# A Comparative Study of Hexagonal and Extended Constellations in PAPR Reduction by Tone Injection Technique

Pooja G. Wadibhasme<sup>1</sup>, Abhay Satmohankar<sup>2</sup>, Santhosh Banoth<sup>3</sup>

<sup>1,2,3</sup> Department of Electronics Engineering, Wainganga College of Engineering and Management, Nagpur, India

Abstract-Orthogonal Frequency Division Multiplexing (OFDM) is efficient system mainly used for the transmission of simultaneous large data with higher data rate without signal interference. However, the main drawback of OFDM systems is the high peak-to average power ratio (PAPR), which leads to inferior performance inefficiency output and undesired noise at receiver end. Over the period several methods are designed to reduce PAPR. Out of these, Tone Injection (TI) technique has recently gain popularity for its efficient PAPR reduction capability without incurring data rate loss or extra side information. The basic idea of TI is to increase the constellation size so that each information unit can be mapped into one of several equivalent points, these extra degrees of freedom can be exploited for PAPR reduction. In this paper equivalent points in Extended Constellation (EC) and Hexagonal Constellation (HC) are employed for the Tone Injection of suitable frequency and phase. It is observed that both EC and HC tone injection methods significantly reduces the PAPR. The PAPR reduction by HC technique is better than EC due to symmetric distribution of points in the space of former.

*Index Terms*-Extended Constellation (EC) and Hexagonal Constellation (HC), Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Tone Injection (TI).

# I. INTRODUCTION

Recently, Orthogonal Frequency Division Multiplexing (OFDM) has been receiving considerable attention for high-speed wireless communication systems [1-3]. In OFDM system orthogonally located sub carriers are being used to carry the data from the transmitting device to the receiving device. OFDM gives the higher data rate, high spectral efficiency, immunity to frequency selective fading channel. It eliminates the intersymbol interference (ISI) and offers good protection with respect to noise and co-channel interfaces. Due to all these advantages OFDM is used on large scale for many applications which includes Digital Television and Audio Broadcasting, 3G and 4G Mobile communication using LTE, Wireless network communication such as WLAN and Wi MAX, Internet access with DSL etc.

However, one of the major drawbacks of OFDM is the high peak-to-average power ratio (PAPR) of the transmit signal [4, 5]. A number of approaches have been proposed to deal with the PAPR problem [6-10]. One class of PAPR reduction techniques uses nonbijective constellations [11, 12]. In a nonbijective constellation, N bits are mapped to more than 2N different signal points. Thus, a given set of data bits can be mapped to multiple constellation points. Thus, appropriately choosing the suitable constellation points among the allowable set of points, the PAPR can be significantly reduced without a data rate loss requiring extra side information. One or implementation of this idea is tone injection (TI) [13-17], which uses a cyclic extension of quadrature amplitude modulation (QAM) constellations to offer alternative encoding with a lower PAPR. However, the TI technique requires solving a hard integerprogramming problem, whose complexity grows exponentially with the number of subcarriers. Therefore, suboptimal solutions are typically used [18].

In this paper, we propose the use of extended and hexagonal constellation in the TI technique. We can have more signal points in a given area by using EC and HC instead of quadrature amplitude modulation (QAM) and this extra degree of freedom is utilized for PAPR reduction without increasing signal power in the TI technique [13]

II. PEAK-TO-AVERAGE POWER RATIO (PAPR) In a typical OFDM system, data symbols modulated by phase shift keying (PSK) or quadrature amplitude modulation (QAM) are transmitted independently on

the subcarriers. A complex baseband, x(t)

representation of OFDM signal consisting of N subcarriers is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j 2\pi k (\Delta f)t}, \ 0 \le t < NT,$$
(1)

where the vector,  $X = \{X_0, X_1, X_2, \dots, X_{N-1}\}T$ is the input data block, *N* is the number of subcarriers in the OFDM system, *j* is the complex quantity,  $\Delta f$ is the subcarrier spacing, and *NT* denotes the data block period. The PAPR of the transmitted signal is defined as

$$PAPR = \frac{0 \le t < NT |x(t)|^{2}}{\left(NT\right)^{-1} \int_{0}^{NT} |x(t)|^{2} dt}$$
(2)

where

$$\left(NT\right)^{-1}\int_{0}^{NT}\left|x(t)\right|^{2}dt$$

is the expectation value (average value). For practical OFDM system, x(t) is oversampled *L*-times. Therefore, the *L*-times oversampled OFDM signal can be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi nk/LN}, n = 0, 1, 2, \dots, LN-1$$
(3)

The PAPR computed from the *L*-times oversampled time domain signal samples can be formulated as

$$PAPR = \frac{0 \le n < LN - 1|x(t)|^2}{\left(NT\right)^{-1} \int_{0}^{NT} |x(t)|^2 dt}$$

Following [19], for  $L \ge 4$ , the model in (3) is accurate to approximate continuous-time PAPR.

(4)

## III. TONE INJECTION (TI) SCHEME

A. Extended Constellation (EC)

The main idea of the tone injection technique [13] is to expand the original QAM constellation with several equivalent points so that the same information can be mapped to several points. Thus, these extra degrees of freedom can be exploited to generate OFDM symbols with low PAPR. This method is called tone injection because replacing a basic constellation to a new larger constellation is equivalent to injecting a tone with appropriate frequency and phase in the OFDM signal.



Fig 1:An example of 16QAM constellation expansion with eight replications of a given symbol

For an *M*-ary square QAM, the real and imaginary

parts of  $X_k$  can take values from the set

$$\left\{\pm d/2, \pm 3d/2, \dots, \pm \left(\sqrt{M}-1\right)d/2\right\}$$
 (5)

where  $\sqrt{M}$  and *d* represent the number of levels per dimension and the minimum distance between constellation points, respectively. Mathematically, the objective of tone injection is to send the symbols

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left( X_k + p_k \cdot \Delta + jq_k \cdot \Delta \right) e^{\left(\frac{j2\pi nk}{LN}\right)}$$
(6)

where  $p_k$  and  $q_k$  are integers and  $(p_k + jq_k) \cdot \Delta_{can}$ be seen as the extra freedom constellation. In order not to increase BER at the receiver, the value of  $\Delta$ should be at least  $d\sqrt{M}$  [13]. In order not to increase BER at the receiver, the value of  $\Delta$  should be at least  $d\sqrt{M}$  [13]. An example of 16-QAM constellation with eight replications of a given symbol is illustrated in Fig.1. Since each symbol can be mapped into one of nine equivalent constellation points, these extra freedom degrees can be exploited to reduce the PAPR. However, finding the optimal

constellation to obtain the lowest PAPR for  $X_n$  is a nondetermined-polynomial hard problem. Therefore, suboptimal solutions are required [18]. To prevent a greater power increase, only constellations located on the outer ring could be shifted, and the corresponding equivalent constellations are nearly symmetrical about the origin [12]. This extended constellation (EC) for 16- QAM is illustrated in Fig 1. Therefore, the transmitted signal with the modified TI scheme can be written as

$$x_n(\mathbf{b}) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} M(\mathbf{X}_k, b_k) e^{\left(\frac{j2\pi nk}{LN}\right)}$$
(7)

where  $\mathbf{b} = [b_0, \dots, b_{N-1}]$  is a binary selection sequence whose entries  $b_k \in \{0,1\}$  determine whether the corresponding signal symbol  $X_k$  is shifted or not, and  $M(\mathbf{X}_k, b_k)$  is given by

$$M(\mathbf{X}_{k}) = \begin{cases} S(\mathbf{X}_{k}) & k \in S_{EC} \\ \mathbf{X}_{k} & \text{otherwise} \end{cases}$$
(8)

where  $S_{EC}$  represents the index set of the TI subcarriers. Note that  $M(X_k)$  shows the new mapping relationship between the original QAM constellations

$$X_{k} = \left(\frac{d}{2}\right)p_{k} + j\left(\frac{d}{2}\right)p_{k} \tag{9}$$

and the corresponding equivalent point, which can be expressed as [12]

$$M = \begin{cases} -(d/2) p_{k} - j(d/2)M'', (p_{k} > -M', q_{k} = M') \\ -(d/2) p_{k} + j(d/2)M'', (p_{k} < -M', q_{k} = -M') \\ -(d/2)M'' - j(d/2)q_{k}, (p_{k} = M', q_{k} < M') \\ (d/2)M'' - j(d/2)q_{k}, (p_{k} = -M', q_{k} > M') \\ X_{k} & \text{otherwise} \end{cases}$$
(10)

where  $M' = \sqrt{M} - 1$  and  $M'' = \sqrt{M} + 1$ . As shown in Fig 1, the original constellation points in the outer ring are duplicated cyclically onto the surrounding region in the complex plane. Therefore, the modified constellation includes 12 alternative sub-constellation points and these points are spaced by the extension-size  $\Delta k$  along the real or imaginary axes (Fig 2).



Fig 2: The cyclically extended 16-QAM constellation diagram

In order to facilitate the analysis and design the proposed algorithm, considering that the extended constellation is nearly symmetric, the transmit signal in Fig. 2 can be written as

$$x_{n_{EC}} = x_{n_{EC}} - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\alpha_k X_{k_{EC}} + \Delta_k) e^{j2\pi nk/LN}$$
(10)

where  $x_{n_{EC}}$  is the same as  $x_n, \alpha_k$  and  $|\Delta_k|$  are defined as

$$\begin{cases} \alpha_k = 2, \ \Delta_k = d\sqrt{M}, \ k \in S_{EC} \\ \alpha_k = 0, \ \Delta_k = 0, \ \text{otherwise} \end{cases}$$
(11)

Note that only  $k \in S_{EC}$  contributes to the peak canceling signal.

B. Hexagonal Constellation (HC)

Hexagonal lattice is the densest packing of regularly spaced points in two dimensions. In addition, the average power is very important in the design of the hexagonal constellation.



# Fig 3: The 91 points hexagonal constellation

Therefore, in order to reduce the PAPR without increasing the signal power, a hexagonal constellation (HC) is introduced to TI [14]. In this case, the area of the decision region for each constellation point is  $\sqrt{3}d/2$  and the new transmit signal is given by

$$x_{n_{HC}} = x_{n_{HC}} - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\alpha_k \mathbf{X}_{k_{HC}}) \mathrm{e}^{j2\pi nk/LN}$$
(12)

where  $\alpha_k$  is defined in (14). Fig. 3 illustrates an example of the 91-points hexagonal constellation (91-HC). The points marked from '1' to '64' are utilized to take six information bits.

Furthermore, note that there are two representations for the symbols '38' to '64' in the outer ring, which can be used to reduce the PAPR. Thus, the 91-HC has the same throughput and minimum distance as the square 64-QAM. Since the ratio of the decision region between square 64-QAM constellation and the 91-HC can be calculated as  $d2/(\sqrt{3}d2/2) = 2/\sqrt{3}$ , the BER of the 91- HC is slightly worse than that of the square 64-QAM. In this case, the average power of the 91-HC points is 10.36d2 [14], while that of the square 64-QAM is 10.50d2. Therefore, there is no average power increase by using 91-HC instead of square 64-QAM. In addition, if we reduce the number of '2 representations' constellation points, the average power of 91-HC can be further reduced. According to Figs. 2 and 3, since only one equivalent constellation to choose to be shifted or not, the PAPR problem of EC and HC TI can be written as the following integer optimization problem

$$\min f(\alpha) = \left| x_n(\alpha_k) \right|^2, \ \alpha \in \{0, 2\}^N$$
(13)

The optimization problem (12) has been proved to be a non-determined-polynomial hard problem [2]. Suboptimal solutions, such as cross-entropy TI and parallel tabu search TI, are thus employed.

#### IV. PAPR REDUCTION

A. Index Set for EC-TI

In this section, we first determine S, the index set of EC-TI subcarriers which contribute to the peak canceling signal, and then introduce the proposed TI algorithm for PAPR reduction. According to (11), the size of  $S_{EC}$ , reads as

$$\left| x_{n_{EC}} \right| = \left| x_n - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\alpha_k \mathbf{X}_k + \Delta_k) e^{\frac{j2\pi nk}{LN}} \right|$$
$$\geq \left| x_n \right| - \frac{1}{\sqrt{N}} \sum_{k \in \mathcal{S}_{EC}} \left| 2\mathbf{X}_k + \Delta_k \right|$$
(14)

In order to satisfy the PAPR restriction ( $\chi_{n_{EC}}$  smaller than the PAPR threshold), a necessary condition is that S<sub>EC</sub> must satisfy

$$\frac{1}{\sqrt{N}} \sum_{k \in S_{EC}} \left( 2 |\mathbf{X}_k| + |\Delta_k| \right) \ge |x_n|_{\max} - A \tag{15}$$

where *A* threshold value, is determined by the saturation level of the power amplifier. For further simplification,  $X_k$  is replaced by its mean value. Let  $\overline{\varepsilon}_{EC} = E\{|X_k|\}$ , then for square M-ary QAM [20-21]

$$\bar{\varepsilon}_{EC} = \frac{4}{M} \sqrt{\frac{6}{M-1}} \sum_{p=1,q=1}^{\frac{M/2}{2}} \sqrt{\left(p - \frac{1}{2}\right)^2 + \left(q - \frac{1}{2}\right)^2}$$
(16)

Therefore, the minimum size SEC of the EC-TI scheme that satisfies (16) can be calculated as

$$N_{S_{EC}} = \left[\frac{\sqrt{N}\left(\left|x_{n}\right|_{\max} - A\right)}{2\overline{\varepsilon}_{EC} + \left|\Delta_{k}\right|}\right]$$
(17)

where  $|x_n|$  represents the smallest integer greater than  $x_n$ .

Clipping method limits the peak envelope of the input signal to a predetermined threshold *A*, Thus, the clipping noise can be calculated as

$$f_n' = \begin{cases} x_n - Ae^{j\phi} & |x_n| > A\\ 0 & |x_n| \le A \end{cases}$$
(18)

where  $\phi$  represents the phase of  $x_n$ . In this paper, we take the entire samples of peaks higher than A as the clipping noise [20-21]

$$f_n = \begin{cases} x_n & |x_n| > A \\ 0 & |x_n| \le A \end{cases}$$
(19)

The difference between (19) and (20) is that the former only contains the part of samples larger than A, while the latter contains the whole samples that exceed A. Although both equations, (19) and (20) can work with the proposed algorithms. In order to obtain the peak canceling signal, we can project the frequency domain clipping noise  $F_k$  to  $X_k$ 

$$f_{n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (P_{k} \mathbf{X}_{k}) e^{j2\pi nk/LN}$$
(20)

where  $P_k = R_e \left[ P_k X_k^* \right] / |X_k|^2$ ,  $R_e$  represents the real part of x, and (·)\* is the complex conjugate operation.

To further minimize the transmit signal,  $f_n$  can be scaled by a factor  $\beta$ ,

$$x_{n} = x_{n} - \beta f_{n} = x_{n} - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} (\beta P_{k} X_{k}) e^{\frac{j2\pi nk}{LN}}$$
(21)

Here, we choose the suboptimal solution [6] by minimizing the out-of-range power to obtain the optimal  $\beta$ ,

$$\overset{\min}{\beta} \sum_{|x_n| > A} \left( \left| x_n \right| - A \right)^2$$
(22)

If  $\beta$  is a real number, the optimal solution can be calculated as

$$P_{k} = R_{e} \left[ \sum f_{n} f_{n}^{*} \right] / \sum \left| f_{n} \right|^{2}$$
(23)

The mean rounding error of (11) and (22) is upper bounded as

$$\varepsilon_{EC} = E\left\{ \left\| x_{n_{EC}} - x_{n} \right\| \right\}$$

$$= \frac{1}{LN} \sum_{n} \frac{1}{\sqrt{N}} \left\| \sum_{\substack{k \in S_{EC}}} \left[ \left( 2 - \beta P_{k} \right) X_{k} + \Delta_{k} \right] e^{\frac{j2\pi nk}{LN}} \right\|$$

$$- \sum_{\substack{k \notin S_{EC}}} \beta P_{k} X_{k} e^{\frac{j2\pi nk}{LN}} \right\|$$

$$\leq \frac{\left[ \sum_{\substack{k \in S_{EC}}} \left[ \left| \left( 2 - \beta P_{k} \right) X_{k} \right| + \left| \Delta_{k} \right| \right] + \sum_{\substack{k \notin S_{EC}}} \beta P_{k} X_{k} \right]}{\sqrt{N}} \qquad (24)$$

By minimizing the mean rounding error  $\mathcal{E}_{EC}$  to achieve the optimal position of  $S_{EC}$ .

B. Index Set of HC-TI

The difference between extended and hexagonal constellation is that the former is nearly symmetric, while the latter is perfectly symmetric. With the similar analysis of EC-TI, the index set ( $S_{HC}$ ) of HC-TI subcarriers for PAPR reduction according, reads as

$$\begin{vmatrix} x_{n_{HC}} \end{vmatrix} = \begin{vmatrix} x_{n_{HC}} - \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \alpha_k X_{k_{HC}} e^{j2\pi nk/LN} \end{vmatrix}$$
$$\geq \begin{vmatrix} x_{n_{HC}} \end{vmatrix} - \frac{1}{\sqrt{N}} \sum_{k \in S_{EC}} \begin{vmatrix} 2X_{k_{HC}} \end{vmatrix}$$
(25)

and the necessary condition satisfies the PAPR restriction can be calculated as

$$\frac{1}{\sqrt{N}} \sum_{k \in S_{HC}} \left| 2X_{k_{HC}} \right| \ge \left| x_{n_{HC}} \right|_{\max} - A$$

Finally, the minimum size of  $S_{HC}$  in HC-TI is given by

$$N_{S_{HC}} = \left[\frac{\sqrt{N} \left|x_{n_{HC}}\right|_{\max} - A}{2\overline{\varepsilon}_{HC}}\right]$$
(27)

(26)

where  $\overline{\varepsilon}_{HC} = E\{X_{k_{HC}}\}$ . Similar to EC-TI, the position of  $S_{HC}$  in HC-TI is also made up of the smallest as

$$\varepsilon_{HC} = E\left\{ \left\| x_{n_{HC}} - \dot{x_{n}} \right\| \right\}$$

$$= \frac{1}{LN} \sum_{n} \frac{1}{\sqrt{N}} \left\| \sum_{\substack{k \in S_{HC}}} \left[ \left( 2 - \beta P_{k} \right) X_{k_{HC}} + \Delta_{k} \right] e^{\frac{j2\pi nk}{LN}} - \sum_{\substack{k \notin S_{HC}}} \beta P_{k} X_{k_{HC}} e^{\frac{j2\pi nk}{LN}} \right]$$

$$\leq \frac{\left[ \sum_{\substack{k \in S_{HC}}} \left[ \left| \left( 2 - \beta P_{k} \right) X_{k_{HC}} \right| + \left| \Delta_{k} \right| \right] + \sum_{\substack{k \notin S_{HC}}} \beta P_{k} X_{k_{HC}} \right]}{\sqrt{N}}$$

$$(28)$$

PAPR performance by EC and HC tone injection has been evaluated by calculating complementary cumulative distributive function (CCDF) by designed suitable algorithm in Matlab environment. Results obtained from computer simulation are depicted in Fig 4 and Fig 5.

## V. SIMULATION RESULT

The performance of the proposed modified PAPR Reduction techniques is evaluated by Complementary Cumulative Distribution Function (CCDF) of PAPR with respect to threshold  $PAPR_0$ . The CCDF or  $Pr[PAPR > PAPR_0]_{denotes}$  the probability of the signals having a PAPR greater than threshold  $PAPR_0$ . The result shown in Fig 4 is corresponding to  $N_{sym} = 1024$  (carrier frequency),  $N_{sub} = 64$  (number of subcarrier), L=4 (oversampling factor), M=16(data block size) and 16-QAM modulation is used on each subcarrier. Programme has been executed for various values of iteration numbers, that is, T = 4, T = 3, T = 2.Result depicted in Fig 5 is obtained for  $N_{sub} = 128$  keeping all other parameters identical as applied in first case. Comparative results of PAPR by EC and HC tone injection techniques at  $CCDF = 10^{-3}$  are shown in Table 1 and 2.



Table	1:	Comparative	result	of	PAPR	at	different
numbe	ers (	of iteration for	$N_{sub} =$	= 64			

TI	PAPR	TI	PAPR	HC-EC		
EC2	8.1	HC2	6.3	1.8		
EC3	7.3	HC3	6.1	1.2		
EC4	7.0	HC4	5.3	1.7		

Table 2: Comparative result of PAPR at different numbers of iteration FOR  $N_{sub} = 128$ 

Π	PAPR	TI	PAPR	HC-EC
EC2	7.6	HC2	5.8	1.8
EC3	6.8	HC3	5.6	1.2
EC4	6.6	HC4	4.8	1.7

From above figures and tables, it is observed that PAPR has significantly lowered for higher number of subcarriers. There is lowering of PAPR by the approximately magnitude of 0.5 dB as the number of subcarriers are increased from 64 to 128. The simulation result also shows that PAPR is lower for Hexagonal Constellations (HC) than Extended Constellations (EC) by the factor of 1.2-1.8 dB. Additional information generated from the simulation plot is that PAPR is lower if one allows more numbers of iteration. Although the exact trend in reduction of PAPR magnitude cannot be predicted as it is decreasing randomly with increased in iteration number.

# VI .CONCLUSION

In this paper how the module of Tone Injection (TI) works in the reduction of peak-to-average power ratio (PAPR) has been discussed in considerable details. To grasp the finer details of proposed exercise TI explore clipping noise to find the optimal equivalent outer ring extended constellations (EC) and hexagonal constellations (HC). For this purpose, a suitable algorithm in Matlab environment has been designed and executed programme for varying number of subcarriers, transmit antennas and iteration number. The proposed method (EC and HC) reveals efficient performance on PAPR reduction. The PAPR reduction performance of TI method employing

Hexagonal Constellations (HC) better than Extended Constellations (EC) has also been established.

# REFERENCES

- R. Van Nee and R. Prasad, OFDM for Wireless Multimedia Communications, Boston, MA: Artech House, 2000.
- [2] J. A. C. Bingham, "Multicarrier modulation for data transmission: an idea whose time has come," IEEE Commun. Mag., Vol. 28, pp. 5-14, May 1990.
- [3] S. H. Han and J. H. Lee, "An overview of peakto-average power ratio reduction techniques for multicarrier transmission," IEEE Wireless Commun., Vol. 12, No. 2, pp. 56–65, Apr. 2005.
- [4] S. H. Muller, R.W. Bauml, R. F. H. Fischer, and J. B. Huber, "OFDM with reduced peak-toaverage power ratio by multiple signal representation," Annals of Telecomm, Vol. 52, No. 1-2, pp. 58–67, Feb. 1997.
- [5] L. Wang and C. Tellambura, "An overview of peak to average power ratio reduction techniques for OFDM systems," Proc. IEEE ISSPIT'06, pp. 840-845, Aug. 2006.
- [6] L. Wang and C. Tellambura, "Clipping-noise guided sign-selection for PAR reduction in OFDM systems," IEEE Trans. Signal Process", Vol. 56, No. 11, pp. 5644-5653, Nov. 2008.
- [7] L. Wang and C. Tellambura, "Analysis of clipping noise and tone reservation algorithms for peak reduction in OFDM systems," IEEE Trans. Veh. Technol, Vol. 57, No. 3, pp. 1675-1694, May 2008.
- [8] S. H. Han and J. H. Lee, "PAPR reduction of OFDM signals using a reduced complexity PTS technique," IEEE Sigal Processing Lett., Vol.11, pp. 887–890, Nov. 2004.
- [9] T. Jiang and Y. Wu, "An overview: peak-toaverage power ratio reduction techniques for ofdm signals," IEEE Transactions on broadcasting, Vol. 54, No. 2, p. 257, 2008.
- [10] L. J. Cimini, Jr. and N. R. Sollenberger, "Peakto-average power ratio reduction of an OFDM signal using partial transmit sequences," IEEE Commun. Lett., Vol. 4, pp. 86–88, Mar. 2000.
- [11] B. S. Krongold and D. L. Jones, "PAPR reduction in OFDM via active constellation extension," IEEE Trans.Broadcast., Vol. 49, No. 3, pp. 258–268, Sept. 2003.

- [12] M. Ohta, Y. Ueda, and K. Yamashita, "PAPR reduction of OFDM signal by neural networks without side information and its FPGA implementation," Inst. Elect. Eng. J. Trans. Electron. Inf. Syst., Vol. 126, No. 11, pp. 1296-1303, Nov. 2006.
- [13] J. Tellado, "Peak to average power reduction for multicarrier modulation," Ph.D. Dissertation, Stanford Univ., Stanford, CA, Sep. 1999.
- [14] S. H. Han, J. M. Cioffi, J. H. Lee, "Tone injection with hexagonal constellation for peakto-average power ratio reduction in OFDM," IEEE Commun. Lett., Vol. 10, No. 9, pp. 646-648, Sep. 2006.
- [15] C. Tuna and D. L. Jones, "Tone injection with aggressive clipping projection for OFDM PAPR reduction," in Acoustics Speech and Signal Processing (ICASSP), 2010 IEEE International Conference on. IEEE, 2010, pp. 3278–3281.
- [16] T. Wattanasuwakull and W. Benjapolakul, "Papr reduction for ofdm transmission by using a method of tone reservation and tone injection," in Information, Communications and Signal Processing, 2005 Fifth International Conference on IEEE, 2005, pp. 273–277.
- [17] C. Tuna and D. L. Jones, "Tone injection with aggressive clipping projection for OFDM PAPR reduction," in Proc. IEEE ICASSP'2010, Dallas, TX, United states, 2010.
- [18] P.N. Kota, Priyanka Vilas Shivasharan, "Comparative Study Of Suboptimal Algorithms For Tone Injection", IJEEDC, Vol.-2, March 2014.
- [19] C. Tellambura, "Computation of the continuoustime PAR of an OFDM signal with BPSK subcarriers," IEEE Commun. Lett., Vol. 5, No. 5, pp. 185-187, May 2001.
- [20] F. Glover and M. Laguna, Tabu Search. Norwell, MA: Kluwer Academic Publishers, 1997.
- [21] T. Matsumura, M. Nakamura, S. Tamaki, K. Onaga, "A parallel Tabu search and its hybridization with genetic algorithms," in Proc. ISPAN' 2000, pp. 18-22, Dec. 2000.