model

Dynamics of quad rotor has been performed in many researches. Quad rotor modeling by method of deriving and kinematic relation describes various methods, it is performed by Lagrange–Euler method, Newton-Euler Method and Kinetic Relations are described by Quaternion method or Euler's angles. The development of a suitable attitude controller for the quadrotor prototype required an accurate dynamic model to be developed. A Newtonian modeling method was chosen to define the quadrotor dynamics for control purposes. The Newtonian method is the most popular choice for modeling rigid bodies in six degrees of freedom and has been used extensively for the modeling of traditional helicopters. The figure 1. Shows a schematic of a quadrotor.

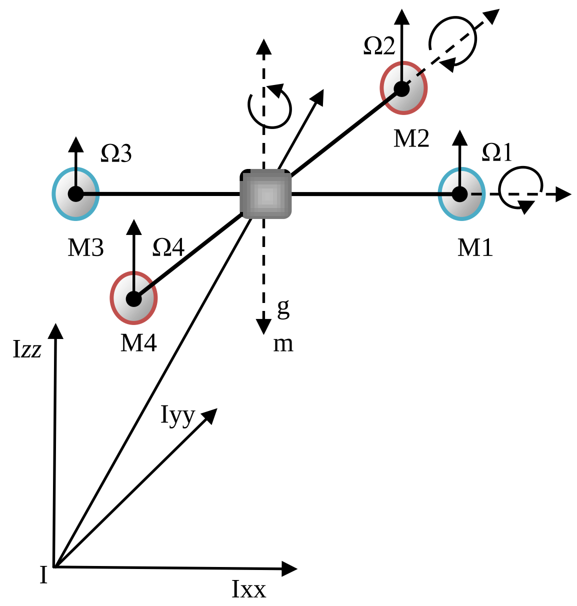

Fig.1.The quadrotor schematic.

Modeling of a UAV such as a quadrotor is not an easy task because its complex structure and it induces the non-linear aerodynamical equations of the UAV along with the actuator's dynamics and saturation limits. The aim is to develop a model of UAV as realistically as possible; the model is represented as follows [1]

$$\ddot{X} = (\sin\Psi \sin\Phi + \cos\Psi \sin\theta \cos\Phi)\frac{U_1}{m} \quad (1)$$

$$\ddot{Y} = (-\cos\Psi \sin\Phi + \sin\Psi \sin\theta \cos\Phi)\frac{U_1}{m} \quad (2)$$

$$\ddot{Z} = -g + (\cos\theta \cos\Phi)\frac{U_1}{m} \quad (3)$$

$$\dot{p} = \frac{I_{yy}-I_{zz}}{I_{xx}}qr - \frac{U_2}{I_{xx}} \quad (4)$$

$$\dot{q} = \frac{I_{zz}-I_{xx}}{I_{yy}}pr - \frac{U_3}{I_{yy}} \quad (5)$$

$$\dot{r} = \frac{I_{xx}-I_{yy}}{I_{zz}}pq - \frac{U_4}{I_{zz}} \quad (6)$$

Where X, Y and Z are the positions of the center of mass with respect to inertial frame. The first three equations describe the linear acceleration of the vehicle in the direction of x, y and z axes respectively. p, q and r are the body rates, (Φ,θ,Ψ) represent the Euler's angles about the body axes(X,Y,Z) respectively. Ixx, Iyy and Izz are the inertial components about the X-axis, Y-axis and Z-axis respectively is the mass of the system.

The actuator's dynamics are described by the following equations,

Where **Ui (i=1, 2, 3, 4)** represents input of the system.

$$U_1 = \sum_{i=1}^{4} T_i = b(\Omega_1^2+\Omega_2^2+\Omega_3^2+\Omega_4^2) \quad (7)$$

$$U_2 = (-T_2 + T_4) = bl(-\Omega_2^2 + \Omega_4^2) \quad (8)$$

$$U_3 = (-T_1 + T_3) = bl(-\Omega_1^2 + \Omega_3^2) \quad (9)$$

$$U_4 = (-1)^i \sum_{i=1}^{4} M_{D_i} = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \quad (10)$$

Where "l" length of the arm holding the propeller (i.e. radius of the quadrotor frame),U1 represents the total thrust, U2 represents the pitch moment, U3 represents the roll moment while U4 represents the yaw moment and finally "b" is the thrust factor in hover and "d" is the drag factor is hover. Increasing or decreasing of the speed of the four propellers together will be responsible for the altitude change in position and velocity while varying the speed of one pair of propellers (Ω3, and Ω1) will cause the aircraft to tilt about the y-axis which is denoted as pitch angle theta "θ". Similarly varying the speed of the propellers pair (Ω4, and Ω2) will cause the aircraft to tilt about x-axis which is denoted as roll angle phi "Φ". Finally the vector summation of the reaction moment produced by the rotation of the pair (Ω3, and Ω1) and the reaction moment produced by the rotation of the pair (Ω4, and Ω2) will cause the quadrotor to spin about its axis (z-axis) which is denoted as yaw angle epsi "ψ" these are the six degrees of freedom of the system consisting of the position (x, y, z) and the orientation (Φ, θ, ψ) [16].

B. *Controller Design*

Control system for the quadrotor can be designed by various methods as LQR, PID, Sliding Mode, Backstepping, Model Preductive Control (MPC) etc. In this paper, the PID controller for the quadrotor is developed based on the fast response. Using this approach as a recursive algorithm for the control-laws synthesis, all the calculation stages concerning the tracking errors are simplified. One other aspect of the controller selection depends on the method of control of the UAV. It can be mode based or non-mode based. For the mode based, controller, independent controllers for each state is needed, and a higher level controller decides how these interact. On the other hand for a non-mode based controller, a single controller controls all of the states together. The traditional implementation can be defined as follows [14],

$$Output(t) = K_p\, e(t) + K_i \int_0^t e(\tau)dt \; + K_d \frac{d}{dt}\, \mathrm{e}\,(\mathrm{t}) \quad (11)$$

Where Kp is the proportional gain, Ki is the integral gain and Kd represents derivative gain. These are 3 Algorithms in PID Controller, they are P, I and D respectively depends on Present Error .I depends on the accumulation of Past Errors .D is a prediction of Future Errors Based on Current rate of

Change gain and e (t) is defined as the difference between the actual controlled variable value and a desired valued (error). The Per axis PID structure is shown in Figure 2.

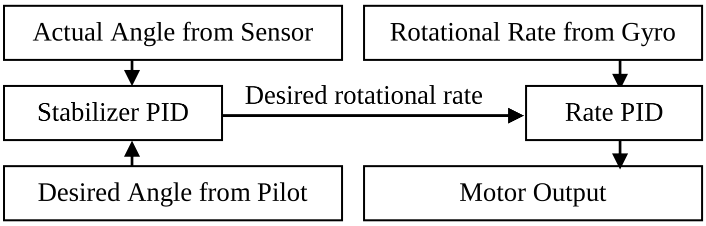

Fig. 2. Per axis PID Control Algorithm[17].

The quadrotor controller will be targeted at forcing the attitude angles to desired set points. As a Result e (t) can be defined as

- e $(t)_{ROLL}$= Set point - Measured Roll
- e $(t)_{PITCH}$ = Set point - Measured Pitch
- e $(t)_{YAW}$ = Set point - Measured Yaw

To have any kind of constant over the Quadrotor, We need to be able to Measure the Quadrotor Sensor Output (e.g. Pitch Angle).So we can estimate the Error (i.e. how far we are from the desired Pitch angle e.g. Horizontal,0 Degree we can then Apply the 3 Control Algorithms to the Error, to get the next Outputs for the Motors aiming to Control Error).To achieve this the rotational speed for each motor based on these command signals is fused using inverse kinematics as follows

$$\Omega_1=\frac{U_1}{4b}-\frac{U_3}{2lb}-\frac{U_4}{4d} \tag{12}$$

$$\Omega_2=\frac{U_1}{4b}-\frac{U_2}{2lb}+\frac{U_4}{4d} \tag{13}$$

$$\Omega_3=\frac{U_1}{4b}+\frac{U_3}{2lb}-\frac{U_4}{4d} \tag{14}$$

$$\Omega_4=\frac{U_1}{4b}+\frac{U_3}{2lb}+\frac{U_4}{4d} \tag{15}$$

$$\lambda_{desired}=\begin{bmatrix} X \\ Y \\ Z \\ \Psi \end{bmatrix} \tag{16}$$

It can be noticed the proposed controller structure requires four set points as shown in vector λ (Eq-16).There must be six PID controllers are used for X-Position-Position, Altitude, Yaw, Pitch, Roll movements. For the roll and pitch desired angles are generated from Y and X position PID controllers respectively, based on a cascade controller structure. Based on several tests, tuning and simulation following parameters cause the behavior of the quadrotor.

1) Proportional gain Coefficient

- Quadrotor can fly relatively stable without other parameter but this one, this coefficient determines which is more important known control or the values measured by the gyroscopes, the higher the coefficient, the higher the quadrotor seems more sensitive and sensitive to angular change.
- If it is too low, the quadrotor will appear sluggish and will appear sluggish and will be harder to keep Steady.
- It can be seen quadrotor starts to oscillate with frequency when P gain is too high.

2) Integral Gain Coefficient

- This coefficient can increase the precision of the angular position.

  E.g.: When Quadrotor is disturbed and its angle Changes 15 Degrees .This term is especially useful in Irregular winds and ground effect i.e. Turbulence from motors.

- If I value is too High

  Quadrotor might begin to have Slow Reaction and a decrease effect of the proportional gain as consequence it will start Oscillating like having high P gain but with a lower Frequency.

3) Derivative Gain Coefficient

- It allows the quadrotor to reach more quickly the desired attitude .Like accelerator parameters because it amplifies the user Input.
- It also decrease control action fast when error decreasing fast.
- In practice, it will increase the reaction speed and in certain cases it Increase the effect of P gain.
- D gain makes QR more sensitive .i.e. when quadrotor oscillator this parameter makes oscillation worse.

III. RESULTS & SIMULATION STUDY

The simulation was completely built by using MATLAB software. All the states of the quadrotor can be monitored and plotted after the simulation is completed. The simulation platform was the integration of three subsystems sensor subsystem, controller subsystem and quadrotor subsystem.

a) The Sensor Subsystem

In the sensors subsystem, the model has been designed to simulate measurement of position, orientation and altitude of the quadrotor. In this scenario the triple-axis MEMS gyroscope/Accelerometer in the MPU-60X0 includes a wide range of features. Digital-output X-, Y-, and Z-Axis angular rate sensors (gyroscopes) with a user-programmable full-scale range of ±250, ±500, ±1000, and ±2000°/sec. Digital-output triple-axis accelerometer with a programmable full scale range of ±2g, ±4g, ±8g and ±16g.It features Orientation detection and signaling, Tap detection. The ultrasonic sensors are integrated with the system to detect any obstacle available in the environment to avoid the collision. A collision avoidance behavior was practically obtained after numerous tests and tuning. Once the obstacle is detected, a pitch reference is given to fly away the quadrotor from the obstacle.

b) The Controller subsystems

In the Controller subsystems, different algorithms can be utilized and their functionality and performance in avoiding obstacles and driving the quadrotor to the target location can be examined within the indoor environments. The proposed control algorithm as shown in figure 2 is composed of all the controllers, inputs, speed reference and the inner relationships of the thrust. The simulation results show that the PID controllers are able to robustly stabilize the quadrotor and move it to a desired position with a desired yaw angle while keeping the pitch and the roll angles zero, and here in this design, it is easy and with a fast response time, can get the Theta (Pitch angle) to its desired value the results are shown from figure 3 to figure 15. The reason for using the PID controllers in this system is to control z, which is sensitive to the changes for the other parameters. While PID tuning following setting must be considered,

For Aerobatic Flight

- It requires a slightly higher **P**.
- Slightly lower **I.**
- Increase **D**.

For Gentle Smooth Flight

- Slightly lower **P.**
- Slightly higher **I.**
- Decrease **D.**

c) The Quadrotor Model Subsystem

In quadrotor model's subsystem, the nonlinear equations of motion are provided to simulate the quadrotor's response to a given control commanded from the controller subsystem.

IV. CONCLUSION

This paper presented the design of a PID controller algorithm to control the quadrotor system. The integration of the navigation sensors, controllers, quadrotor dynamics and the navigation environment, provide an interesting platform which can be utilized for obstacles avoidance algorithms, localization, or mapping of navigated area surrounding the quadrotor.

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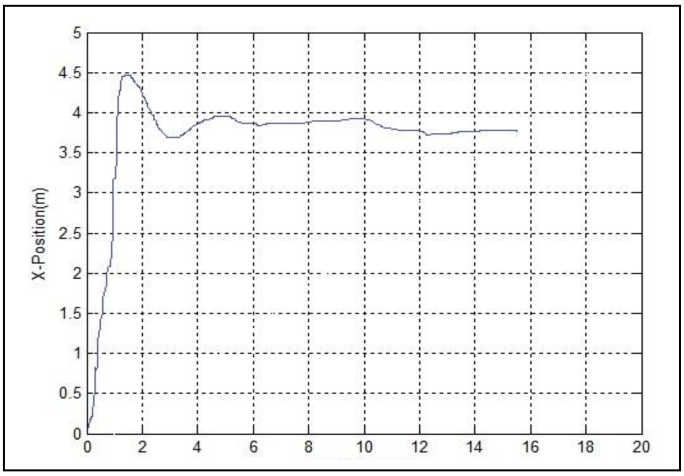

Fig. 3. X-position

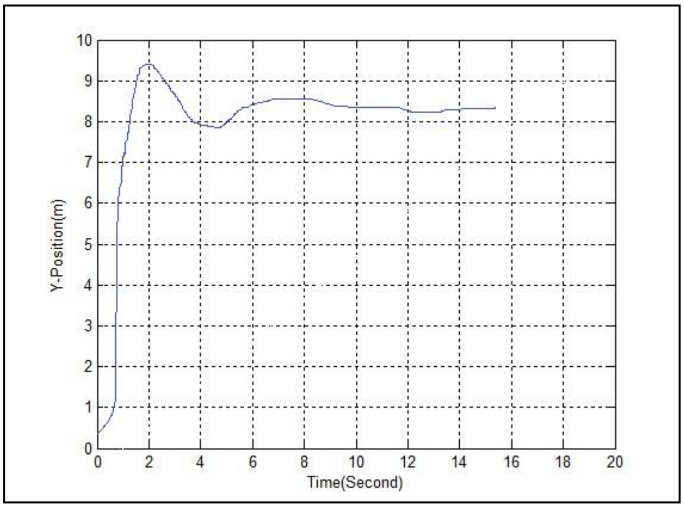

Fig. 4. Y-position

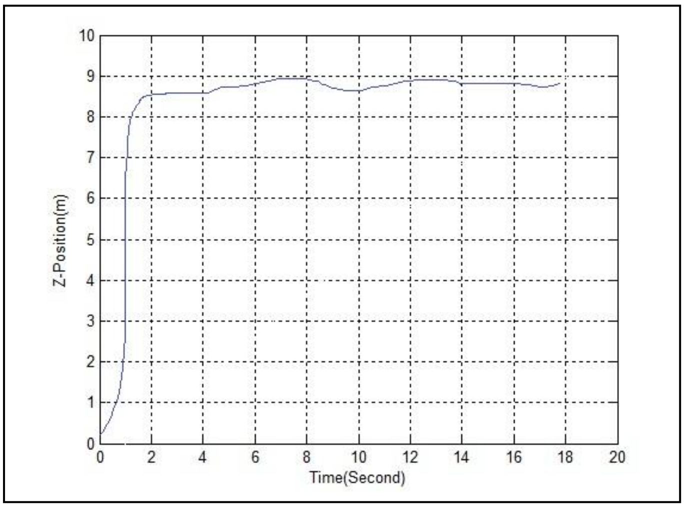

Fig. 5. Z-position

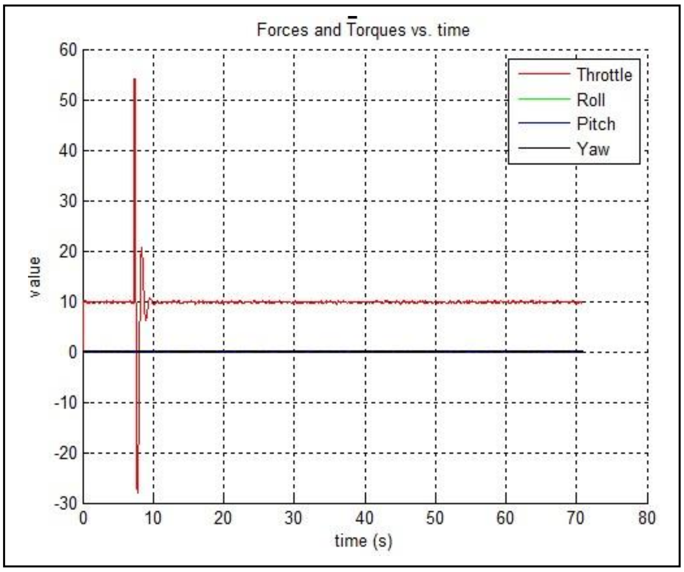

Fig. 6. Force and Torque vs Time

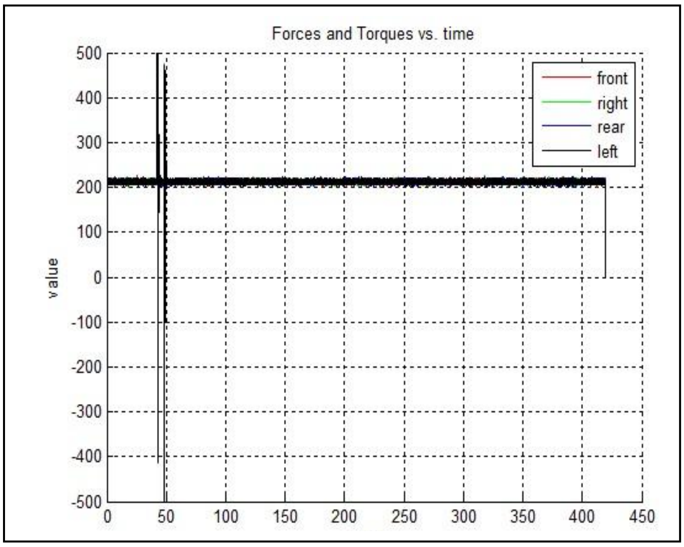

Fig. 7. Motor Force and Torque vs Time

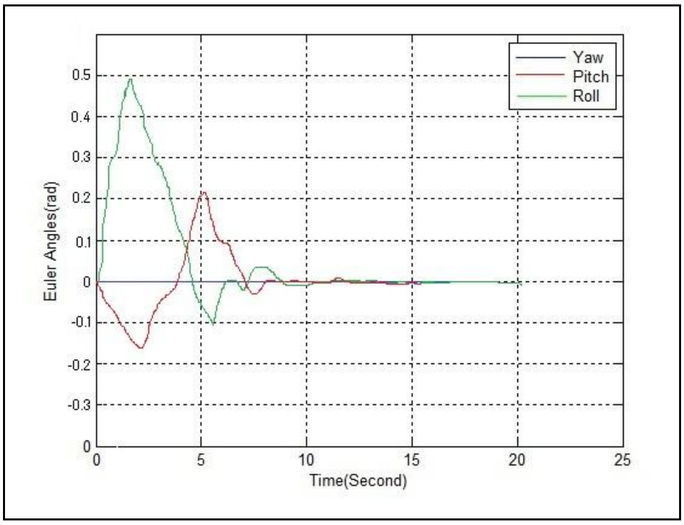

Fig. 8. Euler Angles Yaw,Pitch and Roll

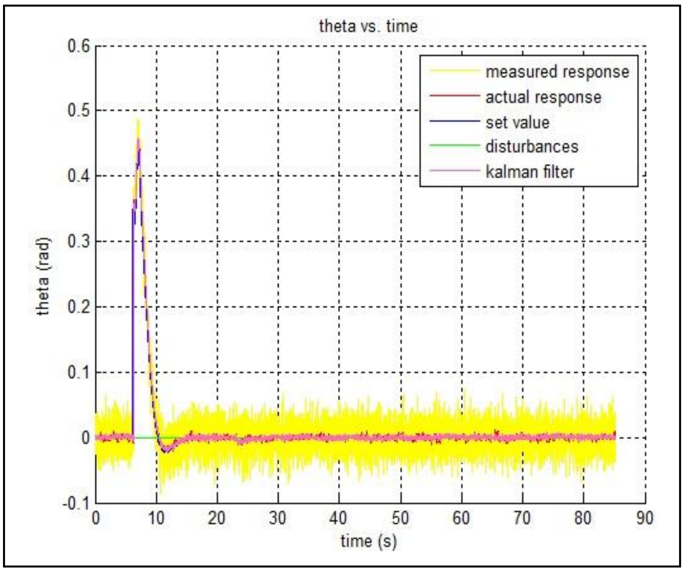

Fig. 9. Angle Theta vs Time

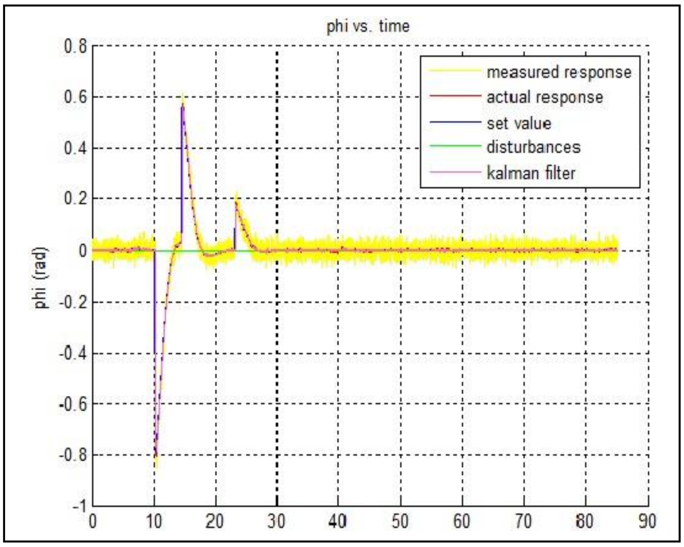

Fig. 10. Angle Phi vs Time

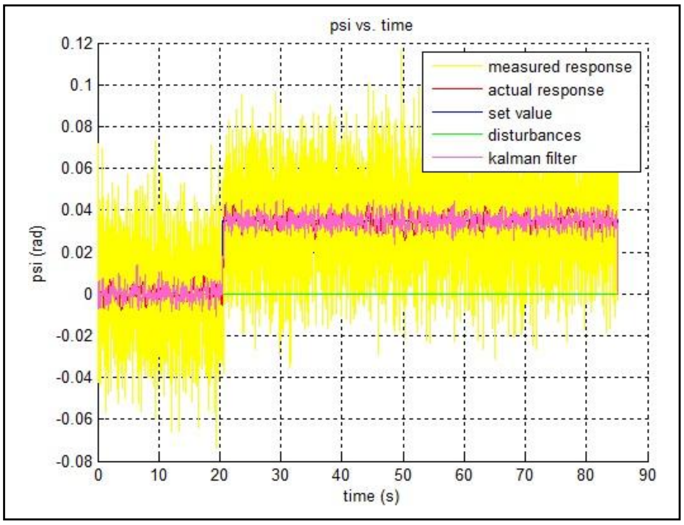

Fig. 11. Angle Psi vs Time

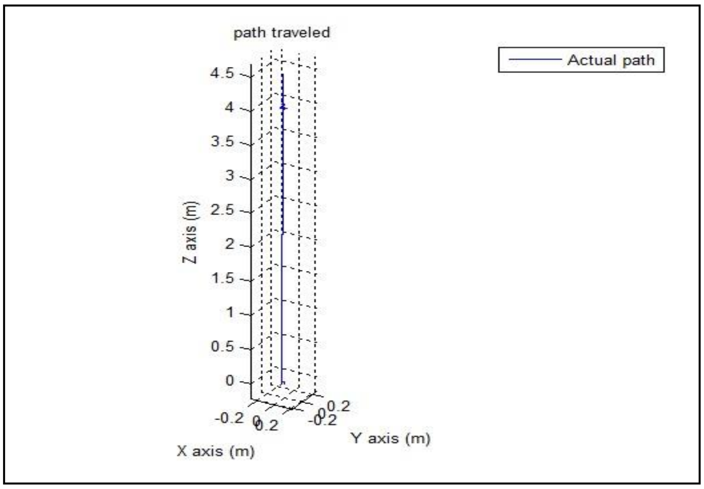

Fig. 12. Flight Path

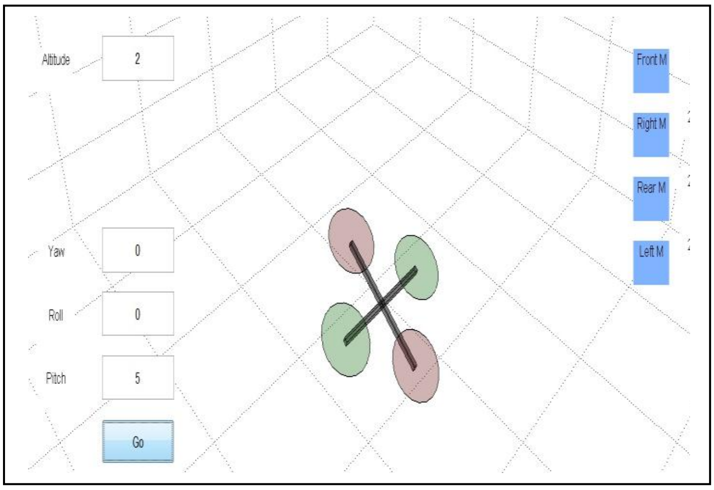
Altitude 2
Yaw 0
Roll 0
Pitch 5

Fig. 13. . Quadrotor Movement (Pitch)

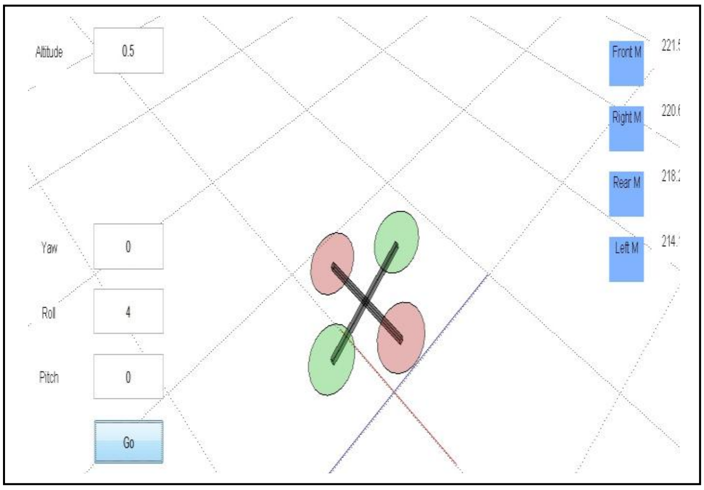
Altitude 0.5
Yaw 0
Roll 4
Pitch 0

Fig. 14. Quadrotor Movement (Roll)

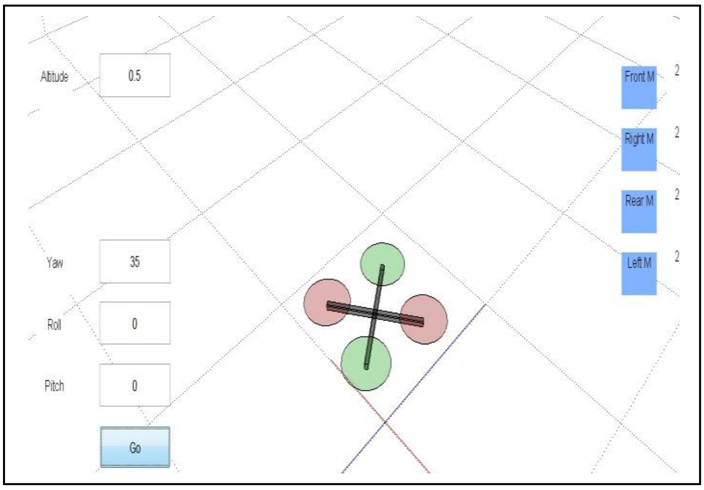
Altitude 0.5
Yaw 35
Roll 0
Pitch 0

Fig. 15. Quadrotor Movement (Yaw)

REFERENCES

[1] Muhannad A.R. Al-Omari. “Integrated Simulation Platform for Indoor Quadrotor Applications”, Proceedings of the 9th International Symposium on Mechatronics and its Applications (ISMA13), Amman, Jordan, April 9-11, 2013.

[2] Li, Borui, Chundi Mu, and Botao Wu. "A survey of vision based autonomous aerial refueling for Unmanned Aerial Vehicles." Intelligent Control and Information Processing (ICICIP), 2012 Third International Conference on. IEEE, 2012.

[3] Saggiani, G. M., and Teodorani, B., 2004: Rotary wing UAV potential applications: an analytical study trough a matrix method. Aerospace Division –University of Bologna, Published on Aircraft Engineeringg and Aerospace Technology Vol. 75 N°1, 2004.

[4] M. Sakti Alvissalim, Big Zaman, Ahmad Hafizh Z M, Anwar Ma’sum, Grafika Jati, Wisnu Jatmiko, Petrus Mursanto,” Swarm Quadrotor Robots for Telecommunication Network Coverage Area Expansion in Disaster Area”, SICE Annual Conference 2012, August 20-23, 2012, Akita University, Akita, Japan.

[5] Paul Infant Teenu Mohan Das, Suraj Swami and James M. Conrad, “An Algorithm for Landing a Quadrotor. Unmanned Aerial Vehicle on an Oscillating Surface”, 2012 IEEE.

[6] Integrated Guidance Navigation and Control for a Fully Autonomous Indoor UAS Girish Chowdhary, Michael Sobers, Chintasid Pravitra, Claus Christmann, Allen Wu, Hiroyuki Hashimoto, Chester Ong, Roshan Kalghatgi, Eric N. Johnson, AIAA Guidance Navigation and Control Conference, Portland, OR, August, 2011.

[7] Slawomir Grzonka, Giorgio Grisetti, and Wolfram Burgard, “A Fully Autonomous Indoor Quadrotor” IEEE Transactions On Robotics, Vol. 28, No. 1, February 2012.

[8] Manohar B. Srikanth, Zachary T. Dydek, Anuradha M. Annaswamy, and Eugene Lavretsky,” A Robust Environment for Simulation and Testing of Adaptive Control for Mini-UAVs”, 2009 American Control Conference.

[9] Chen Xiao-long, Tang Qiang, and Che Jun,” An Indoor Quadrotor Locating and Object-Following Algorithm using Monocular Vision”, Proceedings of the 10th World Congress on Intelligent Control and Automation, July 6-8, 2012, Beijing, China

[10] Tommaso Bresciani,” Modelling, Identification and Control of a Quadrotor Helicopter” MASTER THESIS, Department of Automatic Control,Lund University, October 2008.

[11] S. Bouabdallah, P. Murrieri, and R. Siegwart, Design and Control of an Indoor Micro Quadrotor. In Proc. ICRA, New Orleans, Louisianna, 2004.

[12] Puri, A. (2004); A survey of Unmanned Aerial Vehicles (UAV) for Traffic Surveillance,Technical Report 2004.

[13] Gabriel M. Hoffmann, Haomiao Huang, Steven L. Waslander, and Claire J. Tomlin, Quadrotor Helicopter Flight Dynamics and Control: Theory and Experiment, In Proc. AIAA GNC, Hilton Head, South Carolina, 2007.

[14] Jun Li, Yuntang Li,” Dynamic Analysis and PID Control for a Quadrotor”, Proceedings of the 2011 IEEE International Conference on Mechatronics and Automation, August 7 - 10, Beijing, China.

[15] http://www.quadrotors.net/quadrotors-introduction.

[16] A. Y. Elruby,” Dynamic Modelling and Control for Quadrotor vehical”, Arab Academy forscience and Technology, Egypt.

[17] http://blog.oscarliang.net/quadcopter-pid-explained-tuning.