On SK Normal and Con SK Normal Bimatrices

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Abstract- **We consider S-K normal bimatrix and conjugate SK normal bimatrix. We discuss some of the most important properties of Sk normal bimatrix and conjugate S-K normal bimatrix. Also secondary K normal bimatrices are obtained**

Index Terms- **Normal, SK – normal, Bimatrix, Sk-normal Bimatrices moore-Penrorse inverse. Ams classification 15A09, 15A57, 11B25, 11C20.**

I. INTRODUCTION

Matrices provide a very powerful tool for dealing wit h linear models. Bimatrices are still a powerful and a n advanced tool which can handle over one linear model at a time. Bimatric es are useful when time bound comparisons are need ed in the analysis of amodel A square complex matrix $A_B \in C_{n\times n}$ is called normal if $A_B A_B^* = A_B^* A_B$ where $A_B^* = \overline{A}_B^T$ denotes the conjugate transpose of $A[2]$. There are many equailenet conditions in the literature for a square matrix to be normal $[3]$. For an mxn complex matrix A, the Moore Penrose inverse $A^\dagger$ of $A[2]$ is the unique nxm matrix X satisfying the following four Penrose equations:

(i) $A_B X_B A_B = A_B$

(ii) $X_B A_B X_B = X_B$

(iii) $(A_B X_B)^* = A_B X_B$

(IV) $(X_B A_B)^* = X_B A_B.\,(2)$

Recently, Hill and Waters $[3]$ have developed a theory for k-real and k-hermitian matrices. Ann Lee $[1]$has initiated the study of Secondary symmetric matrices, that is matrices whose entries are symmetric about the (Skew) Secondary diagonal. Ann Lee has shown that for complex matrix, A_B^S are related as $A_B^S = V_B A_B^T V_B$ where 'V' is the permutation matrix with units in the secondary diagonal. The concept of s-normal matrices is introduced by S. KJrishnamoorthy & R. Vijayakumar [6] and the concept of k-normal matrices introduced by S. Krishnamoorthy and R. Subhash [7]. In this paper characterization of s-k normal bimatrices are discussed.

We introduced Mingle the Bimatrices and Sk-normal matrices.

II. PRELIMINARIES AND NOTATIONS

Let Cnxn be the space of nxn complex matrices of order n. For $A_B \in C_{nxn}$, let $A_B^T, \overline{A}_B A_B^*, A_B^S$ and A_B^θ denote the transpose, Conjugate, Conjugate transpose secondary transpose and conjugate secondary transpose of matrix A_B respectively. Throughout, let 'k' be a fixed product of disjoint transpositions in S_n the set of all permutations on $\{1,2,3,...,n\}$ and K be the associated permutation matrix with units in the secondary diagonal. $'K'_B$ and $'V'_B$ clearly satisfies the following properties.

$$\overline{K}_B = K_B^T = K_B^S = K_B^* = \overline{K_B^S} = K_B; \qquad K_B^2 = 1$$

$$\overline{V}_B = V_B^T = V_B^S = V_B^* = \overline{V_B^S} = V_B; \qquad V_B^2 = 1$$

III. DEFINITION

1) A Bimatrix A_B is define as union of two rectangular array of numbers $A_1 \& A_2$ arranged into rows and columns.It is written as $A_B = A_1 \cup A_2$

2)A matrix $A_B \in C_{nxn}$ is said to secondary k-normal (s-k normal) bimatrix if

$$A_B\left(K_B V_B A_B^* V_B K_B\right) = \left(K_B V_B A_B^* V_B K_B\right) A_B$$
$$= (K_1 \cup K_2)(V_1 \cup V_2) A_1^* \cup A_2^* (V_1 \cup V_2)(K_1 \cup K_2)(A_1 \cup A_2)$$

Where A_B, V_B, K_B be the bimatrices.

IV. EXAMPLE

$$\overline{K}_B = K_B, \overline{V}_B = V_B$$
$$A_B\left(K_B V_B A_B^* V_B K_B\right) = \left(K_B V_B A_B^* V_B K_B\right) A_B$$
$$A_1 \cup A_2\left[(K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2)\right] =$$
$$\left[(K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2)\right]\left[A_1 \cup A_2\right]$$
$$\Rightarrow (A_1 \cup A_2)\left[K_1 V_1 A_1^* V_1 K_1\right] \cup \left[K_2 V_2 A_2^* V_2 K_2\right] = \left[K_1 V_1 A_1^* V_1 K_1\right] \cup \left[K_2 V_2 A_2^* V_2 K_2\right] \left[A_1 \cup A_2\right]$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \cup \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \left[\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \cup \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}\right] = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \cup \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \cup \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 3 & 2 \\ 3 & 2 & 3 \\ 2 & 3 & 3 \end{pmatrix} \cup \begin{pmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 2 \\ 3 & 2 & 3 \\ 2 & 3 & 3 \end{pmatrix} \cup \begin{pmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{pmatrix}$$

$$A_1\left(K_1 V_1 A_1^* V_1 K_1\right) \cup A_2\left(K_2 V_2 A_2^* V_2 K_2\right) = (K_1 V_1 A_1^* V_1 K_1) A_1 \cup \left(K_2 V_2 A_2^* V_2 K_2\right) A_2$$

V. REMARK

The concept of S-K normal bimatrices is analogous to that of normal matrices.

VI. SOME OF THE THEOREMS

Theorem :6.1

The transpose of an S-K normal bimatrix is S-K normal bimatrix.

Proof:

Set $A_B \in C_{n \times n}$

A_B is SK normal bimatrix

$$A_B\left(K_B V_B A_B^* V_B K_B\right) = \left(K_B V_B A_B^* V_B K_B\right) A_B$$
$$\left(A_B\left(K_B V_B A_B^* V_B K_B\right)\right)^T = \left(\left(K_B V_B A_B^* V_B K_B\right) A_B\right)^T$$

$$\left(K_B V_B A_B^* V_B K_B\right)^T A_B^T = A_B^T \left(K_B V_B A_B^* V_B K_B\right)^T$$

$$K_B V_B \left(A_B^*\right)^T V_B K_B \, A_B^T = A_B^T \; K_B V_B \, A_B^T \, V_B K_B$$

$$\left(K_1 \cup K_2\right)\left(V_1 \cup V_2\right)\left(A_1^T\right)^* \cup \left(A_2^T\right)^* \left(V_1 \cup V_2\right)\left(K_1 \cup K_2\right)\left(A_1^T \cup A_2^T\right)$$

$$= A_1^T \cup A_2^T \left(K_1 \cup K_2\right)\left(V_1 \cup V_2\right)\left(A_1^T\right)^* \cup \left(A_2^T\right)^* \left(V_1 \cup V_2\right)\left(K_1 \cup K_2\right)$$

$$\left[\left(A_1^T\right)^* \cup \left(A_2^T\right)\right]\left[\left(K_1 V_1 \left(A_1^*\right)^T V_1 K_1\right) \cup \left(K_2 V_2 \left(A_2^*\right)^T V_2 K_2\right)\right]$$

$$= \left(K_1 V_1 \left(A_1^*\right)^T V_1 K_1\right) \cup \left(K_2 V_2 \left(A_2^*\right)^T V_2 K_2\right)\left(A_1^T\right) \cup \left(A_2^T\right)$$

Therefore A^T is SK – normal bimatrix.

Theorem :6.2

The secondary transpose of an SK normal Bimatrix is SK normal bimatix

Proof:

$$A_B\left(K_B V_B A_B^* V_B K_B\right) = \left(K_B V_B A_B^* V_B K_B\right) A_B$$

$$\left(A_B\left(K_B V_B A_B^* V_B K_B\right)\right)^S = \left(\left(K_B V_B A_B^* V_B K_B\right) A_B\right)^S$$

$$\left(K_B V_B \left(A_B^*\right)^S V_B K_B\right) A_B^S = A_B^S \left(K_B V_B \left(A_B^*\right)^S V_B K_B\right)$$

$$\left(K_1 \cup K_2\right)\left(V_1 \cup V_2\right)\left(A_1^S\right)^* \cup \left(A_2^S\right)^* \left(V_1 \cup V_2\right) A_1^S \cup A_2^S$$

$$= A_1^S \cup A_2^S \left(K_1 \cup K_2\right)\left(V_1 \cup V_2\right)\left(A_1^S\right)^* \cup \left(A_2^S\right)^* \left(V_1 \cup V_2\right)\left(K_1 \cup K_2\right)$$

$$\left(A_1^S \cup A_2^S\right)\left(K_1 V_1 \left(A_1^S\right)^* V_1 K_1\right) \cup \left(K_2 V_2 \left(A_2^S\right)^* V_2 K_2\right)$$

$$= \left(K_1 V_1 \left(A_1^S\right)^* V_1 K_1\right) \cup \left(K_2 V_2 \left(A_2^S\right)^* V_2 K_2\right) A_1^S \cup A_2^S$$

Thus A_B^S is SK normal Bimatrix.

Theorem: 6.3

The conjugate of an SK normal Bimatrix is SK normal Bimatrix.

Proof:

$$\overline{A_B\left(K_B V_B A_B^* V_B K_B\right)} = \overline{\left(K_B V_B A_B^* V_B K_B\right) A_B}$$

$$A_B\left(K_B V_B A_B^* V_B K_B\right) = \left(K_B V_B A_B^* V_B K_B\right) A_B$$

$$\bar{A}_B\left(\overline{K_B}\,\overline{V_B}\,\overline{A_B^*}\,\overline{V_B}\,\overline{K_B}\right) = \left(\overline{K_B}\,\overline{V_B}\,\overline{A_B^*}\,\overline{V_B}\,\overline{K_B}\right)\overline{A_B}$$

$$\bar{A}_B\left(\overline{K_B}\,\overline{V_B}\,\overline{A_B^*}\,\overline{V_B}\,\overline{K_B}\right) = \left(\overline{K_B}\,\overline{V_B}\,\overline{A_B^*}\,\overline{V_B}\,\overline{K_B}\right)\overline{A_B}$$

$$\overline{\left(A_1 \cup A_2\right)}\left(K_1 \cup K_2\right)\left(V_1 \cup V_2\right)\overline{\left(A_1^* \cup A_2^*\right)}\left(V_1 \cup V_2\right)\left(K_1 \cup K_2\right) = \left(K_1 \cup K_2\right)\left(V_1 \cup V_2\right)\overline{\left(A_1^* \cup A_2^*\right)}\left(V_1 \cup V_2\right)\left(K_1 \cup K_2\right)$$

$$\left(\overline{A_1} \cup \overline{A_2}\right)\left(K_1 \cup K_2\right)\left(V_1 \cup V_2\right)\left(\overline{A_1}\right)^* \cup \left(\overline{A_2}\right)^* \left(V_1 \cup V_2\right)\left(K_1 \cup K_2\right)$$

$$= \left(K_1 \cup K_2\right)\left(V_1 \cup V_2\right)\left(\overline{A_1}\right)^* \cup \left(\overline{A_2}\right)^* \left(V_1 \cup V_2\right)\left(K_1 \cup K_2\right)\left(\overline{A_1} \cup \overline{A_2}\right)$$

$$\Rightarrow \left(\overline{A_1} \cup \overline{A_2}\right)\left(K_1 \cup K_2\right)\left(V_1 \cup V_2\right)\left(A_1^* \cup A_2^*\right)\;\left(V_1 \cup V_2\right)\left(K_1 \cup K_2\right)$$

$$= \left(K_1 \cup K_2\right)\left(V_1 \cup V_2\right)\left(A_1^* \cup A_2^*\right)^* \left(V_1 \cup V_2\right)\left(K_1 \cup K_2\right)$$

$$\overline{A_1} \cup \overline{A_2}\left(K_1V_1A_1^* \cup V_1K_1\right) \cup \left(K_2V_2A_2^* \cup V_2K_2\right) = \left(K_1V_1A_1^* \cup V_1K_1\right) \cup \left(K_2V_2A_2^* \cup V_2K_2\right)$$

Hence $\overline{A_B}$ is SK normal Bimatrix

Theorem : 6.4

Real secondary K – symmetric bimatrix are SK normal bimatrix.

Proof:

Let $A_B \in C_{n\times n}$

Let A_B be a real S-K symmetric bimatrix

Thus $A_B = K_BV_BA_B^TV_BK_B = K_BV_BA_B^*V_BK_B$

$$A_1 \cup A_2 = (K_1 \cup K_2)(V_1 \cup V_2)(A_1 \cup A_2)^T(V_1 \cup V_2)(K_1 \cup K_2)$$

which implies that $= (K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2)$

$$A_B\left(A_BV_BA_B^*V_BK_B\right) = \left(A_BV_BA_B^*V_BK_B\right)A_B$$

$$\Rightarrow A_1 \cup A_2 = \left(K_1V_1A_1^TV_1K_1\right) \cup \left(K_2V_2A_2^TV_2K_2\right) = \left(K_1V_1A_1^*V_1K_1\right) \cup \left(K_2V_2A_2^*V_2K_2\right)$$

$$A_1 \cup A_2\left(K_1V_1A_1^*V_1K_1\right) \cup \left(K_2V_2A_2^*V_2K_2\right) = \left(K_1V_1A_1V_1K_1\right) \cup \left(K_2V_2A_2V_2K_2\right)\left(A_1 \cup A_2\right)$$

Hence $A_B is$ S-K normal bimatrix.

Theorem:6.5

Real secondary K-skew symmetric bimatrices are SK normal bimatrix.

If A is a real S-K Skew symmetric bimatrix then

$$A_B = -\left(K_BV_BA_B^TV_BK_B\right) = -\left(K_BV_BA_B^*V_BK_B\right)$$

$$A_1 \cup A_2 = -\left[(K_1 \cup K_2)(V_1 \cup V_2)(A_1 \cup A_2)^T(V_1 \cup V_2)(K_1 \cup K_2)\right]$$
$$= -\left[(K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2)\right]$$

$$A_1 \cup A_2 = -\left[\left(K_1V_1A_1^TV_1K_1\right) \cup \left(K_2V_2A_2^TV_2K_2\right)\right]$$
$$= -\left[\left(K_1V_1A_1^*K_1V_1\right) \cup \left(K_2V_2A_2^*K_2V_2\right)\right]$$

which is implies that

$$A_1 \cup A_2\left[\left(K_1V_1A_1^*V_1K_1\right) \cup \left(K_2V_2A_2^*V_2K_2\right)\right] = \left[\left(K_1V_1A_1^*K_1V_1\right) \cup \left(K_2V_2A_2^*K_2V_2\right)\right]\left(A_1 \cup A_2\right)$$

Hence $A_B is$ S-K normal.

Theorem :6.6

The real secondary K-orthogonal Bimatrices and S-K normal Bimatrix.

Proof:

Let A_B be a real S-K orthogonal Bimatrix

$$A_B\left(K_BV_BA_B^TV_BK_B\right) = \left(K_BV_BA_B^TV_BK_B\right)A_B = I_B$$

$$A_B^{-1} = K_BV_BA_B^TV_BK_B$$

Since A_B is real

$$A_B = \left(K_B V_B A_B^T V_B K_B\right) = A_B A_B^{-1} = I_B \text{ and}$$

$$\left(K_B V_B A_B^T V_B K_B\right) A_B = A_B^{-1} A_B = I_B$$

$$A_B A_B^{-1} = \left(A_1 \cup A_2\right)\left(A_1 \cup A_2\right)^{-1} = I_1 \cup I_2$$

$$= A_1 A_1^{-1} \cup A_2 A_2^{-1} = I_1 \cup I_2$$

Thus A_B is S-K normal bimatrix.

Theorem:6.7

Secondary K – Hermition bimatrices are S-K normal Bimatrix.

$$\left(K_B V_B A_B^* V_B K_B\right) = A_B$$

$$\Rightarrow A_B\left(K_B V_B A_B^* V_B K_B\right) = A_B^2$$

Also, $\left(K_B V_B A_B{}^* V_B K_B\right) A_B = A_B^2$

$$A_B\left(K_B V_B A_B^* V_B K_B\right) = \left(K_B V_B A_B^* V_B K_B\right) A_B$$

$$\left(A_1 \cup A_2\right)\left(K_1 V_1 A_1^* V_1 K_1\right) \cup \left(K_2 V_2 A_2^* V_2 K_2\right)$$

$$\left(K_1 V_1 A_1^* V_1 K_1\right) \cup \left(K_2 V_2 A_2^* V_2 K_2\right)\left(A_1 \cup A_2\right)$$

Thus A_B is S-K normal bimatrix.

Theorem: 6.8

Secondary K – Skew Hermition bimatrices and SK normal bimatrix.

By Definition, $K_B V_B A_B{}^* V_B K_B = -A_B$

$$\Rightarrow A_B\left(K_B V_B A_B^* V_B K_B\right) = -A_B^2$$

Also $\left(K_B V_B A_B^* V_B K_B\right) A_B = -A_B^2$

$$\Rightarrow \left(A_1 \cup A_2\right)\left(K_1 V_1 A_1^* V_1 K_1\right) \cup \left(K_2 V_2 A_2^* V_2 K_2\right)$$

$$= \left(K_1 V_1 A_1^* V_1 K_1\right) \cup \left(K_2 V_2 A_2^* V_2 K_2\right)\left(A_1 \cup A_2\right)$$

Thus A_B is S-K normal bimatrix.

Lemma:6.9

Let $A_B N_B \in C_{n\times n}$, if N_B is S-K normal bimatrix such that

$A_B N_B = N_B A_B$ & $K_B V_B N_B V_B K_B = N_B$

then $A_B\left(KVN_B^* V_B K_B\right) = \left(K_B V_B N_B{}^* V_B K_B\right) A_B$

Theorem:6.10

If A and C are two S-K normal Bimatrix such that AC =CA.

$$A_B\left(K_B V_B C_B{}^* V_B K_B\right) = \left(K_B V_B C_B{}^* V_B K_B\right) A_B \;\&$$

$\left(K_B V_B A_B^* V_B K_B\right) C = C\left(K_B V_B A_B^* V_B K_B\right)$ then A + C and AC are also S-K normal matrices.

Proof:

Let $A_B, C_B \in c_{n\times n}$

Let $A_B \;\&\; C_B$ be S-K normal bimatrices such that $A_B C_B = C_B A_B$

By Lemma: 10

$$(A_B + C_B)(K_B V_B (A_B + C_B)^* V_B K_B) = (A_B + C_B) K_B V_B (A_B^* + C_B^*) V_B K_B$$
$$= (K_B V_B (A_B + C_B)^* V_B K_B) = (A_B + C_B) K_B V_B (A_B^* + C_B^*) V_B K_B$$
$$= (K_B V_B (A_B^* + C_B^*)\ V_B K_B) A_B + (K_B V_B (A_B^* + C_B^*) V_B K_B) C_B$$
$$= (K_B V_B (A_B^* + C_B^*) V_B K_B) A_B + C_B$$
$$= (K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^* + C_1^* \cup C_2^*)(A_1 \cup A_2) + (C_1 \cup C_2)$$

$$(K_1 V_1 A_1^* V_1 K_1) \cup (K_2 V_2 A_2^* V_2 K_2)(A_1 \cup A_2) + (K_1 V_1 C_1^* V_1 K_1) \cup (K_2 V_2 C_2^* V_2 K_2)(C_1 \cup C_2)$$

$A_B + C_B$ IS S-K normal Bimatrix

Also

$$(A_B C_B)(K_B V_B (A_B C_B)^* V_B K_B) = (K_B V_B C_B A_B V_B K_B) A_B C_B$$
$$(A_1 \cup A_2)[(C_1 \cup C_2)(K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(C_1^* \cup C_2^*)](V_1 \cup V_2)(K_1 \cup K_2)$$

$$= [(K_1 \cup K_2)(V_1 \cup V_2)[(C_1^* \cup C_2^*)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2)(C_1 \cup C_2)(A_1 \cup A_2)]]$$
$$(A_1 \cup A_2)(K_1 V_1 A_1^* C_1^* V_1 K_1) \cup (K_2 V_2 A_2^* C_{2*} V_2 K_2) = (K_1 V_1 C_1^* A_1^* V_1 K_1) \cup (K_2 V_2 C_2^* A_2^* V_2 K_2)(A_1 \cup A_2)$$

Hence

$$A_B C_B = C_B A_B$$

REFERENCES

[1] Hill, R.D., Water, S.R., On k-real and k-hermitian matrices, Linear Alg. Appl., 169(1992), 17-29.

[2] Dr. Elumalai, N. and Manikandan,R. On Spectral Theory at K N Arithmethic mean idempotent matrices on posets.

[3] Dr.Krishnamoorthy, S. and Bhuvanshwari, H. On S-K normal matrices, International journal of Recent Scientific Research.

[4] Krishnamoorthy. S., Vijayakumar. R., On s-normal matrices, journal of Analysis and Computation, Vol 5, No.2 (2009), in press.

[5] Krishnamoorthy. S., and Subash. R., 2011 “On k-normal matrices” international J. of Math. Sci & Engg. Appls Vol.5, no.II, PP 119-130.