Synchronization Likelihood - a Non Linear Multivariate Technique for finding Functional Connectivity

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Abstract- **The brain is a highly complex biological system. The study of complex systems requires the use of analytical tools which can detect statistical dependencies between time series recorded from the interacting subsystems. This work aims to study various signal processing methods to determine the functional connectivity in the brain using multichannel EEG signals. The functional connectivity is analyzed using linear and non linear techniques with correlation and synchronization Likelihood. Functional connection is obtained by computing the correlation among multi channel EEG data by applying different threshold. Coherency of EEG is sensitive only to linear and symmetric interdependencies and cannot deal with non-stationarity. The approach is based upon the theory of non-linear dynamical systems. This discusses the Synchronization Likelihood, a multivariate non-linear technique which measures synchronization between different brain sites.**

Index Terms- **Synchronization Likelihood, Non-linear, Dynamical System**

I. INTRODUCTION

Functional connectivity is a measure of how regions of the brain interact with each other. Brain functional connectivity evaluates the statistical dependencies between spatially distributed brain regions. A central problem in the study of normal and disturbed brain function is the question how functional interactions take place between different specialized networks. Understanding the coordination between brain regions is important in the context of information processing in the healthy brain but also in the case of neurological disease. Loss of neurons and connecting fibre systems may lead to diminished interactions and cognitive dysfunction. Usually, functional interactions are studied by considering time series of electrical potentials (electroencephalogram (EEG)) or magnetic field strengths (Magnetoencephalogram (MEG)) recorded from different brain areas. Similarities between these time series are taken to reflect functional influences between the neuronal networks generating the time series. Similarities between time series are commonly quantified with linear techniques, in particular estimates of the coherency, which is a normalized measure of linear correlation as a function of frequency only sensitive to linear and symmetric interdependencies and cannot deal with non-stationary.

Recently, several algorithms based upon the concept of generalized synchronization have been introduced to overcome some of the limitations of coherency estimates. Similarities between these time series are taken to reflect functional influences between the neuronal networks generating the time series. Similarities between time series are commonly quantified with linear techniques, in particular estimates of the coherency, which is a normalized measure of linear correlation as a function of frequency. To this end we can apply coherence function, which is based on Fourier transform. Word "coherence" is from the Latin word cohaerentia– it means natural or logical connection or consistency. The coherence function allows us to find common frequencies and to evaluate the similarity of signals. However, it does not give any information about time. There are two often used methods to calculate the coherence function: Welch method and MVDR (Minimum Variance Distortion less Response) method. A transformation T is said to be linear if applied to linear combination of signals $ax+by$ gives linear combination of results $aT(x)+bT(y)$. Coherence function is based on Fourier transform, which is a linear transformation.

The Coherence function is defined as

$$C_{xy}(w)=Pxy(w)/\sqrt{Pxx(w).Pyy(w)} \qquad (1)$$

Where P_{xx} and P_{yy} are power spectra of signals x and y ,P_{xy} is cross-power spectrum for these signals, ω is frequency. In case, when $P_{xx}(\omega) = 0$ or $P_{yy}(\omega) = 0$, then also $P_{xy}(\omega) = 0$ and we assume, that value $C_{xy}(\omega)$ is zero. The power spectrum (also called periodogram) and cross-power spectrum is defined as the Fourier transform. It yields the information about frequencies occurring in signals and the dominant frequency for these signals.

First coherency estimates are not suitable to characterize non-stationary data with rapidly changing interdependencies. Possibly, modifications such as event-related coherence can overcome this limitation. Coherency only captures linear relations between time series, and may fail to detect non-linear interdependencies between the underlying dynamical systems.

Recently a variety of methods have been proposed to detect more general types of interactions between dynamical systems. Here, the instantaneous phase of both time series is computed, and interactions are quantified in terms of time-dependent.

II. CORRELATION

Similarities between time series are commonly quantified with linear techniques, in particular estimates of the coherency, which is a normalized measure of linear temporal correlation as a function of frequency. The functional connectivity refers to the pattern of temporal correlations (or, more generally, deviations from statistical independence) that exists between distinct neuronal units. Such temporal correlations are often the result of neuronal interactions along anatomical or structural connections; in some cases observed correlations may be due to common input from an external neuronal or stimulus source. Deviations from statistical independence between neuronal elements are commonly captured in a correlation matrix under certain statistical assumptions, may be viewed as a representation of the system's functional connectivity. Although temporal correlations are perhaps most often used to represent statistical patterns in neuronal networks, other measures such as spectral coherence or consistency in relative phase relationships may also serve as indicators of functional connectivity. The relationship between structural and functional dimensions of brain connectivity is mutual and reciprocal. In the other direction, functional interactions can contribute to the shaping of the underlying anatomical substrate. This is accomplished either directly through activity (correlation) dependent synaptic modification, or, over longer time scales, through effects of functional connectivity on an organism's perceptual, cognitive or behavioral capabilities, which in turn affect adaptation and survival. The reciprocity between anatomical and functional networks deserves emphasis as it captures some of the unique aspects of brain networks.

The brain is inherently a dynamic system, in which the traffic between regions, during behavior or even at rest, creates and reshapes continuously complex functional networks of correlated dynamics. The important goal in neuroscience is to understand these spatiotemporal patterns of brain activity. Two brain sites are said to be functionally connected if their temporal correlation exceeds a positive predetermined value rc, regardless of their anatomical connectivity []. The temporal correlation between two brain sites x1(t) and x2(t) is defined by the mathematical formula [1]

$$r(x1,x2)=\frac{\langle x1(t)x2(t)\rangle-\langle x1(t)\rangle\langle x2(t)\rangle}{\sigma(x1(t))\sigma(x2(t))} \qquad (2)$$

The resulting Correlation matrix is converted to graph by applying a threshold (typically ranging between 0.5-0.8).

Correlations between all pair wise combinations of EEG channels were determined using the temporal correlation. The correlation value ranges between 0 and 1.The end result of computing the correlation for all pair wise combinations of channels is a square matrix of size N x N (the number of EEG channels), where each entry Ni,j contains the value of the correlation for the channels i and j. The resulting synchronization matrix is converted to a binary graph by applying a threshold, T. If the correlation between a pair of channels i and j exceeds T, an edge is said to exist between i and j; otherwise no edge exists between i and j.The corresponding threshold matrix is called 'Adjacency matrix'. A binary graph is a network that consists of elements (also called ''vertices'') and undirected connections between elements (called ''edges'') .The correlation matrix and it's corresponding threshold matrix computed from N channel EEG data is obtained. The binary graph having N vertices/nodes which is nothing but the functional network is also generated.

Degree distribution of Network: The 'degree' of a vertex is defined as the number of edges connected to that vertex.

d_i={ number of edges connected to that vertex }

Similarly the degree of each vertex is found (degree vector)

$$d=(d1, d2, \dots\dots dN)^T \qquad (3)$$

where N is the no. of vertices, d1,d2,...d_N are the degrees of the corresponding vertices 1,2,....N. Now the degree distribution (PD) is found by computing the histogram of the degree vector d.

$$PD = hist(d) \qquad (4)$$

Average Path length: The 'path length (L)' between two vertices is defined as the minimum number of links/edges required to connect to both vertices. The average of all the possible path lengths between any two vertices in a graph is defined as the 'Average Path Length (L_{avg})'. If suppose there are N number of vertices in a graph, the of possible ways in which two nodes can be picked for finding path is nC_2.For calculating the Average path length Floyd's Algorithm (Undirected graph) is employed.

This algorithm is designed to find the least-expensive paths between all the vertices in a graph. It does this by operating on a matrix representing the costs of edges between vertices. Before we invoke Floyd's algorithm a matrix must be built, usually in a two- dimensional array. If there are N vertices in the graph, the size of the matrix will be N x N. Each row in the matrix represents a "starting" vertex in the graph while each column in the matrix represents an "ending" point in the graph [2]. If there is an edge between a starting point i and ending point j in the graph, the cost of this edge is placed in position (i,j) of the matrix. In this analysis all the edges are given the same cost equal to unity. If there is no edge directly linking two vertices, an infinite (or, in practice, very large) value is placed in the (i,j) position of the matrix to specify that it is impossible to directly move from i to j.

The clustering coefficient is a measure of the local interconnectedness of the graph, whereas the path length is an indicator of its overall connectedness. The clustering coefficient Ci for a vertex vi is the proportion of links between the vertices within its neighborhood divided by the number of links that could possibly exist between them. For an undirected graph the edge eij between two nodes i,j is considered identical to eji. Therefore, if a vertex vi has ki neighbours, ki(ki-1)/2 edges could exist among the vertices within the neighborhood. Thus, the clustering coefficient for undirected graphs can be defined as:

$$C_i=\frac{2|\{e_{jk}\}|}{k_{i(k_i-1)}}: V_j V_k \in N_{i,} e_{ij} \in E \qquad (5)$$

The clustering coefficient for the whole graph is given as the average of the clustering coefficient for each vertex:

$$\hat{C}=\frac{1}{2}\sum_{i=1}^{n} C_i \qquad (6)$$

While this approach has produced a large body of knowledge on normal and pathological brain function, it has a number of limitations.

First, coherency estimates are not suitable to characterize non-stationary data with rapidly changing interdependencies. Possibly, modifications such as event related coherence can overcome this limitation.

A more important limitation is that methods such as coherency only capture linear relations between time series, and may fail to detect nonlinear interdependencies between the underlying dynamical systems. Recently a variety of methods have been proposed to detect more general types of interactions between dynamical systems. One line of research is based on the analytical signal concept [3]. Here, the instantaneous phase of both time series is computed, and interactions are quantified in terms of time dependent *n*:*m* phase locking (*n* and *m* being integers). This approach has been successful in the study of EEG seizure data and in the study of synchronization between muscle and cortical activity [4]. However, this approach is only valid when the time series are approximately oscillatory.

III. SYNCHRONIZATION LIKELIHOOD

Synchronization likelihood (SL) is general approach based upon the theory of non-linear dynamical systems is used to determine the functional network of brain. The variation of SL with coupling strength between identical and non-identical systems, ability of SL to detect nonlinear coupling, using multivariate synthetic data, sensitivity of SL to time dependence of SL on Pref , noise and application to epileptic EEG data, eyes opened and eyes closed are studied thoroughly. The first step in finding the synchronization likelihood is to obtain the embedded vectors using Taken's theorem. Then the correlation integral is evaluated using the embedded vectors. The Synchronization Likelihood is obtained from correlation Integral. Correlations between all pair wise combinations of EEG channels were determined with the synchronization likelihood. The resulting synchronization matrix is converted to a binary graph by applying a threshold, and then the cluster coefficients and path lengths are computed as a function of threshold or as a function of degree k. The properties of nonlinear technique (SL) are studied in detail by first working on the synthetic Henon data followed by real EEG data. The variation of SL with coupling strength between identical and non- identical systems, ability of SL to detect nonlinear coupling, using multivariate synthetic data, sensitivity of SL to time dependence of SL on Pref , noise and application to epileptic EEG data, eyes opened and eyes closed are studied thoroughly. All this work provides sufficient information for analyzing the connectivity of the human brain and we have discussed this for case of epileptic seizure and also for different states of the subject (relaxed, eyes closed and eyes opened).

Information theoretic approach for studying synchronization phenomena in experimental bivariate time series is presented. This exists between two dynamical systems X and Y when the state of the response system Y is a function of the state of the driving system X: $Y = F(X)$. When F is continuous, and x_i , x_j are two points on the attractor of X which are very close together, then the corresponding points y_i , y_j on Y will also be close together. An important feature of generalized synchronization is that the corresponding time series need not resemble each other.

Synchronization Likelihood (SL) is one of the successful algorithms for analyzing the generalized synchronization. It is a straightforward normalized estimate of the dynamical interdependencies between two or more simultaneously recorded time series. The SL ranges between P_{ref} (a small number close to 0) in the case of independent time series and 1 in the case of maximally synchronous signals. The basic principle of the SL is to divide each time series into a series of ''patterns'' (roughly, brief pieces of time series containing a few cycles of the dominant frequency) and to search for a recurrence of these patterns. The SL is then the chance that pattern recurrence in one time series coincides with pattern recurrence in another time series; Pref is the small but nonzero likelihood of coincident pattern recurrence in the case of independent time series. This measure can also be computed in a time dependent way, making it suitable for the analysis of non-stationary data. An algorithm for calculating synchronization likelihood is explained in the paper [5].An algorithm for the determination of synchronization likelihood.

Forming Embedded vectors:

Consider an M simultaneously recorded time series $x_{k,i}$, where k denotes channel number $(k = 1, . . .,M)$ and i denotes discrete time $(i = 1, . . .,N)$. From each of the M time series embedded vectors $X_{k,i}$ are reconstructed with time delay embedding.

$$X_{k,i} = (x_{k,i} , x_{k,i+l} , x_{k,i+2l} , . . ., x_{k,i+(m.1)l})$$

where l is the lag and m is the embedding dimension.

Finding Critical distances:

For each time series k and each time i we define the probability $P^{\varepsilon}_{k,i}$ that embedded vectors are closer to each other than a distance ε:

$$P_{k,i}^{\varepsilon}=\frac{1}{2(w2-w1)}\sum^{N}_{\substack{j=1\\ w_1<|i-j|<w_2}} \theta(\epsilon_{k,i}-|X_{k,i}-X_{k,j}|) \quad (7)$$

Here the | · | is the Euclidean distance and θ is the Heaviside step function, $\theta(x) = 0$ if $x \leq 0$ and $\theta(x) =1$ for $x > 0$. Here $w1$ and $w2$ are two windows; $w1$ is the Theiler correction for autocorrelation effects and should be at least of the order of the autocorrelation time, *w*2 is a window that sharpens the time resolution of the synchronization measure and is chosen such that $w1 << w2 << N$. Now for each *k* and each *i* the critical distance $\varepsilon k,i$ is determined for which $P\varepsilon k,1$

$$k_{,i} = \text{pref} \qquad \text{where } pref<<1. \qquad (8)$$

Formation of H matrix:

Now for each discrete time pair (*i*, *j*) within our considered window $(w1 < |i\ .\ j\ | < w2)$ the number of channels $Hi\ ,j$ *is* determined, where the embedded vectors Xk,i and Xk,j will be closer together than this critical distance $\varepsilon k,i$:

$$H_{i,j} = \sum_{k=1}^{M} \theta\left(\epsilon_{k,i} - \left|X_{k,i} - X_{k,j}\right|\right) \qquad (9)$$

This number lies in a range between 0 and *M*, and reflects how many of the embedded signals “resemble” each other.

Determination of Synchronization Likelihood(SL) Sk,i :

The Synchronization Likelihood Sk,i,j for each channel *k* and each discrete time pair (*i*, *j*) is defined as

$$\text{if}\left|X_{k,i} - X_{k,j}\right| < \epsilon_{k,i} : S_{k,i,j} = \frac{H_{i,j-1}}{M-1} \qquad (10)$$

$$\text{if}\left|X_{k,i} - X_{k,j}\right| < \epsilon_{k,i} : S_{k,i,j} = 0 \qquad (11)$$

The synchronization likelihood Sk,i is obtained by averaging over all *j*

$$S_{k,i} = \frac{1}{2(w_2 - w_1)} \sum_{\substack{j=1 \\ w_1 < |j-i| < w_2}}^{N} S_{k,i,j} \qquad (12)$$

The synchronization likelihood (SL) $S_{k,i}$ is a measure which describes how strongly channel k at time i is synchronized to all the other M1 channels. The synchronization likelihood takes on values between pref and 1. $S_{k,i}$ = pref corresponds with the case where all M time series are uncorrelated and $S_{k,i}$ = 1 corresponds with maximal synchronization of all M time series. The value of pref can be set at an arbitrarily low level, and does not depend on the properties of the time series, nor is it influenced by the embedding parameters.

Determination of functional connectivity using synchronization likelihood:

1. Correlations between all pair wise combinations of EEG channels were determined with the synchronization likelihood. The SL ranges between Pref(a small number close to 0) and 1.

2. The end result of computing the SL for all pair wise combinations of channels is a square matrix of size N x N (the number of EEG channels), where each entry Ni,j contains the value of the SL for the channels i and j.

3. The resulting synchronization matrix is converted to a binary graph by applying a threshold.

4. The cluster coefficients and path lengths were computed as a function of threshold or as a function of degree K.

Recently a variety of methods have been proposed to detect more general types of interactions between dynamical systems. Here, the instantaneous phase of both time series is computed, and interactions are quantified in terms of time-dependent n : m phase locking (n and m being integers). This approach has been successful in the study of EEG seizure data in the study of synchronization between muscle and cortical activity. However, this approach is only valid when the time series are approximately oscillatory.

A more general approach is based upon the theory of non-linear dynamical systems. It was demonstrated in the 1980s and early 1990s that, contrary to intuition, two interacting chaotic systems can also display synchronization phenomena [1-6]. Initially synchronization was understood as identical synchronization, implying equality of the variables of the coupled systems. In the context of uni-directionally coupled driver response systems Rulkov et al. [7] introduced the wider concept of generalized synchronization. Generalized synchronization exists between two dynamical systems X and Y when the state of the response system Y is a function of the state of the driving system X : Y = F(X) . When F is continuous, and x i,x j are two points on the attractor of X which are very close together, then the corresponding points y i,y j on Y will also be close together. An important feature of generalized synchronization is that the corresponding time series need not resemble each other. Since the concept of generalized synchronization was introduced, several algorithms have been proposed to detect this type of interdependencies in experimental time series.

In this paper we propose a synchronization likelihood measure S which avoids the bias pointed out by Pereda et al. and gives a straightforward normalized estimate of the dynamical interdependencies between two or more simultaneously recorded time series. The measure is closely related to the concept of generalized mutual information as introduced by Pawelzik and co-workers [8][9] and this measure can also be computed in a time-dependent way, making it suitable for the analysis of non-stationary data.

IV. FUNCTIONAL NETWORK

A 16 channel EEG data, with a sampling frequency of 128, is collected from a healthy subject during eyes closed states and each channel is of length 2624 samples and is of length. A 16x16 correlation matrix is obtained by computing the correlation among these 16 channels. The resulting correlation matrix is converted to graphs by applying a threshold with 0.7.The following figure illustrates the correlation matrix and its corresponding threshold matrix.

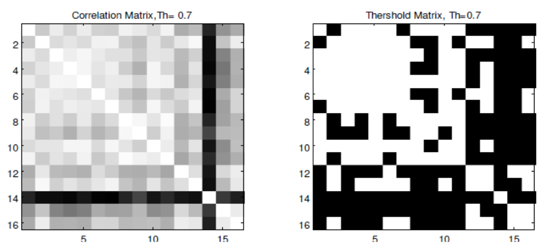

Figure 1: Correlation and Threshold matrix.

Functional Network generated from a 16 Channel EEG data of eyes closed state

A graph having 16 vertices/nodes, computed from a 1 6 channel EEG data is shown in figure 2 below.

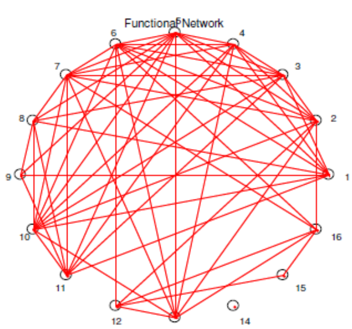

Figure 2: A graph having 16 vertices/nodes

It is observed from above in figure 1 that the correlation matrix shown all the nodes between 1 to 11 are highly correlated (bright region) and the threshold matrix shows that the nodes with correlation coefficient less than threshold 0.7 are zero(black). The figure 2 gives the graphical representation of threshold matrix in the form of functional network or binary graph. The node 14th has zero degree which gives a disconnected graph. The resulting functional networks shown above has high cluster coefficient and small 'mean shortest path length' which gives the about functional network is a small world network.

Degree distribution of Network:

A plot of degree distribution (PD) for a 100 vertex graph, against degree, is shown figure 3. We can observe from the following figure that the distribution clearly follows a power law distribution $P(k) \sim k^{-\gamma}$

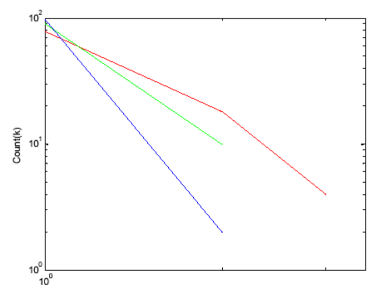

Figure 3: A plot of degree distribution (PD) for a 100 vertex graph

Influence of threshold on connectivity in the case of a relaxed subject:

An 8 channel EEG data is collected from a healthy relaxed subject. The 8 channels from different locations on the scalp C3, C4, F3, F4, O1, O2, P3, and P4 are denoted as 1, 2, 3, 4, 5, 6, 7, and 8 respectively. Each channel data is of length 750 samples. The correlation matrix is obtained by computing the correlation among these 8 channels. The correlation matrix is converted to 3. Threshold matrices by applying various thresholds Th=0.4, 0.5, and 0.6 respectively. The binary graphs are then extracted from the threshold matrices which are shown in figures 4, 5, and 6 below.

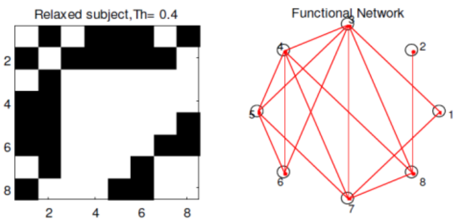

Figure 4: The 'Threshold matrix' and its 'Functional network' with Th = 0.4

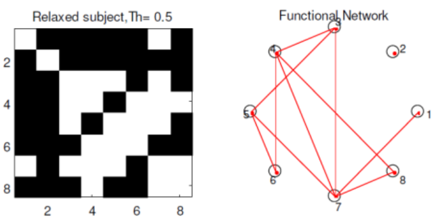

Figure 5: The 'Threshold matrix' and its 'Functional network' with Th = 0.5

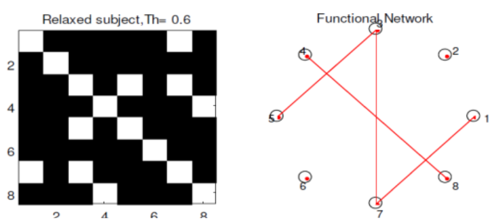

Figure 6: The 'Threshold matrix' and its 'Functional network' with Th = 0.6

The average path length < L >, average clustering coefficient < C > and average degree of the network < k > are calculated and compared for each case. The results are shown in the following table

Threshold	< L >	< C >	< K >
0.4	1.5714	0.5208	3.7500
0.5	2.5009e+003	0.3292	2.5000
0.6	7.4996e+003	1.2500e-005	1

Table:1

Observation:

1. With the increase in threshold the connectivity is found to decrease.
2. The average path length has increased drastically with the increase in threshold.
3. The average clustering coefficient has decreased with the increase in threshold

which indicates that the connectivity is decreasing.

4. The average degree is decreasing gradually with the increase in threshold.

Functional connectivity during epileptic seizure:

A 10 channel EEG data is collected from subject suffering from epileptic seizure. The 10 channels from different locations on the scalp C3, C4, F3, F4, O1, O2, P3, P4, T3 and T4 are denoted as 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 respectively. Each channel data is of length 230 samples. A 10x10 correlation matrix is obtained by computing the correlation among these 10 channels. The correlation matrix is converted to a threshold matrix by applying the thresholds Th = 0.5. The binary graph is then extracted from the threshold matrix which is shown in figure 7 below.

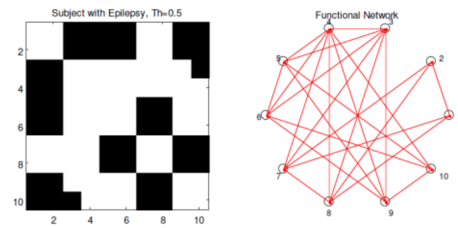

Figure 7: The 'Threshold matrix' and its 'Functional network' in an epileptic subject with Th = 0.5

The average path length < L >, average clustering coefficient < C > and average degree of the network < k > are calculated and the results are shown in the table 2 below.

Threshold	<L>	<C>	<K>
0.5	1.6444	0.8071	4.8000

Table: 2

Comments:

1. The average degree <K> of the network is 4.8 which mean that the connectivity is very high during seizure.
2. Comparing < K > = 4.8 in this case with that of relaxed subject < K > = 2.5 we can

conclude that connectivity is more when the subject is suffering from epilepsy.

Functional connectivity during eyes closed and eyes open states:

A 16 channel EEG data, with a sampling frequency of 128, is collected from a healthy subject during eyes closed and eyes open states. During eyes closed condition, each channel is of length 2624 samples and is of length 1472 in the case of eyes open. A 16x16 correlation matrix is obtained by computing the correlation among these 16 channels. The correlation matrix is converted to a Threshold matrix by applying the thresholds Th = 0.8 in each case. The binary graphs are then extracted from respective threshold matrices which are shown in figures 8 and 9 below.

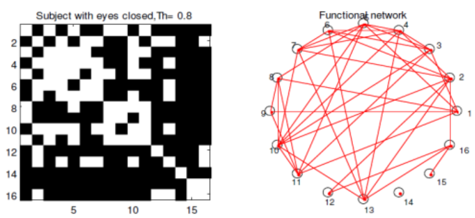

Figure 8: The ‘Threshold matrix’ and its ‘Functional network’ when the subject has closed his eyes, Th = 0.8

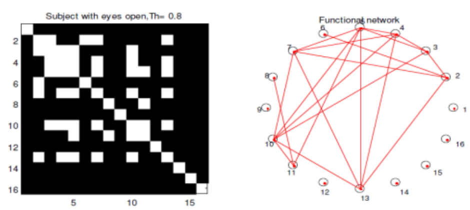

Figure 9: The ‘Threshold matrix’ and its ‘Functional network’ when the subject has opened his eyes, Th = 0.8

The average path length < L >, average clustering coefficient < C > and average degree of the network < k > are calculated and compared for each case. The results are shown in the following table 3.

Task	Th	<L>	<C>	<K>
Eyes closed	0.8	1.2517e+003	0.0208	4.8750
Eyes open	0.8	6.2501e+003	6.2500e-006	2.5000

Table: 3

Observations:

1. Both the graphs in this case are unconnected as it is expected since the correlation is very high (=0.8) this time.
2. The high value of path length indicates the possibility of an isolated node which can be observed from the above figures.
3. The average path length < L > is more in the case of Eyes opened where as the Clustering coefficient < C > is more for eyes closed state. This indicates that the connectivity is more in eyes closed condition.
4. The average degree of the network < K > is more in the case of subject with his eyes closed which means that the former(eyes closed) is densely connected where as the later(eyes open) is sparsely connected.

5. Analysis of functional connectivity using real data for synchronization likelihood (SL):

Functional connectivity in the case of a relaxed subject:

An 8 channel EEG data is collected from a healthy relaxed subject. The 8 channels from different locations on the scalp C3, C4, F3, F4, O1, O2, P3, and P4 are denoted as 1, 2, 3, 4, 5, 6, 7, and 8 respectively. Each channel data is of length 750 samples. The SL matrix is obtained by computing the SL among these 8 channels pair wise. The SL matrix is converted in to the threshold matrix by applying the threshold Th=0.05,the black square in the above threshold matrix indicates ‘ZERO’ which means the SL between the considered pair has fallen below the threshold and a white square indicates ‘ONE’ (Th>0.05).The corresponding functional network is then extracted from the threshold matrix which is shown in figure 2.

Relaxed subject, Th= 0.05

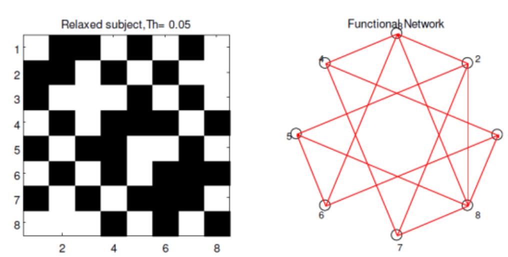

Figure 10: The ‘Threshold matrix’ and its ‘Functional network’ in relaxed subject with Th = 0.05

The average pathlength < L >, average clustering coefficient < C > and average degree of the network < k > are calculated and are shown in the following table 1.

Threshold	< L >	< C >	< K >
0.05	1.5714	0.1042	2.8750

Table: 4

Functional connectivity during epileptic seizure:

A 10 channel EEG data is collected from subject suffering from epileptic seizure. The 10 channels from different locations on the scalp C3, C4, F3, F4, O1, O2, P3, P4, T3 and T4 are denoted as 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 respectively. Each channel data is of length 230 samples. A 10x10 SL matrix is obtained by computing the pair wise SL among these 10 channels. The SL matrix is converted to a threshold matrix by applying the thresholds Th = 0.04. The binary graph is then extracted from the threshold matrix which is shown in figure 11 below.

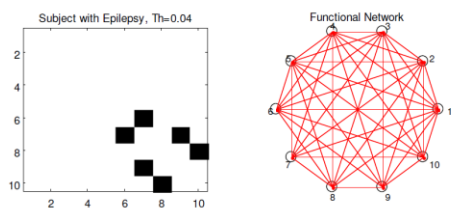

Figure 11: The 'Threshold matrix' and its 'Functional network' in an epileptic subject with Th = 0.04

The average path length < L >, average clustering coefficient < C > and average degree of the network < k > are calculated and the results are shown in the following table 5 below.

Threshold	< L >	< C >	< K >
0.5	1.0667	0.9321	8.4000

Table: 5

Comments:

1. The average degree of the network is given by 8.4 which mean that the connectivity is very high during seizure.

2. Comparing < k > = 8.4 in this case with that of relaxed subject < K > = 2.875 we can conclude that connectivity is more when the subject is having a epileptic seizure.

Functional connectivity during eyes closed and eyes open:

A 16 channel EEG data, with a sampling frequency of 128, is collected from a healthy subject during eyes closed and eyes open states. During eyes closed condition, each channel is of length 2624 samples, it is of length 1472 in the case of eyes open. A 6x16 correlation matrix is obtained by computing the correlation among these 16 channels. The correlation matrix is converted to a Threshold matrix by applying the thresholds Th = 0.5 in each case. The functional network is then extracted from respective threshold matrices which are shown in figures 12 and13below.

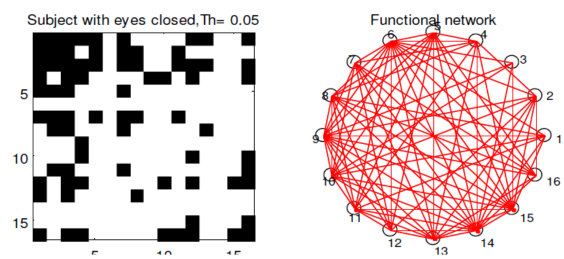

Figure 12: The 'Threshold matrix' and its 'Functional network' when the subject has closed his eyes, Th = 0.05

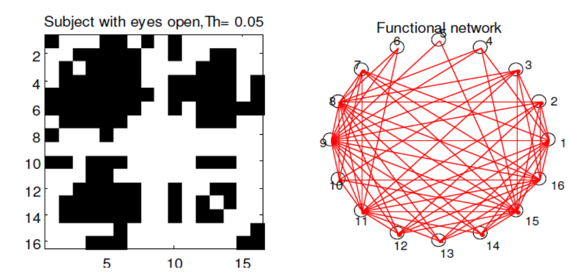

Figure 13: The 'Threshold matrix' and its 'Functional network' when the subject has opened his eyes, Th = 0.05

The average path length < L >, average clustering coefficient < C > and average degree of the network

< k > are calculated and compared for each case. The results are shown in the following table 3.

Task	Th	<L>	<C>	<K>
Eyes closed	0.05	1.2917	0.7721	10.1875
Eyes open	0.05	1.4500	0.5119	7.9375

Table: 6

Observations:

1. Both the graphs in this case are connected.
2. The average path length < L > is more in the case of Eyes opened. Since <L> is inversely proportional to connectivity we can conclude that the connectivity is less during eyes opened condition.
3. The Clustering coefficient < C > is more for the eyes closed condition. The more the clustering coefficient the more is the connectivity of that graph. This also indicates that the connectivity is more in the eyes closed condition.
4. The average degree of the network < K > is more in the case of subject with his eyes closed which means that the former(eyes closed) is densely connected where as the later(eyes open) is sparsely connected.

V CONCLUSION

The functional connectivity in brain is analyzed using correlation, a linear technique and synchronization Likelihood(SL), a nonlinear technique. The functional connectivity is extracted using functional network based on the obtained adjacency matrix. The attributes of the functional network like average pathlength (Lavg),clustering coefficient(Cavg) and degree distribution are evaluated for both the cases. The power law is observed from the degree distribution results which conclude that the human brain network is scale free. The functional connectivity is studied and compared for varies cases of interest.. The connectivity is found more in the case of epileptic seizure compared to a normal recording and more in eyes closed state compared to eyes opened state. The limitation of linear technique and the advantage of nonlinear technique were explained clearly. All this work provides sufficient information for analyzing the connectivity of the human brain.

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