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Čech D-Closed Sets in Closure Spaces

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***Abstract*- In this paper we introduced the concept Čech D-closed and Čech D-open sets in Čech closure spaces and investigate their characterizations.**

***Index Terms*- Čech D-closed sets, Čech D-open sets.**

I. INTRODUCTION

Čech spaces were introduced by Eduard Čech[1] in 1996. Generalized closed sets in Čech closed spaces were introduced by Chawalit Boonpok[2], and studied by many authors[4][6].In this paper we introduce the concept of Čech D-closed sets and Čech D-open sets and discuss their properties.

II. PRELIMINARIES

A map $k:P(X)\to P(X)$ defined on the power set $P(X)$ of a set X is called a closure operator on X and the pair (X, k) is called a closure space if the following axioms are satisfied.

1. $k(\phi)=\phi$.
2. $A\subseteq k(A)$ for every $A\subseteq X$
3. $k(A\cup B)=k(A)\cup k(B)$ for all $A,B\subseteq X$.

A closure operator k on X is called idempotent if $k(A)=k[k(A)]$ for all $A\subseteq X$.

Definition 2.1:[1] A subset of a Čech-closure space (X,k) will be called Čech closed if k(A)=A and Čech open if its complement is closed. i.e., k(X-A)=X-A.

Definition 2.2:[9] A subset A of a Čech closure space (X,k) is said to be

1. Čech regular open if A= int (k(A)) and Čech Regular closed if A=k(int(A))
2. Čech pre-open if $A\subseteq int(k(A))$ and Čech pre-Closed if $k(int(A))\subseteq A$
3. Čech semi open if $A\subseteq k(int(A))$
4. Čech α-open if $A\subseteq int(k(int(A)))$ and Čech α- closed if $k(int(k(A))\subseteq A$
5. Čech β-open if $A\subseteq k(int(k(A)))$ and Čech β- Closed if $int(k(int(A))\subseteq A$

Definition 2.3:[10] Let (X, k) be a Čech closure space. A subset $A\subseteq X$ is called a Čech w-closed set if $k(A)\subseteq G$ whenever $A\subseteq G$ and G is semi-open Subset of (X,k).A subset A of X is called a w-open set if its complement is w-closed subset of (X, k).

Definition 2.4: Let (X,k) be a Čech closure space. A subset $A\subseteq X$ is called g-closed set if $k(A)\subseteq G$ whenever $A\subseteq G$ and G is Čech- open subset of (X, k).A subset $A\subseteq X$ is called a generalized open set, briefly a g-open set, if its complement is g-closed.

Definition 2.5:[5] Let (X,k) be a Čech closure space . A subset $A\subseteq X$ is called a Čech αψ-closed if $k(A)\subseteq G$ whenever $A\subseteq G$ and G is α-open subset of (X, k).

Definition 2.6:[1] Let (X,k) be a Čech closure space. A subset $A\subseteq X$ is called a J -Čech closed set if $k_\alpha(A)\subseteq G$, whenever $A\subseteq G$ and G is semi-open subset of (X,k),where $k_\alpha(A)$ is the smallest α-closed set containing A.

Definition 2.7:[6] Let (X,k) be a Čech closure space. A subset $A\subseteq X$ is called a Čech πgβ-closed set if $k(A)\subseteq G$ whenever $A\subseteq G$ and G is π-open subset of (X, k).

III.ČECH D-CLOSED SETS

Definition 3.1: Let (X, k) be a closure space. A subset $A\subseteq X$ is called a Čech D-closed set $k_\beta(A)\subseteq G$ whenever $A\subseteq G$ and G is pre-open subset of (X,k), where $k_\beta(A)$ is the smallest β-closed set containing A.

A subset A of X is called a Čech D-open if its complement is a Čech D-closed subset of (X, k).

Theorem 3.2: Every Čech closed set is Čech D-closed set.

Proof: Let G be a pre-open subset of (X, k) such that A ⊆ G. Since A is Čech closed k(A)=A. Therefore $k_\beta(A) \subseteq k(A) = A \subseteq G$. (i.e.,)$k_\beta(A) \subseteq G$, where G is pre-open. Therefore A is Čech D-closed set. Hence Every Čech closed set is D-closed set.

Remark 3.3: Converse of the above theorem need not be true which can be seen from the following example.

Example 3.4: Let X= {a,b,c,d}
And define a closure k on X by
k{ϕ}=ϕ,k{a}={a},k{b}=k{a,b}={a,b},k{b,c}= k{a,b,c}={a,b,c},k{c}=k{a,c}={a,c},k{d}=k{a,d}= k{a,b,d}={a,b,d},k{b,d}=k{c,d}=k{a,c,d}= k{b,c,d}=kX=X.
Čech closed sets of X: {ϕ,X,{a},{a,b},{a,c},{a,b,c},{a,b,d}}
Čech D-closed of X:
{ϕ,X,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d}, {a,b,c},{a,b,d}}
Then A={a,d} is Čech D-closed set but not Čech closed set.

Theorem 3.5:
(a)Every Čech w-closed set is Čech D-closed set.
(b)Every Čech g-closed set is Čech D-closed set.
(c)Every Čech αψ-closed set is Čech D-closed set.
(d)Every J-Čech closed set is Čech D-closed set.
(e)Every Čech πgβ-closed set is Čech D-closed set.

Proof: (a) Let A be a Čech w-closed set. Then $k_\beta(A) \subseteq G$ whenever A ⊆ G, G is pre-open in X. But $k(A) \subseteq k_\beta(A)$ whenever A ⊆ G,G is pre-open in G. Now we have $k_\beta(A) \subseteq G$,G is pre-open. Therefore A is Čech D-closed set.

Note: Proof is obvious for others.

Remark 3.6: Converse of the above theorem need not be true which can be seen from the following example.

Example 3.7: Let X={a,b,c,d}
And define a closure k on X by
k{ϕ}=ϕ,k{a}={a},k{b}=k{a,b}={a,b},k{b,c}= k{a,b,c}={a,b,c},k{c}=k{a,c}={a,c},k{d}=k{a,d}= k{a,b,d}={a,b,d},k{b,d}=k{c,d}=k{a,c,d}=

k{b,c,d}=kX=X.
Čech w-closed set of X: {ϕ,X,{a},{b},{a,b},{a,c},{b,c},{a,b,c},{a,b,d}}.
Čech D-closed of X:
{ϕ,X,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d}, {a,b,c},{a,b,d}}
Then A={a,d} is Čech D-closed set but not in Čech w-closed set.

Example 3.8:Let X={a,b,c,d}
And define a closure k on X by
k{ϕ}=ϕ,k{a}={a},k{b}=k{a,b}={a,b},k{b,c}= k{a,b,c}={a,b,c},k{c}=k{a,c}={a,c},k{d}=k{a,d}= k{a,b,d}={a,b,d},k{b,d}=k{c,d}=k{a,c,d}= k{b,c,d}=kX=X.
Čech g-closed set of X:
{ϕ,X,{a},{b},{a,b},{a,c},{a,d},{b,c},{a,b,c}, {a,b,d}}
Čech D-closed of X:
{ϕ,X,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d}, {a,b,c},{a,b,d}}
Then A={c} is Čech D-closed set but not in Čech g-closed set.

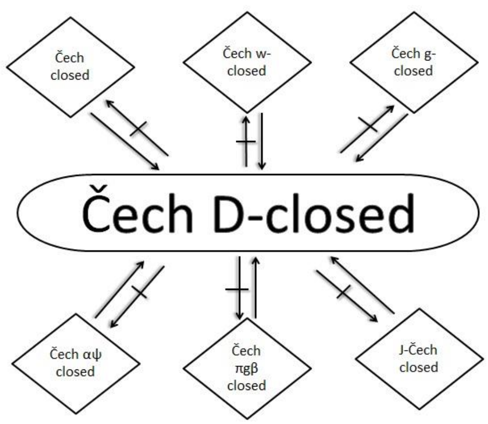
Example 3.9: Let X={a,b,c,d}
And define a closure k on X by
k{ϕ}=ϕ,k{a}={a},k{b}=k{a,b}={a,b},k{b,c}= k{a,b,c}={a,b,c},k{c}=k{a,c}={a,c},k{d}=k{a,d}= k{a,b,d}={a,b,d},k{b,d}=k{c,d}=k{a,c,d}= k{b,c,d}=kX=X.
Čech αψ-closed set of X:
{ϕ,X,{a},{b},{a,b},{a,c},{a,d},{b,c},{a,b,c}, {a,b,d}}
Čech D-closed of X:
{ϕ,X,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d}, {a,b,c},{a,b,d}}
Then A={b,d} is Čech D-closed set but not in Čech αψ-closed set.

Example 3.10: Let X={a,b,c}
And define the closure operator on X by
k{ϕ}=ϕ,k{a}={a},k{b}={b,c},k{c}={c}, k{a,c}={a,c}=kX=X.

J- Čech closed set of X :
{ϕ, X,{a},{c},{a,c}}
Čech D-closed set of X:
{ϕ,X,{a},{b},{c},{a,b},{a,c},{b,c}}
Then A={a,b} is Čech D-closed set but not in J-Čech closed set.

Example 3.11: Let X={a,b,c,d}
And define a closure operator on X by
k{ϕ}=ϕ,k{a}={a},k{b}=k{a,b}={a,b},k{b,c}= k{a,b,c}={a,b,c},k{c}=k{a,c}={a,c},k{d}=k{a,d}= k{a,b,d}={a,b,d},k{b,d}=k{c,d}=k{a,c,d}= k{b,c,d}=kX=X.
Čech πgβ-closed set of X:
{ϕ,X,{a},{b},{a,b},{a,d},{b,c},{a,b,c},{a,b,d}}
Čech D-closed set of X:
{ϕ,X,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d}, {a,b,c},{a,b,d}}
Then A={c} is Čech D-closed set but not in Čech πgβ-closed set.

Remark 3.12: From the above results we have the following implications.



Theorem 3.13: Let (X, k) be a closure space. If A and B are Čech D-closed subsets of (X, k) then A∪B is Čech D-closed set.

Proof: Let G be a pre-open subset of (X, k) such that A∪B⊆G, then A⊆G, B⊆G. Since A and B are Čech D-closed sets, $k_\beta(A) \subseteq G$ and $k_\beta(B) \subseteq G$ that implies $k_\beta(A) \cup k_\beta(B) \subseteq G$. Therefore (A∪B) is Čech D-closed set.

Remark 3.14: The intersection of two Čech D-closed sets need not be Čech D-closed as can be seen by the following example.

Example 3.15: Let X={a,b,c,d} And define the closure operator k on X by

k{ϕ}=ϕ,k{a}=k{d}=k{a,b}=k{a,d},=k{a,b,d}={a,b, d},k{b}={b},k{c}=k{b,d}=k{c,d}=k{b,c,d}={b,c,d}, k{a,c}= k{a,c,d}={a,c,d}=kX=X.
Čech D-closed set of X:
{ϕ,X,{b},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c}, {a,b,d},{a,c,d},{b,c,d}}
If A={a,b} and B={a,c} ,then A∩B={a,b}∩{a,c}={a} which is not Čech D-closed set.

Theorem 3.16: Let (X, k) be a Čech closure space and Let A⊆X. If A is a Čech D-closed set, then $k_\beta(A)$-A contains no non empty Čech pre-closed.

Proof : Let A be Čech D-closed set. Let F be a nonempty Čech pre-closed set$\subseteq k_\beta(A)$-A. That implies $F \subseteq k_\beta(A) \cap A^c$. (i.e.,)$F \subseteq k_\beta(A)$ and $F \subseteq A^c$. $F \subseteq A^c \Rightarrow A \subseteq F^c$.Since F is Čech pre-closed, F^c is Čech pre-open. Thus we have $k_\beta(A) \subseteq F^c$. Consequently $F \subset [k_\beta(A)]^c$.Hence we get $F \subseteq k_\beta(A) \cap [k_\beta(A)]^c = \phi$. Hence $k_\beta(A)$-A contains no non empty Čech pre-closed set.

Theorem 3.17: Let A be a Čech D-closed set. Then A is Čech β-closed if and only if $k_\beta(A)$-A is Čech pre-closed set.

Proof: Suppose that A is Čech D-closed set and Čech β-closed set. Since $A = k_\beta(A)$ we have $k_\beta(A) - A = \phi$, which is Čech pre-closed. Conversely, Suppose that A is Čech D-closed set and $k_\beta(A)$-A contains no non empty Čech pre-closed set. Then $k_\beta(A)$-A is itself Čech pre-closed $\Rightarrow k_\beta(A) - A = \phi$. Hence A is Čech pre-closed.

Proposition 3.18: Let (X,k) be a Čech closure space. If A is Čech D-closed and F is Čech pre-closed in (X, k) then A∩F is Čech D-closed.

Proof: Let G be a Čech pre-open subset of (X, k) such that A∩F ⊆ G, Then A⊆ G∪(X-F). And so, since A is Čech D-closed ,$k_\beta(A) \subseteq G \cup (X-F)$,Then $k_\beta(A) \cap F \subseteq G$, since F is Čech pre-closed, $k_\beta(A) \cap F \subseteq G$. Therefore A∩F is Čech D-closed.

Proposition 3.19: Let (Y, l) be a closed subspace of (X, k).If F is a Čech D-closed subset of (Y, l), then F is a Čech D-closed subset of (X, k).

Proof: Let G be a Čech pre-open set of (X, k) such that F⊆G. Since F is Čech D-closed and G∩Y is Čech pre-open k_β(F)∩Y ⊆ G, But Y is closed subset of (X, k) and k_β(F)⊆G, where G is a Čech pre-open set. Therefore F is a Čech D-closed of (X, k).

Proposition 3.20: Let (X, k) be a Čech closure spaces and let k be idempotent. If A is Čech D-closed subset of (X, k) such that A⊆B⊆ k_β(A),then B is a Čech D-closed subset of (X, k).

Proof: Let G be a Čech pre-open subset of (X, k) such that B⊆G. Then A⊆ G, since A is Čech D-closed, k_β(A)⊆G. As k is idempotent, k_β(B)⊆k_β(k_β(A))= k_β(A)⊆G. Hence B is Čech D-closed.

IV.ČECH D-OPEN SETS

Definition 4.1: A Subset A in Čech closure space (X,k) is called Čech D-open if its complement is Čech D-closed set.

Theorem 4.2: A subset A in Čech closure space (X,k) is called Čech D-open set if and only if F⊆X-k_β(X-A) whenever F is pre-closed and F⊆A.

Proof : Suppose that A is Čech D-open and F be a pre-closed subset of (X, k) such that A⊆F then X-A⊆X-F. But X-A is Čech D-closed set and X-F is pre-open. That implies k_β(X-A)⊆X-F. (i.e.,) F ⊆ X-k_β(X-A) .Conversely, Let F be a pre-closed set, F⊆ A and F⊆X-k_β(X-A) that implies k_β (X-A) ⊆ X-F, X-F is pre-open that implies X-A is Čech D-closed set and so A is Čech D-open.

Theorem 4.3: If A and B are Čech D-open subsets of (X, k) then A∩B is Čech D-open set.

Proof: Let F be a pre-closed subset of (X, k) such that F⊆ A∩B. Then X-(A∩B) ⊆ X-F. This implies that (X-A)∪(X-B)⊆X-F. (X-A)∪(X-B) is Čech D-closed set. Thus k_β[(X-A)∪(X-B)]⊆X-F. Hence k_β[X-(A∩B)]⊆X-F. That implies F⊆X-k_β[X-(A∩B)] that implies A∩B is Čech D-open.

Theorem 4.4: Let (X, g) be a closure space and let (Y, h) be a closed subspace of (X, g).If G is Čech D-open subset of (X, g) then G∩Y is Čech D-open subset of (Y, h).

Proof: Let G be a Čech D-open subset of (X, g).Then X-G is a Čech D-closed subset of (X, g).Since Y is a closed subset of (X, g), (X-G)∩Y is a Čech D-closed set of (X, g). But (X-G) ∩ Y=Y-(G∩Y). Therefore Y-(G∩Y) is a Čech D-closed subset of (X, g).Hence G∩Y is a Čech D-open subset of (X, g).

Proposition 4.5: Let (X,k) be Čech closure space. If A is D-open and B is pre-open in (X,k) then A∪B is D-open.

Proof: Let F be a pre-closed subset of (X, k) such that F⊆A∪B. Then X-(A∪B) ⊆ X-F. Hence (X-A) ∩ (X-B) ⊆ X-F. We have (X-A) ∩ (X-B) is D-closed. Therefore k [(X-A) ∩ (X-B)] ⊆ X-F. Consequently, F⊆X-k [(X-A) ∩ (X-B)] = X-k [X (A∩B)]. Since F⊆X-k [(X-A)], then A is D-open. Therefore A∪B is D-open.

Remark 4.6: The union of two D-open sets need not be D-open.

Example 4.7: Let X={a,b,c,d}
And define a closure k on X by
k{ϕ}=ϕ,k{a}={a},k{b}=k{a,b}={a,b},k{b,c}=
k{a,b,c}={a,b,c},k{c}=k{a,c}={a,c},k{d}=k{a,d}=
k{a,b,d}={a,b,d},k{b,d}=k{c,d}=k{a,c,d}=
k{b,c,d}=kX=X.
Čech D-open of X:
{ϕ,X,{c},{d},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,d},
{a,c,d},{b,c,d}}
Then A={a,c} and B={b,c} then A∪B={a,b,c,} which is not Čech D-open set.

Proposition 4.8: Let (X,k) be a Čech closure space. If A is D-open subset of (X,k) then X=G whenever G is pre-open and (X-k(X-A))∪(X-A)⊆G.

Proof: Suppose that A is D-open. Let G be an pre-open subset of (X,k) such that (X-k(X-A))∪(X-A)⊆G. Then X-G⊆X-[(X-k(X-A))∪(X-A)].Therefore X-G ⊆ k(X-A)∩A implies that X-G ⊆ k(X-A)-(X-

A).But X-G is pre closed and X-A is D-closed. Then by proposition 3.17 X- G=ϕ, consequently X=G.

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