Comparison of PAPR Reduction Companding Techniques of OFDM Signal

Jyoti Gurjar[1], Rohit Bhayre[2], Shivraj Singh[3]
[1]*Research Schcolers, Technocrats Institute of Technology, Bhopal, India*
[2]*Research Schcolers, National Institute Of Technology, Jaipur, India*
[3]*Assistant Professor , Technocrats Institute of Technology, Bhopal, India*

Abstract **- the major drawbacks which is yet to be solved in orthogonal frequency division multiplexing (OFDM) system is its high peak-to-average power ratio (PAPR). In this paper, different companding transform scheme are compared which are nonlinear in nature and which transforms the Gaussian-distributed OFDM signals into a desired piecewise function. variable slopes and inflexion points is introduced in the target probability density function, to impart more flexibility in the companding form and an effective trade-off between the PAPR and bit error rate performances can be achieved along with a relatively low complexity. The theoretical analyses of different techniques and trade-off between PAPR reduction and bit error ratio (BER) performance are presented well.**

Index Terms - orthogonal frequency division multiplexing (OFDM), nonlinear companding transform (NCT), peak-to-average power ratio (PAPR).

I. INTRODUCTION

Though orthogonal frequency division multiplexing (OFDM) offered significant advantages, the fluctuating envelope with high peaks is the major drawback of OFDM, which give rise to high peak to-average-power ratio (PAPR) in the transmitted signal. The transmitter's power amplifier (PA) drives into the saturation or nonlinear regions of operation because of High peaks , hence causing in-band distortions and out-of-band radiation. They also require wide dynamic ranges of analog-to-digital converters (ADC). Many techniques to reduce PAPR have been proposed in the literature, such as clipping and filtering, selective mapping ,companding transforms (CTs), partial transmit sequences, tone reservation, tone injection and linear block coding [1]–[3]. Complementary cumulative distribution function (ccdf) is used to measure PAPR reduction capability, which is defined as the probability that the signal's PAPR exceeds a specific threshold. In most methods, PAPR is reduced at the expense of increasing the bit error rate (BER), complexity, or data overhead. due to their flexibility and low complexity the CTs form an attractive and widely used PAPR reduction technique, with N subcarriers in an OFDM system, the complex baseband representation of OFDM signal is given by.

$$x(t) = \frac{1}{N}\sum_{k=0}^{N-1} X_k\, e^{\frac{j2\pi kt}{T}} \qquad 0 \le t \le T \ (1)$$

Irrespective of the number of OFDM signal subcarriers. CTs amplify the low amplitudes and attenuate the high peaks, so, by enlarging small signals and compressing large ones, both immunity of small signals from noise and the PAPR reduction can be achieved [6]. However, it is important to note that NCT is extra pre-distortion operation which is, applied to transmitted signal, which results in increased sensitivity to the HPA and performance degradation.

It was noticed in [8] that, due to the disadvantages of nonlinear distortion, such transform should be designed judiciously such that the clipped signal is as little as possible For this reason, how to reallocate the statistics of OFDM signal as well as the power more reasonably to reduce the companding distortion impact is the key challenge for a to design NCT method. Moreover, a flexible and effective trade-off among the overall performance of OFDM system with respect to PAPR reduction (power efficiency), spectral re-growth (bandwidth efficiency), bit error rate (BER) and also the implementation complexity should be considered.

In this paper, further motivated by the observation above the NCT algorithms are compared in which the Gaussian distributed signal is transformed into a desirable distribution form defined by piecewise functions with inflexion points. Compared to the methods discussed and the impact of companding distortion on the BER performance is studied. In addition the achievable PAPR reduction analytical expressions, the selection criteria of transform parameters and signal attenuation factor studied. The rest of this paper is outlined as follows. Section II briefly gives the idea about the statistical characteristic of OFDM signal, while in Section III the generic formulas for different algorithm are shown. performance study is given in Section IV section V shows comparison result, followed by the conclusions in Section V

Notation: The expectation and maximal element operator are denoted by E {.} and max {.}. We use $[.]^T$, $(.)^{-1}$ and $|.|$ to denote the transpose, inverse and modulus operation, respectively. sgn(.) stands for the sign function IFFT$_N${.} represents the N -point inverse fast Fourier transform (IFFT) operation. Prob{A} is the probability of the event A.

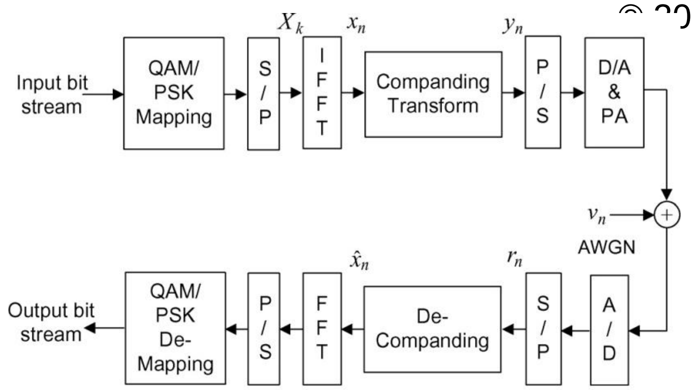

Fig. 1 Block diagram of OFDM system with companding transform

II. CHARACTERIZATION OF OFDM SIGNAL

Generally, an OFDM signal is the addition of N independent data symbols modulated by quadrature amplitude modulation (QAM) or phase-shift keying (PSK). In discrete-time domain, since the peaks of the continuous-time signal may not be represented by the Nyquist rate samples, so it is preferred to approximate on an oversampled signal. The time-domain oversampled OFDM symbols x can be calculated as

$$x = [x_0 , x_1 , \ldots . x_{JN-1}]^T$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_k \, e^{\frac{j2\pi kn}{T}} \quad ; 0 \le t \le T \quad (2)$$

where n=0,1,2…..JN-1 is time index and J is the oversampling ratio. Usually, J>=4 is used for accurate PAPR of the continuous-time signal. This process of oversampling can be achieved by performing a JN-point IFFT with extending X to a JN-length vector by inserting (j -1) N extra zeros in its middle, i.e.

$$X_e = \left[X_0 \ldots . . X_{\frac{N}{2}-2}, 0 \ldots 0 \, , X_{\frac{N}{2}} \ldots . X_{N-1} \right]^T \qquad (3)$$

It is clear that $x = IFFT_{JN}\{X_e\}$. For a large N (i.e. N>=64), the real and imaginary parts of x_n may be approximated as Gaussian random variables with zero mean and a variance σ^2.Based on this assumption, the signal amplitude $|x_N|$ follows a Rayleigh distribution with the probability density function (PDF) as

$$f_{|x_n|}(x) = \frac{2x}{\sigma^2} e^{-\frac{x^2}{\sigma^2}} , \; x \ge 0 \qquad (4)$$

The cumulative density function (CDF) of $|x_n|$ is therefore

$$F_{|x_n|}(x) = prob\{|x_n| \le x\} = \int_0^x \frac{2y}{\sigma^2} ep\left(-\frac{y^2}{\sigma^2}\right) dy = 1 - \exp\left(-\frac{x^2}{\sigma^2}\right), \qquad x \ge 0 \qquad (5)$$

The PAPR of OFDM signal in a given frame is defined as

$$PAPR(x(t)) = \frac{\max\left[|x(t)|^2 \right]}{P_{av}} \qquad (6)$$

Considering the PAPR as a random variable is more helpful and utilizing a statistical description given by the complementary cumulative density function (CCDF), it is defined as the probability that the PAPR of exceeds an assigned level yo>=0.i.e

$$CCDF_x(\gamma_0) = \Pr ob(PAPR_x > \gamma_0) \approx 1 - (1 - \exp(-\gamma_0))^N \qquad (7)$$

The NCT principle is described as follows. The original signal x_n is companded before converted into analog waveform and amplified by the HPA. The signal after companding is denoted as yn=h(xn), where h(.) is the companding function that only changes the amplitude of x_n . In additive Gaussian white noise (AWGN) channel case the received signal rn=yn+vn can be recovered using the de-companding function h-1(.), namely, $x_n' = h^{-1}(y_n + v_n) = x_n + h^{-1}(v_n)$ where channel noise is v_n .

III.THE GENERIC FORMULAS FOR DIFFERENT ALGORITHM DESCRIPTION

Companding functions for different technique are as follows Pdf function for companding functions are represented by f(.) and companding function by h(.) [27], [28].

$$f_{1|y_n|}(x) = \begin{cases} k_1 x, & 0 \le x \le cA \\ k_2 x + (k_1 - k_2) cA, & cA < x \le A \end{cases} \qquad (8)$$

$$h_1(x)=\begin{cases}\operatorname{sgn}(x)\sqrt{\dfrac{2}{k_1}\left(1-\exp^{\frac{-|x|^2}{\sigma_2}}\right)}, & |x|\le X_0\\ \operatorname{sgn}(x)\dfrac{1}{k_2}\left((k_2-k_1)cA+\sqrt{(k_2-k_1)k_1c^2A^2+2k_2\left(1-\exp^{\frac{-|x|^2}{\sigma_2}}\right)}\right), & |x|>X_0\end{cases} \qquad (9)$$

$$f_{2|y_n|}(x)=\begin{cases}\dfrac{2x}{\sigma^2}\exp\left(\dfrac{x^2}{\sigma^2}\right), & 0\le x\le c\sigma\\ kx+\dfrac{2c}{\sigma}\exp-c^2-kc\sigma, & c\sigma<x\le A\end{cases} \qquad (10)$$

$$h_2(x)=\begin{cases}x, & |x|\le c\sigma\\ \operatorname{sgn}(x)\dfrac{1}{k}\left(kc\sigma-\dfrac{2c}{\sigma}\exp^{-c^2}+\sqrt{\dfrac{4c^2}{\sigma^2}\exp^{-2c^2}-\exp^{\frac{-|x|^2}{\sigma^2}}}\right), & |x|>c\sigma\end{cases} \qquad (11)$$

IV PERFORMANCE STUDY

In this section, the theoretical performance of algorithms are characterized with two main evaluation criteria the achievable reduction in PAPR and the impact of companding distortion on the BER performance at the receiver.

A. . Achievable Reduction in PAPR

By making appropriate substitution in (6), the ultimate PAPR of the companded signal with the new algorithm is given by

$$PAPR_y=\frac{\max\left[|y_n|^2\right]}{E\left[|y_n|^2\right]} \qquad (12)$$

Furthermore, a transform gain G is defined as the ratio of the PAPR of original signal to that of the companded signal [8], i.e.

$$G(db)=10\log_{10}\frac{PAPR_X}{PAPR_Y} \qquad (13)$$

These algorithms offer an adequate flexibility in the PAPR reduction by adjusting the values of k, k_2 and c. Consequently, the ultimate PAPR of the companded signal can be effectively confined in the interval [4.1 dB, 5.7 dB], or in other words, the achievable transform gain G in the PAPR is from 6 dB to 7.7 dB.

B. Impact of Companding Distortion

In essence, NCT is an extra nonlinear operation applied to the transmitted signal. For this reason, how to minimize the impact of companding distortion on the BER performance is the key in choosing the optimal companding form and parameters. Based on the analysis results for the Gaussian signals in [15] and [16], two performance criteria: signal attenuation and companding noise can be used to characterize this impact. Obviously, smaller value of companding distortion gives the reduced BER performance.

.

V. COMPARISON RESULTS

To compare the overall system performance of algorithms, on an OFDM system with N 1024 subcarriers 16 QAM modulation is used under AWGN channel.

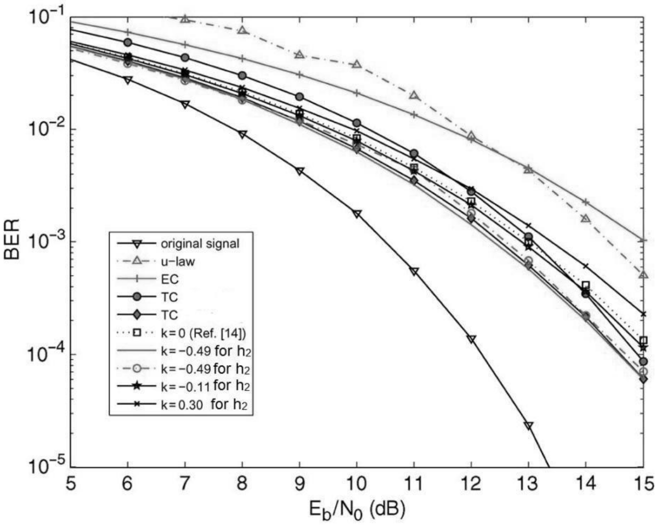

Fig. 2 Block BER performance of different transforms under an AWGN channel for N=1024 the OFDM system with and 16 QPSK modulation. For h_1 companding function

.

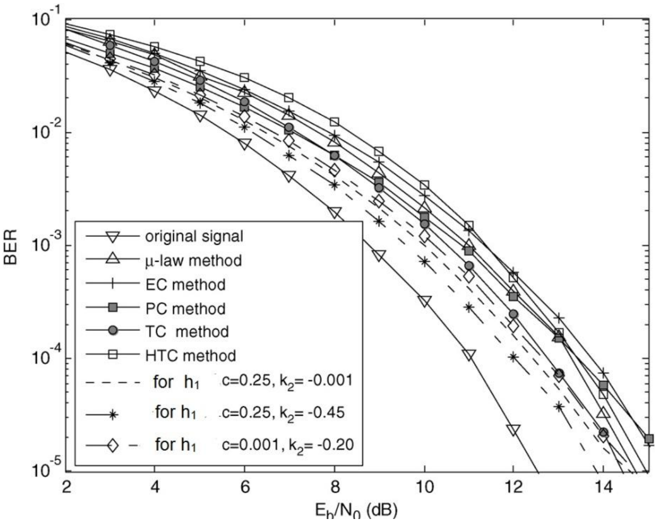

Fig. 3 BER performance of different transforms under an AWGN channel for N=1024 the OFDM system with and 16 QPSK modulation.For h_2 companding function.

For companding function h_1 and h_2 BER VS E_b/N_0 table is shown below

TABLE I

BER **(dB)**	$\frac{E_b}{N_0}$ **for** h_1 **(C=0.25,** k_2 **=-0.45)**	$\frac{E_b}{N_0}$ **for** h_2 **K=-0.45**
11	10^{-2}	$10^{-1.6}$
12	10^{-3}	$10^{-2.8}$
13	$10^{-3.7}$	$10^{-3.3}$
14	$10^{-4.5}$	10^{-4}

VI CONCLUSIONS

Due to its simplicity and effectiveness, NCT is an attractive solution to reduce the PAPR of OFDM signal. In this paper, we compared NCT algorithm which changes the statistics of original signal from the complex Gaussian to a desirable PDF defined as a linear piecewise function. Thus, an effective and flexible trade-off between the PAPR and BER performance can be achieved to satisfy various system requirements. the discussed algorithm substantially outperforms the existing NCT methods in the overall performance of OFDM system regarding the reduction in PAPR, BER and out-of-band interference under the multipath fading channel or with the HPA.

REFERENCES

[1] Wu Y., W. Y. Zou, “Orthogonal frequency division multiplexing: A multi-carrier modulation scheme,” IEEE Transactions on Consumer Electronics, vol. 41, no. 3, pp. 392 399, Aug. 1995.

[2] Van Nee R., Prasad R., OFDM for wireless Multimedia Communications, Artech House, 2003.

[3] Chang R. W., “Synthesis of band-limited orthogonal signals for multichannel data transmission”, Bell Systems Technical Journal, vol. 46, pp. 1775-1796, Dec. 1966.

[4] Weinstein S. B., Ebert P. M., “Data Transmission for Frequency-Division Multiplexing Using the Discrete Fourier Transform,” IEEE Transactions on Commun. Tech., vol. 19, no. 5, pp. 628–34, Oct. 1971.

[5] Cooley J., Tukey, J., “An Algorithm for the Machine Calculation of Complex Fourier Series”, Math. Comp., vol. 19, pp. 297–301, April 1965.

[6] Chow P.S. Tu J.C. and Cioffi J.M., “Performance Evaluation of a Multichannel Transceiver System for ADSL and VHDSL services", IEEE Journal on Selected Areas in Communications, vol. 9, no. 6, Aug. 1991.

[7] ETSI, “Radio broadcasting systems; Digital Audio Broadcasting (DAB) to mobile, portable and fixed receivers”, European Telecommunication Standard, Standard EN-300-401, May 1997.

[8] ETSI, “Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television”, European Telecommunication Standard, Standard EN-300-744, 2004-2006.

[9] Yang L. L., Multicarrier Communications, 1st edition, John Wiley & Sons Ltd, 2009.

[10] Armstrong J., “Peak-to-average power reduction for OFDM by repeated clipping and frequency domain filtering”, Electronics Letters, vol. 38, no. 5, pp. 246–247, Feb. 2002.

[11] Chen H., Haimmovich A. M., “Iterative estimation and cancellation of clipping noise for OFDM signals,” IEEE Comm. Letters, vol. 7, no. 5, pp.246-247, July 2003.

[12] Wang X. B., Tjhung T. T., Ng C. S., “Reduction of peak-to-average power ratio of OFDM system using a companding technique”, IEEE Transactions on Broadcasting, vol. 45, no. 3, pp. 303–307, Sept. 1999.

[13] Jiang T., Xiang W., Richardson P. C., Qu D., Zhu G., “On the Nonlinear Companding Transform for Reduction in PAPR of MCM Signals”, IEEE Transactions on Wireless Communications, vol. 6, no. 6, pp.2017-2021, June 2007.

[14] Jiang T., Yang Y., Song Y., “Exponential companding transform for PAPR reduction in

[15] OFDM systems”, IEEE Transactions on Broadcasting, vol. 51, no. 2, pp. 244–248, June2005.

[16] Huang X., Lu J. H., Zheng J. L., Letaief K. B., Gu J., “Companding transform for reduction in peak-to-average power ratio of OFDM signals”, IEEE Transactions on Wireless Communications, vol. 3, no. 6, pp. 2030–2039, Nov. 2004.

[17] Aburakhia S. A., Badran E. F., Mohamed D. A. E., “Linear Companding Transform for the Reduction of Peak-to-Average Power Ratio of OFDM Signals”, IEEE Transactions on Broadcasting, vol. 55, no. 1, pp. 195-200, March 2009.

[18] Hou J., Ge J. H., Li J., “Trapezoidal companding scheme for peak-to-average power ratio reduction of OFDM signals”, Electronics Letters, vol. 45, no. 25, pp. 1349-1351, Dec. 2009.

[19] Jeng S. S., Chen J. M., “Efficient PAPR reduction in OFDM system based on a companding techniques with trapezium distribution”, IEEE Transactions on Broadcasting, vol. 57, no. 2, pp. 291-298, June 2011.

[20] Baüml R. W., Fischer R. F. H., Hüber J. B., “Reducing the peak-to average power ratio of multicarrier modulation by selective mapping”, Electronics Letters, vol. 32, no. 22, pp. 2056–2057, Oct. 1996.

[21] Muller S. H., Huber J. B., “OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences”, Electronics Letters, vol. 33, no. 5, pp. 36–69, Feb. 1997.

[22] Zhao Y., Häggman S. G., “Intercarrier interference self-cancellation scheme for OFDM mobile communication systems”, IEEE Transactions on Communications, vol. 49, no.7, pp. 1185-1191, July 2001.

[23] Sathananthan K., Rajatheva R. M. A. P., “Analysis of OFDM in the presence of Frequency Offset and a method to reduce performance degradation”, Proc. of Global telecommunications conference, GlOBECOM 00, San Francisco, USA, vol. 1, pp. 72-76,Nov. 2000.

[24] Yeh H.-G., Chang Y. K., Hassibi B., “A scheme for canceling intercarrier interference using conjugate transmission in multicarrier

communication systems", IEEE Transactions on Wireless Communications, vol. 6, no. 1, pp. 3-7, Jan. 2007.

[25] Simon, M.K., Alouini M.S., Digital Communication over Fading Channels, 2nd edition, Wiley, New York, 2005.

[26] Li Y., Stüber G.L., Orthogonal Frequency Division Multiplexing for Wireless Communications, Springer-Verlag, 2006

[27] Y. Wang, L.-H. Wang, J.-H. Ge, and B. Ai., "An Efficient Nonlinear Companding Transform for Reducing PAPR of OFDM Signals" , IEEE Transactions On Broadcasting, Vol. 58, No. 4, Dec. 2012.

[28] Yong Wang, Jianhua Ge, Lihua Wang, Jing Li, and Bo Ai., "Nonlinear Companding Transform for Reduction of Peak-to-Average Power Ratio in OFDM Systems", IEEE Transactions on Broadcasting, vol. 59, no. 2, pp. 369–375, June . 2013