Comparison of Inverse Kinematic Analysis of Revolute Robotic Arm for Pythagoras's Theorem, Geometric Solution and Algebraic Solution

Savan A Patel[1], Bhavik A. Patel[2], Harshit T. Shah[3]

[1,2]*Student, Mechanical Engineering Department, Faculty of Engineering Technology and Research, Isroli, Bardoli*

[3]*Professor, Mechanical Engineering Department, Faculty of Engineering Technology and Research, Isroli, Bardoli*

***Abstract*- These days robot arms are essential and critical tools in industries for high accuracy and precision by high speed manufacturing systems. One among the most challenging issues in industrial robots is determination inverse kinematics. This paper presents the idea of inverse kinematics analysis, with few different methods. The kinematics problem is outlined because the transformation from the robot's end-effector in Cartesian space to the joint angle of the robotic arms plays important role in functioning of the robot. In this present work, comparison of 3 methods is done to interpret inverse kinematic problem.**

INTRODUCTION

Every day, the importance of arms is increasing in industry and medical applications. It's more accurate and stable for high speed manufacturing [1]. As a result of the development in semiconductor manufacturing and micro assembly of very small products, robotics researches became very important. Sometimes accuracy of assembling tends to be less than 1 mm which requires high accuracy. However Programing these robots always suffers from accuracy problem[2]

The robot kinematics can be divided into forward kinematics and inverse kinematics. Forward kinematics problem is straightforward and there is no complexity deriving the equations. Hence, there is always a forward kinematics solution of a manipulator. Inverse kinematics is a much more difficult problem than forward kinematics. The solution of the inverse kinematics problem is computationally expansive and generally takes a very long time in the real time control of manipulators. Singularities and nonlinearities that make the problem more difficult to solve. [3] The relationship between forward and inverse kinematics is illustrated in figure (1). [2][3]

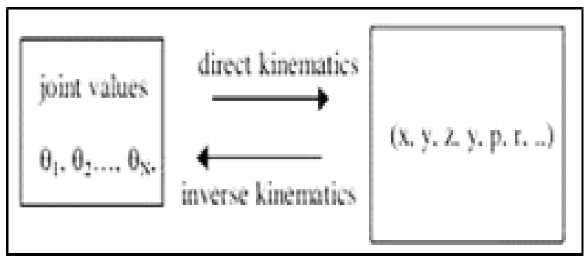

Figure 1.forward and inverse kinematic

Consider once again the door example of Figure, consisting of a single rigid body connected to a wall by a hinge joint. We know that the door has only one degree of freedom, conveniently represented by the hinge joint angleθ. Without the hinge joint, the door would be free to move in three-dimensional space and would have six degrees of freedom. By connecting the door to the wall via the hinge joint, five independent constraints are imposed on the motion of the door, leaving only one independent coordinate (θ). Alternatively, the door can be viewed from above and regarded as a planar body, which has three degrees of freedom. The hinge joint then imposes two independent constraints, again leaving only one independent coordinate (θ). The door's C-space is represented by some range in the interval (0, 2π) over which θ is allowed to vary.

In both cases the joints constrain the motion of the rigid body, thus reducing the overall degrees of freedom. This observation suggests a formula for determining the number of degrees of freedom of a robot, simply by counting the number of rigid bodies and joints. In this section we derive precisely such a formula, called Grubler's formula, for determining the number of degrees of freedom of planar and spatial robots.in figure(2) shows workspace degree of freedom. [4]

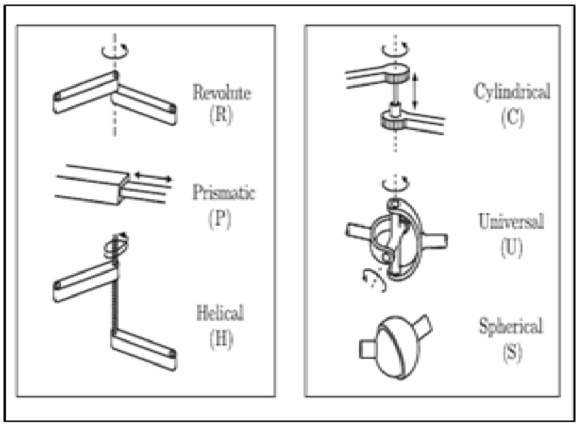

Figure 2 work-space degree of freedom

B. Inverse Kinematics

The inverse kinematics problem of the serial manipulators has been studied for many decades. It is needed in the control of manipulators. Solving the inverse kinematics is computationally expansive and generally takes a very long time in the real time control of manipulators. Tasks to be performed by a manipulator are in the Cartesian space, whereas actuators work in joint space. Cartesian space includes orientation matrix and position vector. However, joint space is represented by joint angles[5]. The conversion of the position and orientation of a manipulator end-effector from Cartesian space to joint space is called as inverse kinematics problem.

The inverse kinematics problem is vice-versa process finding such links configuration for which gripper matches thegiven position and orientation.

There are several ways to solve the inverse kinematics problem. Closed-form solutions are algebraic and geometric[6] Basic numerical ones are the following: the Jacobian transpose method , the Pythagoras's theorem method , the pseudoinverse method, cyclic coordinate descent methods , the Levenberg-Marquardt damped least squares methods , quasi-Newton and conjugate gradient methods , neural net and artificial intelligence methods , and the singular value decomposition.

In these paper we deal compare kinematic problem with different method to fined IK solution. [7]

II.METHODOLY TO FIND IK PROBLEMS

In these paper we present three different method to solve inverse kinematic problem we get the result and compare the each method and get final conclusion.[8]

A. Mathematical Model of Robot Kinematic

Taking into consideration the way the three jointed robot arm of the engineered, the mechanical structure is designated as anthropomorphic articulate (having human-like characteristics).Every joint have one degree of freedom.[9]

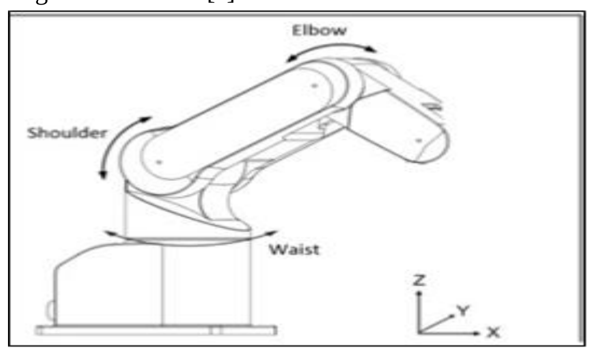

Figure 3 assigned name convention of three joint robot arm[9]

As shown in figure (3) shows the assigned naming convention. Planex-y-z axis are show

B. Pythagoras's Theorem

Referring to the top view of the robot as shown in Figure (4), the waist joint angle of θ1 can be easily resolved. Additionally, it can be seen that wherever the robot moves in any Cartesian position that is permissible for the robot, the waist joint angle could always be calculated by using the same technique that will be discussed in this section

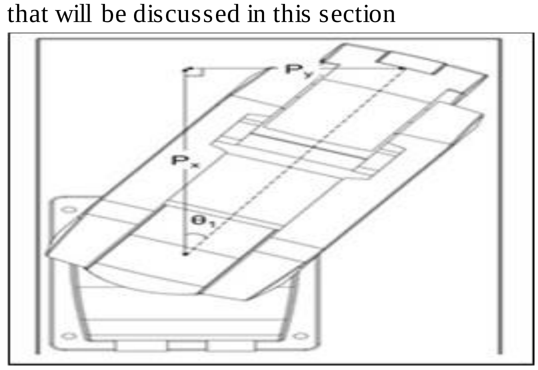

Figure 4. Top view of the robot

Applying trigonometric equation to find θ1.

$$\theta 1 = \tan^{-1}\left(\frac{Py}{Px}\right)$$

In figure (5) shows the front view of robot for findingθ2,θ3 ,θ4. Refer some reference coordinate the fined angle.

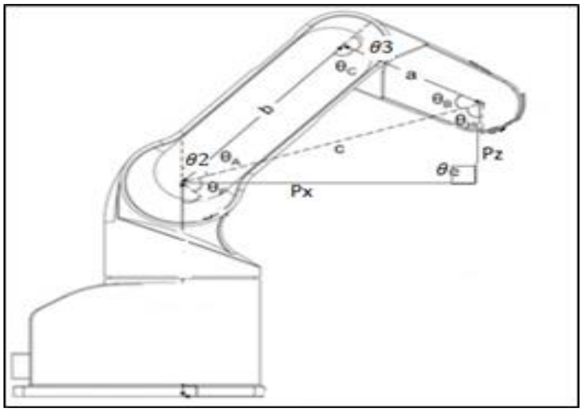

Figure 5. Front view of robot.

Applied law of sine used,

$$\frac{a}{\sin\theta A} = \frac{b}{\sin\theta B} = \frac{c}{\sin\theta C}$$

Now a cosine law,

$$\theta_C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

Onfigure (5). We write

$\theta 3 = 180 - \theta c$

To acquire θA and θB

$\theta_{A = \sin^{-1}\left(\frac{a}{c}\sin\theta_C\right)}$

$\theta_{B = \sin^{-1}\left(\frac{b}{c}\sin\theta_C\right)}$

Next to solve triangle,

$\theta_{P = \tan^{-1}\frac{P_Z}{P_X}}$

Consequently, the remaining joint angles of $\theta 2$

$\theta 2 = 180 - \theta_A - \theta_P$

The inverse kinematics of the robot has been implemented and simulation study has been performed using the MATLAB program. To find out all the angle, with respect to random positions of the x,y, z axis. While Py is fixed so take is zero.

C. Geometric Solution Approach

Geometric solution approach is based on decomposing the spatial geometry of the manipulator into several plane geometry problems.It is applied to2-DOF planer manipulator of the simple robot structures.Figure 6(a). Consider Figure 6(b) in order to derive the kinematics equations for the planar manipulator. The components of the point P (Px and Py) are determined as follows[10]

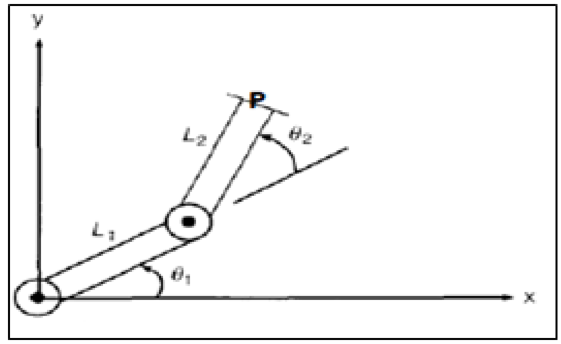

Figure 6(a).2DOF planer manipulator

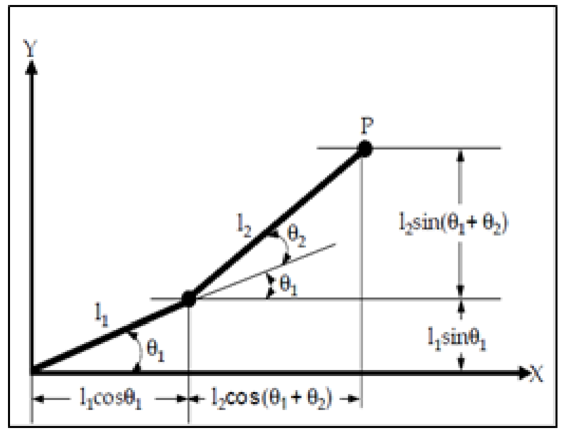

Figure 6(b).2DOF planer manipulator[10]

As shown in figure6 (b) write

$Px = L1C\theta 1 + L2C\theta 12$

$Py = L1S\theta 1 + L2S\theta 12$

Where $C\theta 12 = C\theta 1C\theta 2 - S1S\theta 2$ and $S\theta 12 = S\theta 1C\theta 2 + C\theta 1S\theta 2$

Now

$$P^2x + P^2y = L^2 1(C^2\theta 1 + S^2\theta 1) + L^2 2(C^2\theta 12 + S^2\theta 12) + 2L1L2(C\theta 1C\theta 12 + S\theta 1S\theta 12)$$

But w.k.t. $C^2\theta 1 + S^2\theta 1 = 1$

So,

$P^2x + P^2y = L_1^2 + L_2^2 + 2L1L2C\theta 2$

And so,

$$C\theta 2 = \frac{P^2x + P^2y - L_1^2 - L_2^2}{2L1L2}$$

Similarly

$$S\theta 2 = \pm\left[1 - \left(\frac{P^2x + P^2y - L_1^2 - L_2^2}{2L1L2}\right)^2\right]^{1/2}$$

Finally, two possible solution for$\theta 2$,

$$\theta 2 = Atan2\left(\pm\left[1 - \left(\frac{P^2x + P^2y - L_1^2 - L_2^2}{2L1L2}\right)^2\right]^{1/2}, \left[\frac{P^2x + P^2y - L_1^2 - L_2^2}{2L1L2}\right]\right)$$

Find the solution for $\theta 1$ in terms of link parameters, and the known variable $\theta 2$

$C\theta 1 P^2 x + S\theta 1 PxPy = P_x(L1 + L2C\theta 2)$

$-S\theta 1 PxPy + C\theta 1 P^2 y = PyL2S\theta 2$

$C\theta_1(P^2 x + P^2 y) = P_x(L1 + L2C\theta 2) + PyL2S\theta 2$

And so,

$$C\theta 1 = \frac{P_x(L1 + L2C\theta 2) + PyL2S\theta 2}{P^2 x + P^2 Y}$$

Similarly w.k.t,

$$S\theta 1 = \pm\left[1 - \left(\frac{P_x(L1 + L2C\theta 2) + PyL2S\theta 2}{P^2 x + P^2 Y}\right)^2\right]^{1/2}$$

As a result, two possible solutions for θ1 can be written

$$\theta 1 = Atan2\left(\pm\left[1 - \left(\frac{P_x(L1 + L2C\theta 2) + PyL2S\theta 2}{P^2 x + P^2 Y}\right)^2\right]^{1/2}, \left[\frac{P_x(L1 + L2C\theta 2) + PyL2S\theta 2}{P^2 x + P^2 Y}\right]\right)$$

Although the planar manipulator has a very simple structure, as can be seen, its inverse kinematics solution based on geometric approach is very cumbersome.

D. Algebraic Solution Approach

For the more than two links manipulator and whose arm are three dimension the geometric approach is not fined & complicated ,there the algebraic approach can be used for to find inverse kinematic problem .The coordinate frame assignment is depicted in figure (7),

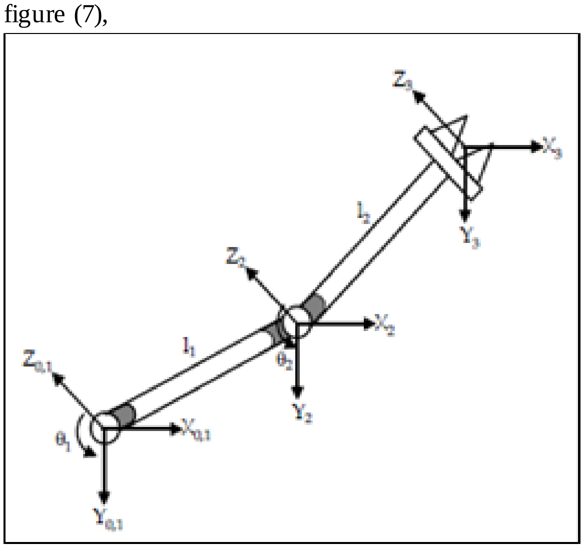

Figure7. Coordinate frame of robot arm[11]

Let us consider the equation of the position and orientation of the end-effector with respect to the base to solve the inverse kinematics of the 2-DOF manipulator.

$$ {}^0_3T = \begin{bmatrix} r11 & r12 & r13 & Px \\ r21 & r22 & r23 & Py \\ r31 & r32 & r33 & Pz \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^0_1T\,{}^1_2T\,{}^3_2T$$

Multiply both side ${}^1_0T^{-1}$ so,

Where, ${}^1_0T^{-1} = \begin{bmatrix} {}^0_1R^T & -{}^0_1R^T P_1 \\ 000 & 1 \end{bmatrix}$

And also knowthe, ${}^1_0T^{-1}\,{}^0_1T = 1$

So, ${}^1_0T^{-1}\,{}^0_3T = {}^1_2T\,{}^3_2T$

Substituting link transformation matrix

$$\begin{bmatrix} C\theta 1 & S\theta 1 & 0 & 0 \\ -S\theta 1 & C\theta 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r11 & r12 & r13 & Px \\ r21 & r22 & r23 & Py \\ r31 & r32 & r33 & Pz \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} C\theta 2 & -S\theta 2 & 0 & 0 \\ S\theta 2 & C\theta 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Simplify this matrix after these, squaring the (1,4) and (2,4) matrix element of both side and then adding the resulting equation

$P^2 x + P^2 y = L_1^2 + L_2^2 + 2L1L2C\theta 2$

So,

$$C\theta 2 = \frac{P^2 x + P^2 y - L_1^2 - L_2^2}{2L1L2}$$

Finally, two possible solution for$\theta 2$,

$$\theta 2 = Atan2\left(\pm\left[1 - \left(\frac{P^2 x + P^2 y - L_1^2 - L_2^2}{2L1L2}\right)^2\right]^{1/2}, \left[\frac{P^2 x + P^2 y - L_1^2 - L_2^2}{2L1L2}\right]\right)$$

Using the trigonometric to find the 1,

$\theta 1 =$

$Atan2(Py\ ,Px)\ \pm$

$Atan2\left(\sqrt{P_y^2 + Px - (l2C\theta 2 + l1)^2}\ , l2C\theta 2 + l1\ \right)*$

[*It should be noted that for all solution in form$-\frac{\pi}{2} < \theta 1 < \frac{\pi}{2}$]

III. CONCLUSIONS AND FUTURE RESEACH

One of the most important problem in new industries using robots is inverse kinematic problem. With increase in number of degree of freedom,it's difficult to solve inverse kinematic problem.Inverse kinematics is the mathematical process of recovering the movements of an object in the world from some other data, such as a film of those movements which is itself making those movements due to this controlling the robot easy and workspace increase. This is useful in robotics and in film animation. For solving this problem we have reviewed the 3 DOF freedom revolute robot arms using three different methods which are presented in this work for solving inverse kinematic problem. We have simplified two links, to find IK.

All simulations can obtained using MATLAB. Finally through the different three methods, Pythagoras's Theorem is more suitable method to find IK. But still one can also choose the most suitable method for inverse kinematics depending on the field of real robot arm applications. This choice can save the calculation time to required level of accuracy.

REFERENCES

[1] Ali T. Hasan, A.M.S. Hamouda, N. Ismail, H.M.A.A and Al-Assadi, "An adaptive-learning algorithm to solve the inverse kinematics," Advances in Engineering Software, pp. 432-438, 2006.

[2] Weidong Wu, and S.S. Rao, "Uncertainty analysis and allocation of joint tolerances in robot," Reliability Engineering and System Safety, pp. 54-64, 2005.

[3] R.C. Luo and K. L. Su,, "a multiphase," science, pp. 11-33, 2017.

[4] Kevin M. Lynch and Frank C. Park, MODERN ROBOTICS MECHANICS, PLANNING, AND CONTROL, Cambridge University, 2017.

[5] M. Andy McEvoya and Nikolaus Corrella, "Distributed Inverse Kinematics for Shape-Changing Robotic," Procedia Technology, pp. 4-11, 2016.

[6] Albert castellet and Federico Thoms, "Using Interval Methods for solving Inverse kinematic Problem".

[7] FrancoBlanchini, GianfrancoFenu, GiuliaGiordano and FeliceAndreaPellegrino, "A convexprogrammingapproachtotheinversekinematicsproblem," EuropeanJournalofControl, pp. 11-23, 2016.

[8] Samuel R. Buss, "Introduction to Inverse Kinematics with".

[9] Mohammad Afif Ayob1,, Wan Nurshazwani Wan Zakaria1,, Jamaludin Jalani2 and Mohd Razali Md Tomari1, "INVERSE KINEMATICS ANALYSIS OF A 5-AXIS RV-2AJ ROBOT," ARPN Journal of Engineering and Applied Sciences, 2015.

[10] V.N. Iliukhina,*, K.B. Mitkovskiib, D.A. Bizyanovaa and A.A. Akopyana, "The modeling of Inverse Kinematics for 5 DOF manipulator," Procedia Engineering, pp. 498-505, 2017.

[11] Gunesh R. Gogate, "Inverse kinematic and dynamic analysis of planar path," Mechanism and Machine Theory, pp. 103-122, 2016.

[12] Ahmed El-Sherbiny, Mostafa A. Elhosseini and Amira Y. Haikal, "A comparative study of soft computing methods to solve inverse," Ain Shams Engineering Journal, 2016.

[13] Dongsheng Zhang, Yundou Xu, Jiantao Yao, Bo Hu and Yongsheng Zhao, "Kinematics, dynamics and stiffness analysis of a novel 3-DOF kinematically/actuation redundant planar parallel mechanism," Mechanism and Machine Theory, pp. 203-219, 2017.