ANALYSIS OF MIMO SYSTEMS WITH DIFFERENT EQUALIZATION TECHNIQUES

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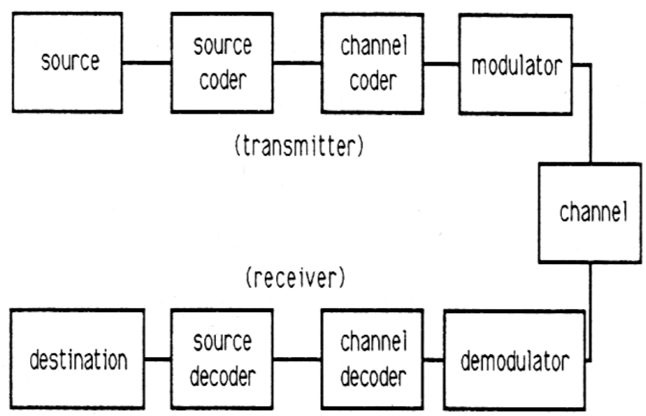
Abstract— **The mobile data applications has enhanced the demand for wireless communication systems giving high throughput, wide coverage, and improved accuracy. The most challenges within the style of wireless communication systems are the reserved resources, like strained transmission power, scarce frequency bandwidth, and restrained implementation complexity—and the impairments of the wireless channels, together with noise, interference, and attenuation effects. Multiple-Input Multiple-Output (MIMO) communication has been shown to be one in every of the foremost rising wireless technologies that may boost the info transmission rate, improve system coverage, and enhance link responsibleness. By using multiple antennas at transmitter and receiver sides, MIMO techniques alter a brand new dimension – the abstraction dimension – that may be utilised in several ways in which to combat the impairments of wireless channels. This text focuses on techniques, for Rayleigh Flat fading channel. Equalization is a technique for combating inter-symbol interference. It is basically a filtering approach that minimizes the error between actual output and desired output by continuous change its filter coefficients. This paper consists of techniques for the analysis of BER in MIMO Systems equalizer like ZF, MMSE, ZF-SIC, MMSE-SIC.**

I. INTRODUCTION

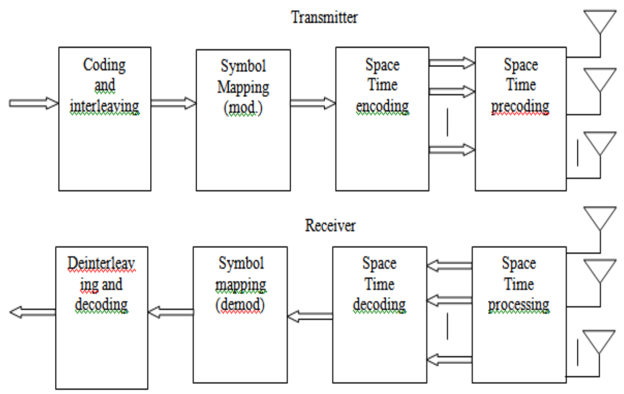
MIMO is use of multiple antennas at the transmitter and receiver in wireless systems. Multipath fading predominates the communication in wireless channels. It is the emergence of the transmitted signal at the receiver through differing angles and differing time delays and or differing frequency. MIMO offers vital increase in throughput and link range without extra transmit power. It achieves this by higher spectral potency, link responsibleness and diversity. MIMO-multiuser, that refers to a configuration that includes a base station with Multiple transmit/receive antennas interacting with multiple users, every with one or additional antennas. The information bits to be transmitted are encoded and interleaved. The interleaved codeword is mapped to symbols (such as bpsk ,qpsk, qam etc.) by the image plotter. These information symbols are input to a space time encoder that outputs one or additional abstraction data streams. The spatial data streams square measure mapped to the transmit antennas by the frame of reference precoding block. The signals launched from the transmit antennas propagate through the channel and reach the receiver antenna array. The receiver collects the signals at the output of every receive antenna component and reverses the transmitter operations so as to rewrite the information receives area time process, followed by space time processing , followed by space time decoding, de-mapping of symbols , deinterleaving and decoding.



II. MIMO SYSTEM MODEL

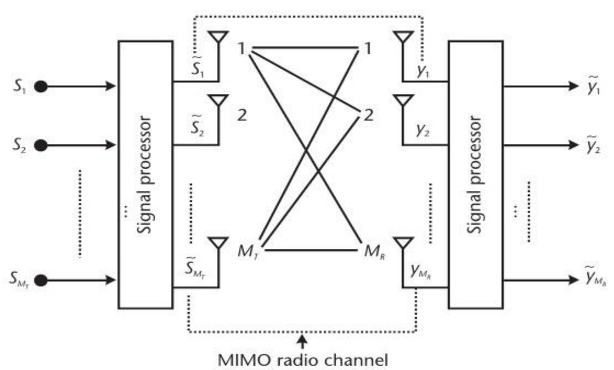
We contemplate a MIMO system with a transmit array of M_T antennas and a receive array of M_R antennas. The diagram of such a system is shown in Figure. The transmitted matrix is a $M_T \times 1$ column matrix s where s_i is the i th element, transmitted from antenna i. We tend to consider the channel to be a gaussian channel such that the components of s are considered to be independent identically distributed gaussian variables. If the channel is unknown at the transmitter, we tend to assume that the signals transmitted from every antenna have equal powers of E_s/M_T . The covariance matrix for this transmitted signal is given by

$$R_{ss} = \frac{E_s}{M_T} I_{M_T} \qquad (1)$$



Where E_s is the power across the transmitter irrespective of the number of antennas M_T and I_{M_T} is an identity matrix. The channel matrix H is a $M_R \times M_T$ complex matrix. The component $h_{i,j}$ of the matrix is the fading coefficient from the j th transmit antenna to the i th receive antenna . We assume that the received power for each of the receive antennas is equal to the total transmitted power E_S. The received signals constitute a $M_R \times 1$ column matrix denoted by r, where each complex component refers to a receive antenna. Since we assumed that the total received power per antenna is equal to the total transmitted power, the SNR can be written as

$$\gamma = \frac{E_S}{N_0} \qquad (2)$$



III. MIMO CHANNELS

B. Classical independent, identically distributed rayleigh fading channel model.

The degree of correlation between the individual M_T M_R channel gains comprising the MIMO channel is a complicated function of the scattering in the atmosphere and antenna spacing at the transmitter and the receiver. Taken into account an extreme condition where all antenna elements at the transmitter are collocated and also at the receiver. In this case, all the condition of H will be totally related (in fact identical) and also the spatial diversity order of the channel is one. De-correlation between the channel elements will increase with antenna spacing. Scattering in the atmosphere in combination with adequate antenna spacing ensures de-correlation of the MIMO channel components. With rich scattering, the typical antenna spacing required or de-correlation is approximately λ/2, where λ is the wavelength corresponding to the frequency of operation. Under ideal conditions when the channel elements are perfectly de-correlated, we tend to get H = Hw, the classical i.i.d. Frequency-flat rayleigh fading MIMO channel.

A. Real World Mimo Channel

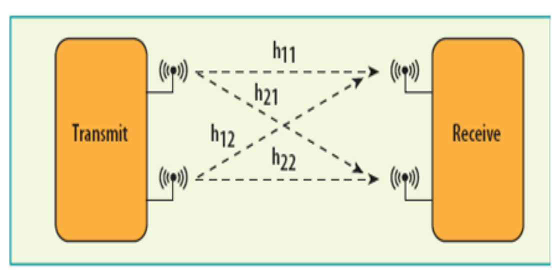
In the presence of an LOS component between the transmitter and the receiver, the MIMO channel may be modeled as the sum of a fixed Component and a fading component.

$$H = \sqrt{\frac{}{1+k}}\,\bar{H} + \sqrt{\frac{}{1+k}}\,H_w$$

IV. EQUALIZATION TECHNIQUES

The diversity schemes at the receiving end are explained below briefly.

1. Selection diversity is the process of taking the signal with highest power i. received signal with highest force will be acknowledged and rest of the signal will be overlooked.
2. Maximal Ratio Combining is a scheme in which the strength of the received signals is used to obtain the corresponding weights and than maximizes the SNR.
3. Equal gain combining is another scheme in which it is required to the weights to vary with the fading signals, the magnitude of which may fluctuate over .several 10s of dB.



MIMO Wireless Communication System

A. Equalization schemes for MIMO systems

In MIMO System equalization is done at the receiving side. There are three equalization schemes, They are Zero Forcing Equalizer, Minimum Mean Square Error and Maximum Likelihood Estimator and are explained below.

1. Zero forcing (ZF) equalizer

In Zero Force equalization frequency response of the channel is inverted. It is a linear equalization process in communication system. ZFE restore the transmitted signal by applying the inverse of the channel to the received signal and brings down the ISI. When ISI is high as compared to the channel noise, this method provides a good way to handle it.

The mathematical model for the system is defined as: the initial equations will remain same for the three schemes of MIMO systems we will discuss here. Lety1 and y2 be two signals received on antenna 1 and 2 y2 respectively, h(1,1), h(1,2), h(2,1) and h(2,2) are the channel parameters demonstrating the relation between transmitting and receive antenna as shown by the figure 5, transmitted signals from antenna 1 and antenna 2 are x1 and x2 respectively and the noise on receiving antenna 1 and antenna 2respectively is n1 and n2 such that:

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \quad h_{1,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

And

$$y_1 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \quad h_{2,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

The matrix form for above equations can be expressed as:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

i.e. Y= Hx + n

Now x can be solved by with the help of the matrix Z such that ZH=1, i.e. Z should be the inverse of the channel matrix H.

$Z= (H^H H)^{-1} H^H$

The term,

$$H^H H = \begin{bmatrix} h_{1,1}{}^* & h_{1,2}{}^* \\ h_{2,1}{}^* & h_{2,2}{}^* \end{bmatrix}$$

$$= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix}$$

For BPSK modulation in Rayleigh fading channel, the bit error rate is derived as,

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{(E_b/N_0)}{(E_b/N_0) + 1}}\right)$$

2. Minimum Mean Square Error

Minimum mean square error (MMSE) is an estimation scheme which minimizes the mean square error and used for quality estimation. This does not removes the ISI but however it reduces or minimizes the components of noise and ISI in the output. The MMSE finds a coefficient M which minimizes criteria:

$$E\{[My - x][My - x]^H\}$$

On solving the above criteria, the mathematical value of M comes out to be:

$$M= [H^H H + N_0 I]^{-1} H^H$$

If we compare the equation of ZFE with MMSE, both the equation seems similar apart from the term NoI that means in the absence of noise, MMSE and ZFE works similar to each other.

3. ML Equalization

The maximum Likelihood equalization schemes finds out the term m, such that Can be minimized. This relation can be further expresses in terms of received signal, channel parameters and m

$$J = |y-Hm|^2$$

$$J = \left\| \begin{bmatrix} y1 \\ y2 \end{bmatrix} - \begin{bmatrix} h1,1 & h1,2 \\ h2,1 & h2,2 \end{bmatrix}\begin{bmatrix} m1 \\ m2 \end{bmatrix} \right\|^2$$

As with BPSK modulation, value of x1 and x2 can be either +1 or -1, hence to find the ML solution, all the four combinations below for x1 and x2 need to be minimized.

$$J_{+1,+1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}\begin{bmatrix} +1 \\ +1 \end{bmatrix} \right|^2$$

$$J_{+1,-1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}\begin{bmatrix} +1 \\ -1 \end{bmatrix} \right|^2$$

$$J_{-1,+1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}\begin{bmatrix} -1 \\ +1 \end{bmatrix} \right|^2$$

$$J_{-1,-1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} \right|^2$$

The estimate of the transmit symbol is chosen based on the minimum value from the above four values i.e

if the minimum is $J_{+1,+1}$ => [1 1]

if the minimum is $J_{+1,-1}$ => [1 0]

if the minimum is $J_{-1,+1}$ => [0 1]
if the minimum is $J_{-1,-1}$ => [0 0]

Zero Forcing with Successive Interference Cancellation (ZF-SIC)

In Zero Forcing (ZF) equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols x_1, x_2, i.e.

$$\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix}=(\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H\begin{bmatrix}y_1\\ y_2\end{bmatrix}.$$

Taken one of the estimated symbols (for example $\hat{x}_2$) and subtract its effect from the received vector y_1 and y_2, i.e.

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix}=\begin{bmatrix}y_1-h_{1,2}\ \hat{x}_2\\ y_2-h_{2,2}\ \hat{x}_2\end{bmatrix}=\begin{bmatrix}h_{1,1}\ x_1+n_1\\ h_{2,1}\ x_1+n_2\end{bmatrix}.$$

By expressing in matrix notation,

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix}=\begin{bmatrix}h_{1,1}\\ h_{2,1}\end{bmatrix}x_1+\begin{bmatrix}n_1\\ n_2\end{bmatrix}$$

$$\boldsymbol{r}=\boldsymbol{h}x_1+\boldsymbol{n}$$

Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply Maximal Ratio Combining (MRC).

The equalized symbol is,

$$\hat{x}_1=\frac{\boldsymbol{h}^H\boldsymbol{r}}{\boldsymbol{h}^H\boldsymbol{h}}.$$

This forms the simple explanation for Zero Forcing Equalizer with Successive Interference Cancellation (ZF-SIC) approach.

Successive Interference Cancellation with optimal ordering

In classical Successive Interference Cancellation, the receiver arbitrarily takes one of the estimated symbols, and subtract its effect from the received symbol and . However, we can have more intelligence in choosing whether we should subtract the effect of first or $\hat{x}_2$first. To make that decision, let us find out the transmit symbol (after multiplication with the channel) which came at higher power at the receiver. The received power at the both the antennas corresponding to the transmitted symbol is,

$$P_{x_1}=|h_{1,1}|^2+|h_{2,1}|^2.$$

The received power at the both the antennas corresponding to the transmitted symbol x_2 is,

$$P_{x_2}=|h_{1,2}|^2+|h_{2,2}|^2.$$

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix}=\begin{bmatrix}y_1-h_{1,1}\ \hat{x}_1\\ y_2-h_{1,2}\ \hat{x}_1\end{bmatrix}=\begin{bmatrix}h_{1,2}\ x_2+n_1\\ h_{2,2}\ x_2+n_2\end{bmatrix}$$

Expressing in matrix notation,

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix}=\begin{bmatrix}h_{1,2}\\ h_{2,2}\end{bmatrix}x_2+\begin{bmatrix}n_1\\ n_2\end{bmatrix}$$

$$\boldsymbol{r}=\boldsymbol{h}x_2+\boldsymbol{n}$$

Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply Maximal Ratio Combining [4] (MRC). The equalized symbol is,

$$\hat{x}_2=\frac{\boldsymbol{h}^H\boldsymbol{r}}{\boldsymbol{h}^H\boldsymbol{h}}.$$

Else if the receiver decides to subtract effect of $\hat{x}_2$from the received vector and , and then re-estimate

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix}=\begin{bmatrix}y_1-h_{1,2}\ \hat{x}_2\\ y_2-h_{2,2}\ \hat{x}_2\end{bmatrix}=\begin{bmatrix}h_{1,1}\ x_1+n_1\\ h_{2,1}\ x_1+n_2\end{bmatrix}$$

Expressing in matrix notation,

$$\begin{bmatrix}r_1\\ r_2\end{bmatrix}=\begin{bmatrix}h_{1,1}\\ h_{2,1}\end{bmatrix}x_1+\begin{bmatrix}n_1\\ n_2\end{bmatrix}$$

$$\boldsymbol{r}=\boldsymbol{h}x_1+\boldsymbol{n}$$

Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply Maximal Ratio Combining [4] (MRC). The equalized symbol is,

$$\hat{x}_1=\frac{\boldsymbol{h}^H\boldsymbol{r}}{\boldsymbol{h}^H\boldsymbol{h}}.$$

Doing successive interference cancellation with optimal ordering ensures that the reliability of the symbol which is decoded first is guaranteed to have a lower error probability than the other symbol. This results in lowering the chances of incorrect decisions resulting in erroneous interference cancellation. Hence gives lower error rate than simple successive interference cancellation.

V. RESULTS

Simulation Model

MATLAB code for all the detectors follow the following procedure-

(a) Produce random paired succession of +1′s and - 1′s.

(b) Bunch them into pair of two symbols and send two symbols in one time space

(c) Multiply the symbols with the channel and after that include white Gaussian noise.

(d) Equalize the symbols received

(e) Perform hard choice translating and check the bit errors

(f) Repeat for various estimations of and plot the reproduction and hypothetical results.

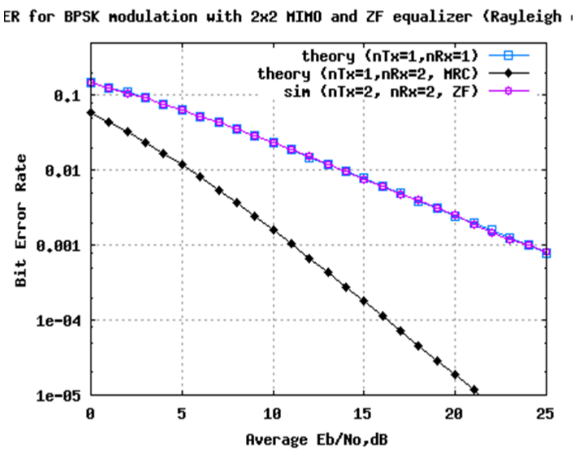
1. MIMO with Zero Forcing equalizer

Result-

1. The results with a 2×2 MIMO framework utilizing BPSK as a part of Rayleigh channel is indicating coordinating results

as got in for a 1×1 framework for BPSK adjustment in Rayleigh channel.

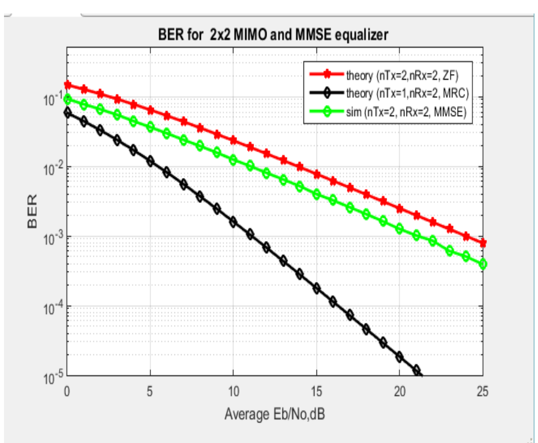
2. The Zero Forcing equalizer is not the most ideal approach to equalize the symbol recieved. The zero constraining equalizer helps us to accomplish the information rate gain , yet not exploit diversity gain (as we have 2 antennas at the receiving end).



2. Minimum Mean Square Error (MMSE) equalizer for 2×2 MIMO channel

Result-

Contrasted with the Zero Forcing equalizer case, at 10^{-3} BER point, it is visualized that the Minimum Mean Square Error (MMSE) equalizer results in around 3dB of change, thus better.

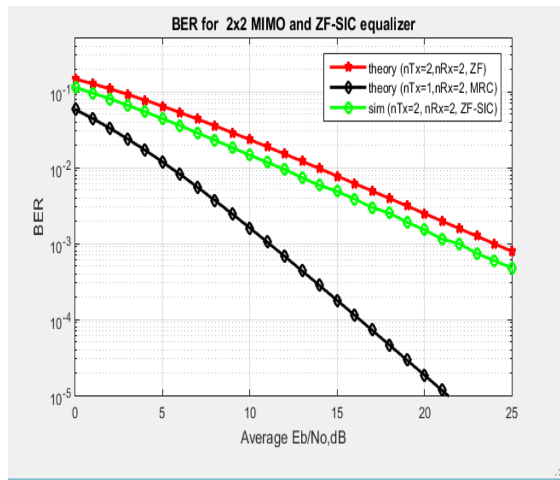


3. Zero Forcing Successive Interference Cancellation equalization

Result-

Contrasted with Zero Forcing adjustment alone case, expansion of interference cancelation results in around 2.2dB of change for BER of 10^{-3} .

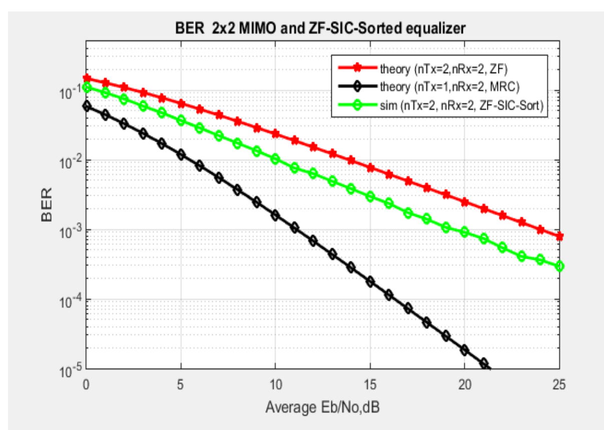
The change is gotten in light of the fact that interpreting of the data from the primary spatial measurement (x_1) has a lower mistake likelihood that the symbol transmitted from the second measurement. Notwithstanding, the presumption is that x_2 is decoded accurately may not be valid when all is said in done.



4. ZF-SIC with optimal ordering

Result-

Contrasted with the Zero Forcing equalize rwith successive interference cancellation case, at 10^{-3}BER point, it is visualized that the Minimum Mean Square Error (MMSE) equalizer results in around 2.0dB of change, thus better.

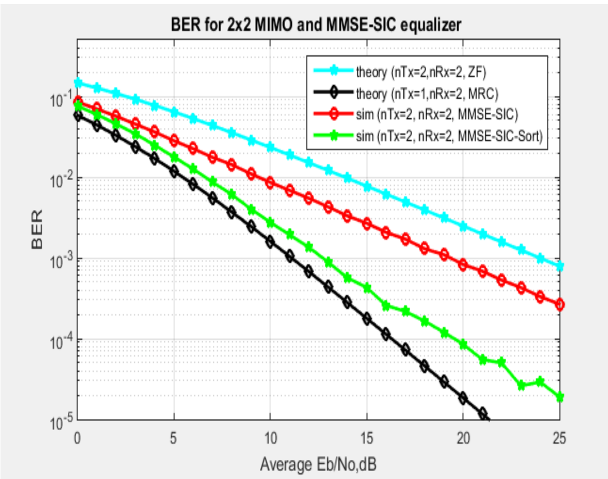


5. MMSE-SIC equalization with and without optimal ordering

Result-

Compared to Minimum Mean Square Equalization with simple successive interference cancellation case, addition of optimal ordering results in around **5.0dB of improvement** for BER of 10^{-3}.

The performance is now closely matching with curve 1 transmit 2 receive antenna MRC case.

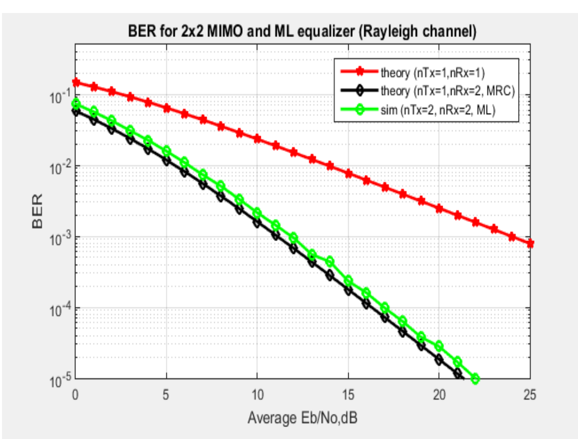


6. Maximum Likelihood equalization

Result-

1. The results for 2×2 MIMO with **Maximum Likelihood (ML) equalization** helped us to achieve a performance closely matching the 1 transmit 2 receive antenna **Maximal Ratio Combining (MRC)** case.

2. If we use a higher order constellation like 64QAM, then computing Maximum Likelihood equalization might become prohibitively complex. With 64QAM and 2 spatial stream we need to find the minimum from $64^2 = 4096$ combinations In such scenarios we might need to employ schemes like sphere decoding which helps to reduce the complexity.



VI. CONCLUSION

In this paper performance of different equalization techniques has been analysed to find out suitable equalizer for 2x2 MIMO channel in Rayleigh multipath fading environment. Zero Forcing equalizer performs well only in theoretical assumptions that are when noise is zero. Its performance degrades in mobile fading environment.

Minimum Mean Square Error (MMSE) equalizer uses LMS (Least Mean Square) as criterion to compensate ISI. The MMSE equalizer results in around 3dB of improvement when compared to zero forcing equalizer.

Zero forcing with Successive interference cancellation improves the performance of equalizer. This process improves the estimator performance on the next component compared to the previous one. Compared to Zero Forcing equalization alone case, addition of successive interference cancellation results in around 2.2dB of improvement for BER.

Zero forcing with Successive interference cancellation with optimal ordering ensures that the reliability of the symbol which is decoded first is guaranteed to have a lower error probability than the other symbol. Compared to Zero Forcing equalization with successive interference cancellation case, addition of optimal ordering results in around 2.0dB of improvement for BER.

Minimum Mean Square Equalization with simple successive interference cancellation case, addition of optimal ordering results in around 5.0dB of improvement for BER.

ML provides the better performance in comparison to all previously discussed equalization techniques, however the complexity of ML decoder goes on increasing as we move to higher modulation schemes

REFERENCES

[1] Digital Communication: Third Edition, by John R. Barry, Edward A. Lee, David G. Messerschmitt

[2] Fundamentals of Wireless Communication, David Tse, PramodViswanath

[3] Wireless communications and Networking by VIJAY GARG.

[4] R. Scholtz, “Multiple Access with Time-Hoping Impulse Modulaton,” IEEE milit. Commun. Conf., vol . 2,pp. 447-450,1993.

[5] Wireless communications and networks : second edition, by Theodore S. Rappaport

[6] R. T. Derryberry, S. D. Gray, D. M. Ionescu, G. Mandyam, and B. Raghothaman, “Transmit diversity in 3G CDMA systems,” IEEE Commun. Mag., vol. 40, no. 4, pp. 68–75, Apr. 2002.

[7] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Boelcskei, “An overview of MIMO communications -A key to gigabit wireless,” Proc. IEEE, vol. 92, no. 2, pp. 198–218, Feb. 2004.

[8] MIMO-OFDM modem for WLAN by, Authors: Lisa Meilhac, Alain Chiodini, Clement Boudesocque, ChrislinLele, Anil Gercekci.

[9] G. Leus, S. Zhou, and G. B. Giannakis, “Orthogonal multiple access over time- and frequency-selective channels,” IEEE Transactions on Information Theory, vol. 49, no. 8, pp. 1942–1950, 2003.

[10] P. A. Bello, “Characterization of randomly time-variant channels,” IEEE Transactions on Communications, vol. 11, no. 4, pp. 360–393, 1963.

[11] B. G. Evans and K. Baughan, "Visions of 4G," Electronics and Communication Engineering Journal, Dec.2002.

[12] “ZERO-FORCING EQUALIZATION FOR TIME-VARYING SYSTEMS WITH MEMORY “ by Cassio B. Ribeiro, Marcello L. R. de Campos, and Paulo S. R. Diniz.

[13] A. Adjoudani, E. Beck, A. Burg, G. M. Djuknic, T. Gvoth, D. Haessig, S. Manji, M. Milbrodt, M. Rupp, D. Samardzija, A. Siegel, T. S. II, C. Tran, S. Walker, S. A. Wilkus, and P. Wolniansky, “Prototype Experience for MIMO BLAST over Third-Generation Wireless System”, Special Issue JSAC on MIMO Systems, vol. 21, pp. 440–451, Apr. 2003.

[14] P. Murphy, F. Lou, A. Sabharwal, J.P. Frantz, “An FPGA based rapid prototyping platform for MIMO systems”, Proc. of Asilomar, vol. 1, pp. 900–904, Nov. 2003.

[15] “System requirements for IEEE 802.20 mobile broadband wireless access systems,” IEEE, Draft IEEE 802.20-PD-06 version 14, July 2004.

[16] T. Kaiser, A. Wilzeck, M. Berentsen, and M. Rupp, “Prototyping for MIMO Systems - An Overview”, in Proc. of the 12th European Signal Processing Conference (EUSIPCO), pp. 681–688, Vienna, Austria, Sept. 2004.