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ON TERNARY QUADRATIC DIOPHANTINE EQUATION $8x^2+5y^2=143z^2$

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***Abstract*- The ternary quadratic Diophantine equation representing cone given by $8x^2+5y^2=143z^2$ is analyzed for its non-zero distinct integer points. A few interesting relations between the solutions and special figurate numbers are obtained.**

***Index Terms*- Ternary quadratic Diophantine, integer solutions, polygonal number**
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NOTATIONS:

1) Polygonal number of rank 'n' with size m:

$$t_{m,n}=n[1+\frac{(n-1)(n-2)}{2}]$$

2) Pronic number of rank 'n':

$$pr_n=n(n+1)$$

3) Octahedral number of rank n:

$$OH_n=\frac{1}{3}n(2n^2+1)$$

4) stella octangular number of rank "n":

$$so_n=n(2n^2-1)$$

5) centered pyramidal number of rank 'n' with size m:

$$cp_{m,n}=[\frac{m(n-1)n(n+1)}{6}]+n$$

I. INTRODUCTION

The quadratic Diophantine equations with three unknowns offer an unlimited field for research because of their variety [1-3]. For an extensive review of various problems on ternary quadratic Diophantine equations representing specific 3 dimensional surfaces, one may refer [4-14].

In this communication, we present general formulas for obtaining sequences of non-zero integer solutions to the ternary quadratic Diophantine equation $8x^2+5y^2=143z^2$. Also, a few interesting relations among the solutions are presented.

II .METHOD OF ANALYSIS

The ternary quadratic equation to be solved is

$$8x^2+5y^2=143z^2 \qquad (1)$$

We present below different patterns of solutions to (1) :

METHOD 1:

Introducing the linear transformations

$$x=X+5T,\ y=X-8T \qquad (2)$$

In (1), it is written as

$$X^2+40T^2=11z^2 \qquad (3)$$

Assume

$$z=4a^2+10b^2 \qquad (4)$$

Write 11 as

$$11=(1+i\sqrt{10})(1-i\sqrt{10}) \quad (5)$$

Substituting (4),(5) in (3) and it is written in the factorizable form as

$$(1+i\sqrt{10})(1-i\sqrt{10})(2a+i\sqrt{10}b)^2(2a-i\sqrt{10}b)^2$$
$$=(X+i2\sqrt{10}T)(X-i2\sqrt{10}T)$$

Equating real and imaginary parts, the values of z and X are

$$X=4a^2-10b^2-40ab$$
$$T=2a^2-5b^2+2ab \quad (6)$$

From (6) and (2) the corresponding integer solution of (1) are given by

$$x(a,b)=14a^2-35b^2-30ab$$
$$y(a,b)=-12a^2+30b^2-56ab$$ and along with (4).

PROPERTIES:

1) $3z(a,a)-y(a,a)-80t_{4,a}=0$

2) Each of the following expressions are a nasty number.

i) $30[3z(a,a)-y(a,a)]$

ii) $6\{14z(a,a)-4x(a,a)\}$

3) Each of the following expressions are a prefect square.

i) $5\{3z(a,a)-y(a,a)\}$

ii) $\{14z(a,a)-4x(a,a)\}$

Also, instead of (2) one may consider the linear transformation as

$$x=X-5T,\ y=X+8T \quad (*)$$

For this choice, the corresponding integer solutions to (1) are given by

$$x(a,b)=-6a^2+15b^2-50ab$$
$$y(a,b)=20a^2-50b^2-24ab$$
$$z(a,b)=4a^2+10b^2$$

METHOD 2:

(3) is written as

$$X^2+40T^2=11z^2=11z^2*1 \quad (7)$$

Write 1 as

$$1=\frac{(3+i4\sqrt{10})(3-i4\sqrt{10})}{169} \quad (8)$$

Substituting (4), (3), (8) in (7) and it is written in the factorization form as,

$$(1+i\sqrt{10})(1-i\sqrt{10})(2a+i\sqrt{10}b)^2(2a-i\sqrt{10}b)^2$$
$$*[\frac{(3+i4\sqrt{10})(3-i4\sqrt{10})}{169}] \quad =(X+i2\sqrt{10}T)(X-i2\sqrt{10}T)$$

Equating real and imaginary parts, the values of X, T are

$$X=\frac{1}{13}[-148a^2+370b^2-280ab]$$
$$z=\frac{1}{13}[14a^2-35b^2-74ab] \quad (9)$$

For X and T to be integers, the values of a & b should be multiples of 13.

Therefore, replacing a by 13A, b by 13B in (4), (9), we have

$$X=-1924A^2+4810B^2-3640AB$$
$$T=182A^2+455B^2-962AB$$
$$z=676A^2+1690B^2 \quad (10)$$

In view of (2), it is seen that

$$x(A,B)=-1014A^2+2535B^2-8450AB$$
$$y(A,B)=-3380A^2+8450B^2-4056AB \quad (11)$$

Thus, (10) &(11) represent the integer solutions to (1).

PROPERTIES:

1) $[5z(a,a)-y(a,a)]$ is a perfect square.

2) $y(a,a)+5z(a,a)-20956t_{4,a}=0$

3) $6\{5z(a,a)-y(a,a)\}$ is a nasty number.

4) $y(a,a)+5z(a,a)-16900t_{4,a}-4056cp_{6,n}=0$

Also, using (*), the corresponding integer solutions to (1) are given by

$$x(A,B) = -2834A^2 + 7085B^2 + 1170AB$$
$$y(A,B) = -468A^2 + 1170B^2 - 11336AB$$
$$z(A,B) = 676A^2 + 1690B^2$$

METHOD 3:

Write 1 as

$$1 = \frac{(3+i2\sqrt{10})(3-i2\sqrt{10})}{49} \qquad (12)$$

put (4), (5),(10) in (7) and it is written in the factorization form

as
$$(1+i\sqrt{10})(1-i\sqrt{10})(2a+i\sqrt{10}b)^2$$
$$(2a-i\sqrt{10}b)^2 * \frac{(3+i2\sqrt{10})(3-i2\sqrt{10})}{49}$$
$$= (X+i2\sqrt{10}T)(X-i2\sqrt{10}T)$$

Equating real and imaginary parts, the values of X and T are

$$X = \frac{1}{7}[-68a^2 + 170b^2 - 200ab]$$
$$T = \frac{1}{7}[10a^2 - 25b^2 - 34ab] \qquad (13)$$

For X and T to be integers, the values of a and b should be multiples of 7. Therefore, replacing a by 7A, b by 7B in (4), (12), we have

$$X = -476A^2 + 1190B^2 - 1400AB$$
$$T = 70A^2 - 175B^2 - 238AB$$

$$z = 196A^2 + 490B^2 \qquad (14)$$

In view of (2), it is seen that

$$x(A,B) = -126A^2 + 315B^2 - 2590AB$$
$$y(A,B) = -1036A^2 + 2590B^2 + 504AB \qquad (15)$$

Thus, (14) & (15) represent the integer solutions to (1).

PROPERTIES:

1) $126z(a,a) - 196x(a,a) - 458248t_{4,a} = 0$

2) $126z(a,a+1) - 196x(a,a+1) - 507640t_{4,a} + 49392\,pr_a = 0$

3) $126z(a,a+1) - 196x(a,a+1) - 507640t_{4,a} + 49392\,OH_n = 0$

4) $126z(a,a+1) - 196x(a,a+1) - 507640t_{4,a} + 49392\,SO_a = 0$

Also using (*), the corresponding integer solutions to (1) are given by

$$x(A,B) = -826A^2 + 2065B^2 - 210AB$$
$$y(A,B) = 84A^2 - 210B^2 - 330AB$$
$$z(A,B) = 196A^2 - 490B^2$$

REMARKABLE OBSERVATIONS:

In this section, geral formulas generating sequences of integers solutions to (1) based on its given solutions are obtained.

Let (x_0, y_0, z_0) be any given integer solution to (1)

CASE (i)

Let $x_1 = 9x_0, y_1 = 9y_0 + 5h, z_1 = 9z_0 + h, h \neq 0$, (16)

Be the 2nd solution of (1) substitute (16) in (1) and performing a few calculations, we have

$h = 5y_0 + z_0$ and thus $y_1 = 34y_0 + 5z_0$, $y_1 = -5y_0 + 4z_0$

Which is written in the form as matrix as

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = M \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \qquad (12)$$

Where $M = \begin{pmatrix} 34 & 5 \\ -5 & 8 \end{pmatrix}$

Repeating the above process, the general solution (x_n, y_n) to (1) is given by $\begin{pmatrix} x_n \\ y_n \end{pmatrix} = M^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ (13)

To find M^n, proceed as follows:

The Eigen values of M are $\alpha = 33$, $\beta = 9$

It is well known that $M^n = \alpha^n$

$$M^n = \frac{\alpha^n}{\alpha - \beta}(M - \beta I) + \frac{\beta^n}{\beta - \alpha}(M - \alpha I)$$

Thus, using the above formula the general solutions (x_n, y_n, z_n) to (1) is given by

$$y^n = \frac{1}{24}[\{25(33^n) - (9^n)\}y_0 + \{5(33^n) - 5(9^n)\}z_0]$$

$$y^n = \frac{1}{24}[\{-5(33^n)+5(9^n)\}y_0 + \{-1(33^n)+25(9^n)\}z_0]$$

$$x_n = 9^n x_0$$

CASE (ii)

Let $x_1 = 15x_0 + 4h$, $y_1 = 15y_0$, $z_1 = 15z_0 + h$

Following the procedure presented in case (i), the corresponding general solution to (1) is given by

$$x_n = \frac{1}{542}[\{256(557)^n + 286(15)^n\}x_0 + \{144(557)^n - 1144(15)^n\}z_0]$$

$$z_n = \frac{1}{542}[\{64(557)^n - 64(15)^n\}x_0 + \{286(557)^n + 256(15)^n\}z_0]$$

$$y_n = 15^n y_0$$

CASE (iii)

Let $x_1 = 13x_0 - h$, $y_1 = 13y_0 - h$, $z_n = 13z_0$

Repeating the above process the corresponding general solution to (1) is given by

$$x_n = \frac{1}{26}[\{10(13)^n + 16(-13)^n\}x_0 - \{10(13)^n + 10(-13)^n\}y_0]$$

$$y_n = \frac{1}{26}[\{-16(13)^n + 16(-13)^n\}x_0 + \{16(13)^n + 10(-13)^n\}y_0]$$

$$z_n = 13^n z_0$$

III. CONCLUSION

In this paper, we have obtained infinitely many non-zero distinct integer solutions to the ternary quadratic Diophantine equation represented by

$$8x^2 + 5y^2 = 143z^2$$

As quadratic equations are rich in variety, one may search for their choices of quadratic equation with variables greater than or equal to 3 and determine their properties through special numbers.

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