

# Dynamic Response of Composite Beam Structure to a Moving Mass Using FEA and Theoretical approach

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**Abstract-** The dynamic response of a composite beam subjected to a moving mass is of very much importance in the designing process of roadway bridges and railway bridges. This moving load problem finds several applications in the field of bridges, Transportation, guide ways, road ways, overhead crane, cable ways, pipe lines and tunnels. These structural elements are designed to support moving masses. Composite materials are gradually becoming more popular for many reason, the main advantage over metal components is the resistance to weight ratio which is important for many applications. In the present work the equation of motion in matrix form for an Euler beam subjected to a concentrated moving mass at a constant speed is formulated by Krylov's function approach. The solution of the declared problem for the case of cantilever beam is evaluated using Krylov's function. The effect of moving mass and its speed on the dynamic response of a cantilever beam for different compositions have been investigated using FEA analysis to support theoretical results, in this the beam is divided into twenty divisions and the deflection of the beam at each point is recorded while a mass moves at a steady velocity over different points. The results are obtained is plotted in the form of graphs.

**Index Terms—** Euler-Bernoulli beam, Krylov's function, Cantilever beam, Composite materials, FEA, Transient analysis.

## I. INTRODUCTION

Rapid technology advances in engineering brought the engineers to a point, where the ability of traditional materials became limited with the limits of capabilities the material failed to solve the requirements of engineer. Researchers in material technology are constantly searching for answers to provide stronger, durable material which will provide solution for the need of engineers. Composite materials are one of the most favorite solutions to this problem in the field by combining the stronger properties of traditional materials and eliminating the disadvantages they contain, it provides limitations like thermal resistance, structural strength and heavy weight are being solved. Due to high specific strength and stiffness, composite materials are used in many engineering applications. Metal Matrix Composite (MMC) is material consisting of a metallic matrix combined with a ceramic (oxides, carbides) or metallic (lead, tungsten,

molybdenum) dispersed phase. E.g. Aluminum, Titanium, Magnesium, Steel, Nickel, Copper, lead. The most important problem facing by the design engineers is dynamic behavior of bridges subjected to moving mass that is the dynamic behavior of bridges subjected to moving mass. This involves moving load problem, evaluation of natural frequency and modes of vibration. It is the oldest problem in the structural dynamics but now a day the mass and speed of the vehicles have been increased. There have been investigations of potential hazards caused by moving mass on structures for several years. The importance of moving mass lies in many applications such as transportation, Tunnel, Highway Bridge, railway track, cable ways and pipe lines and these are needed to be designed to support moving load. Moving load has substantial effect on the dynamic behavior of engineering structure. Dynamic loads are time dependent forces. These forces are applied on the structure either by natural way like earth quake or by human activity like machine vibration, structures affected by the dynamic forces can be simple as beams. In the design and analysis of the structure the time dependent initial force is considered. During its response to dynamic loads, a structure offers resistance to these loads in the form of internal forces some of which related to velocities and displacement these resistance forces also considered in the analysis.

## II. MATERIAL AND METHODOLOGY

Usually the Cantilever beam is made of Aluminum or Stainless steel for the construction of Railway bridge and Roadway bridge and dynamic analysis is done on these materials. In this paper the material chosen is composite material which is combination of Alluminum6061 and Silicon nitride[1], Analysis done on different percentage of Silicon nitride. The beam is divided into 20 divisions that is 21 points and the Modal analysis and Transient analysis is done on each point on the beam to obtain better results.

**Table 2: Engineering Properties of the composite Material**

PROPERTY	UNIT	0% OF Si <sub>3</sub> N <sub>4</sub>	6% OF Si <sub>3</sub> N <sub>4</sub>	10% OF Si <sub>3</sub> N <sub>4</sub>
DENSITY	Kg/m <sup>3</sup>	2700	2744.4	2774
MODULUS OF ELASTICITY	GPa	70.8	85.152	94.72
POISSONS RATIO	--	0.33	0.324	0.32

1. Modeling using solidworks 11. Software
2. Model is imported to ANSYS WORKBENCH
3. Boundary conditions and loads to be applied
4. Analysis and comparison of the results
5. Optimization of results obtained by analysis beams with different structural materials

**III. RESULTS AND TABLES**

By using Krylov's function approach considering the boundary conditions we get the following equations

$$w_i(x) = A_i S(kix) + B_i T(kix) + C_i U(kix) + D_i V(kix),$$

where

$$S(kix) = (\cosh kix + \cos kix)/2,$$

$$T(kix) = (\sinh kix + \sin kix)/2,$$

$$U(kix) = (\cosh kix - \cos kix)/2,$$

$$V(kix) = (\sinh kix - \sin kix)/2$$

Where S, t, U, V are the Krylov's function

1) Then considering the 1st boundary condition

$$A_i = 0 \text{ (a)}$$

2) Now considering 2nd boundary condition

$$B_i = 0 \text{ (b)}$$

3) Now considering 3rd boundary condition

$$\text{i.e. } M_i(L) = EI \partial^2 w_i(L) / \partial x^2 = I_m \omega_i^2 \partial w_i(L) / \partial x$$

taking 2nd derivative of all Krylov's function

$$M_i(L)/EI = -ki^2 [C_i (\cosh kix - \cos kix)/2] + ki^2 [D_i (\sinh kix - \sin kix)/2] \text{ (c)}$$

4) Now considering the 4th boundary condition

$$\text{or } 2Q_i(L)/EI \text{ ki}^3 = C_i (\sinh kix - \sin kix) - D_i (\cosh kix + \cos kix) \text{ (d)}$$

taking equation (c) and (d) and by multiplication of coefficient of  $D_i$  and adding

$$C_i [(\cosh kix - \cos kix)^2 + (\sinh kix - \sin kix)^2] = 2Q_i(L)/EI \text{ ki}^3 (\sinh kix - \sin kix)$$

$$-2M_i(L)/EI \text{ ki}^2 (\cosh kix - \cos kix) \text{ (e)}$$

As  $Q_i(L)$  is the transverse force at the end of the cantilever hence the bending moment will be

$$M(L) = Q_i(L) \cdot L^3 / 3EI$$

Hence equation (e) becomes

$$C_i = 2Q_i(L)/EI \text{ ki}^2 [(\sinh kix - \sin kix) / ki - L^3 (\cosh kix - \cos kix) / 3EI] / (\cosh kix - \cos kix)^2 + (\sinh kix - \sin kix)^2$$

$$\text{(A)}$$

Similarly by cross multiplication by coefficient of  $C_i$  and subtracting

$$D_i = -2Q_i(L)/EI \text{ ki}^2 [(\cosh kix - \cos kix) / ki + L^3 (\sinh kix - \sin kix) / 3EI] \text{ (B)}$$

From all these boundary conditions and their subsequences we can get a 4x4 matrix as follow

By simplifying the equation we will get

$$-ki^2 / 2 \times 2Q_i(L)/EI \text{ ki}^2 [(\cosh kix - \cos kix) (\sinh kix - \sin kix) / ki - L^3 (\cosh kix - \cos kix)^2 / 3EI]$$

$$(\cosh kix - \cos kix)^2 + (\sinh kix - \sin kix)^2 = 2Q_i(0) \cdot L^3 / 3E^2 I^2$$

$$ki = 0 \text{ (C)}$$

$$\text{For } ki = 0 \text{ R.H.S} = \text{L.H.S}$$

$$K_i = \pi \text{ R.H.S} = \text{L.H.S}$$

$$\text{Hence } K_i = n\pi, \text{ where } n = 1, 2, 3, 4, 5, \dots$$

$$\text{Since } K_i^4 = \omega_i^2 \rho A / EI$$

$$\text{Where } K_i = \text{Stiffness coefficient}$$

$$\omega_i = \text{natural frequency}$$

$$\omega_i^2 = K_i^2 \sqrt{EI / \rho A}$$

now taking all the given values and calculated values

**for Al6061 + 0% si3n4**

$$\text{for } K_i = \pi$$

$$\omega_i = 0.69853 \text{ Hz}$$

$$\text{for } K_i = 2\pi$$

$$\omega_i = 0.69949 \text{ Hz}$$

$$\text{for } K_i = 3\pi$$

$$\omega_i = 4.3606 \text{ Hz}$$

$$\text{for } K_i = 4\pi$$

$$\omega_i = 4.3645 \text{ Hz}$$

for Al6061 + 6% si3n4

for  $K_i = \pi$

$\omega_i = 0.66579$  Hz

for  $K_i = 2\pi$

$\omega_i = 0.66686$  Hz

for  $K_i = 3\pi$

$\omega_i = 4.1566$  Hz

for  $K_i = 4\pi$

$\omega_i = 4.1601$  Hz

for Al6061 + 0% si3n4

for  $K_i = \pi$

$\omega_i = 0.61194$  Hz

for  $K_i = 2\pi$

$\omega_i = 0.61314$  Hz

for  $K_i = 3\pi$

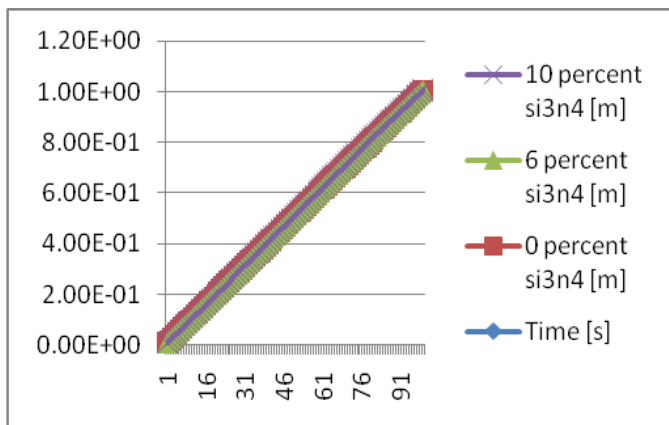
$\omega_i = 3.8212$  Hz

for  $K_i = 4\pi$

$\omega_i = 3.8238$  Hz

and so on

Fig 3: Graph showing deformation v/s time



Time [s]	0 percent si3n4 [m]	6 percent si3n4 [m]	10 percent si3n4 [m]
1.00E-02	1.18E-07	3.25E-07	3.16E-07
2.00E-02	4.92E-07	1.36E-06	1.33E-06
3.00E-02	1.17E-06	3.24E-06	3.16E-06
4.00E-02	2.21E-06	6.24E-06	6.18E-06
5.00E-02	4.26E-06	1.26E-05	1.26E-05
6.00E-02	7.52E-06	2.19E-05	2.18E-05
7.00E-02	1.19E-05	3.43E-05	3.40E-05
8.00E-02	1.75E-05	5.01E-05	4.96E-05
9.00E-02	2.44E-05	6.95E-05	6.86E-05
1.00E-01	3.27E-05	9.27E-05	9.12E-05
0.11	4.24E-05	1.20E-04	1.17E-04
0.12	5.34E-05	1.50E-04	1.48E-04
0.13	6.59E-05	1.86E-04	1.82E-04
0.14	7.99E-05	2.25E-04	2.21E-04
0.15	9.56E-05	2.70E-04	2.65E-04
0.16	1.13E-04	3.18E-04	3.12E-04
0.17	1.32E-04	3.71E-04	3.63E-04
0.18	1.52E-04	4.27E-04	4.18E-04
0.19	1.74E-04	4.87E-04	4.76E-04
0.2	1.98E-04	5.52E-04	5.39E-04

Table 3.1: Total deformation

%Si3N4	total deformation
0 percent	0.0054927
6 percent	0.0036553
10 percent	0.0027002

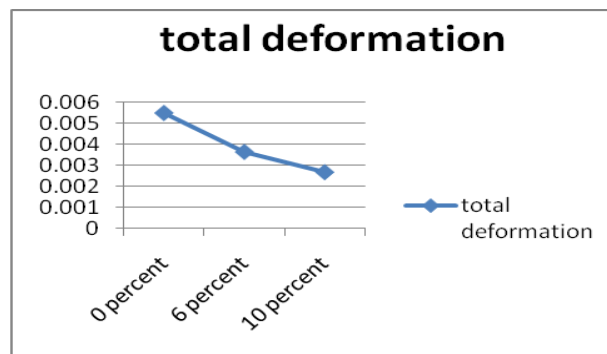
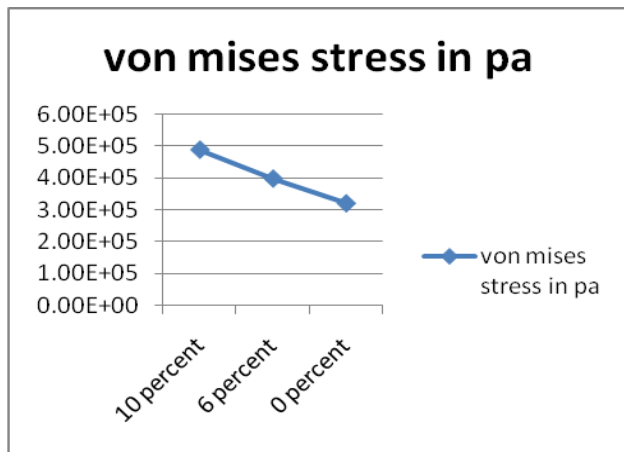


Fig 3.1: Graph showing deformation v/s %Silicon nitride

The above graph shows that the combination of 10% Silicon nitride with aluminum exhibits lesser deformation compared to 0% and 6% Silicon nitride

**Table 3.2: Total stress**

%Si3N4	von mises stress in pa
10 percent	4.90E+05
6 percent	3.99E+05
0 percent	3.22E+05

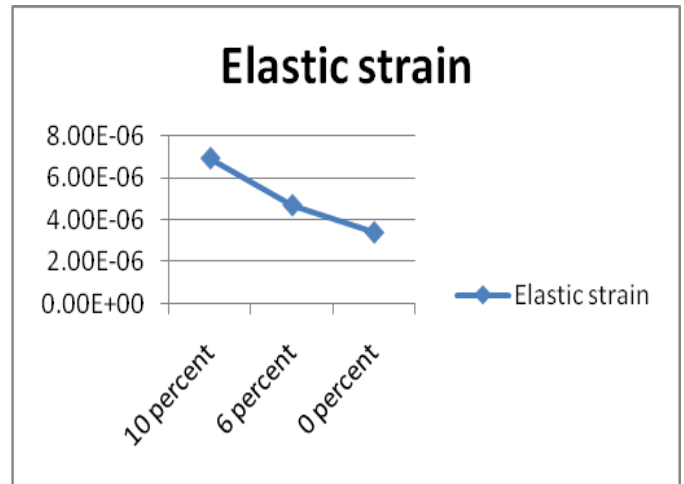


**Fig 3.2: Graph showing stress v/s %Silicon nitride**

By observing the above graph we can come to the conclusion that the combination of 10% Silicon nitride and aluminum shows more stress compared to other two

**Table 3.3: Total Elastic Strain**

%Si3N4	Elastic strain
10 percent	6.93E-06
6 percent	4.70E-06
0 percent	3.41E-06



**Fig 3.3: Graph showing strain v/s %Silicon nitride**

By the above graph we can come to the conclusion that The Elastic strain is more for the combination of 10% Silicon nitride compared to other two

#### IV. CONCLUSION

- The natural frequency obtained for 6% composition of Silicon nitride at 21 positions for 6 modes is relatively lesser when compared 0% Silicon nitride.
- For 10% composition of silicon nitride the natural frequency decreases as compared to 0% and 6% silicon nitride.
- The above result shows that when the percentage of Silicon nitride is increased the natural frequency decreases that is the vibration produced is less i.e. percentage of silicon nitride is inversely proportional to the natural frequency.
- As the Silicon nitride percentage increases Deformation decreases, The value of equivalent stress and elastic strain increases.

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