Design of a Hyperbolic Shell Roof

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Abstract- Shell structures mobilize geometry to activate both the membrane and the flexural internal force systems to efficiently support any distributed loads applied to those structures. Based primarily on their efficiency, these geometric structural forms are employed in a number of applications. The use of hyperbolic paraboloids as a form of thin shell construction was pioneered in the post-war era, as a hybrid of modern architecture and structural engineering. By being both lightweight and efficient, the form was used as a means of minimizing materials and increasing structural performance while also being capable of achieving impressive and seemingly complex designs. By being braced in two directions they experience no bending and are able to withstand unequal loading, whether dead loads (such as equipment hung from the ceiling), or live loads (such as wind).

Index Terms- Shell, Hyperbolic, doubly surface, edge beam, ridge beam.

I. INTRODUCTION

A shell is a type of structural element which is characterized by its geometry, being a three dimensional solid whose thickness is very small when compared with other dimensions, and in structural terms, by the stress resultants calculated in the middle plane displaying components which are both coplanar and normal to the surface.

A shell can be derived from a plate by two means: by initially forming the middle surface as a singly or doubly curved surface, and by applying loads which are coplanar to a plate's plane which generate significant stresses.

These elements, typically curved, are assembled to make large structures. The ideal thin shell structures must be capable of developing both tension and compression.

The most popular types of thin-shell structures are

1. Concrete shell structures
2. Membrane structures
3. Lattice shell structures

In which hyperbolic paraboloid shells included in lattice shell structures type.

Hyperboloid structure

Hyperboloid structures are architectural structures designed using a hyperboloid in one sheet. The hyperboloid geometry's structural strength is used to support an object high off the ground. Hyperboloid geometry is also often used for decorative effect as well as structural economy.

The first hyperboloid structures were built by Russian engineer Vladimir Shukhov (1853–1939). Hyperbolic structures have a negative Gaussian curvature, meaning they curve inward rather than outward or being straight. As doubly ruled surfaces, they can be made with a lattice of straight beams, hence are easier to build than curved surfaces that do not have a ruling and must instead be built with curved beams. Hyperboloid structures are superior in stability towards outside forces compared to “straight” buildings.

A hyperbolic paraboloid is a doubly-curved surface that resembles the shape of a saddle, that is, it has a convex form along one axis, and a concave form on along the other. It is also a doubly-ruled surface, that
is, every point on its surface lies on two straight lines across the surface. Horizontal sections taken through the surface are hyperbolic in format and vertical sections are parabolic. Hyperbolic paraboloids (also known as hypar shells) are surfaces formed by two hyperbolas of opposite curvatures (concave and convex curvatures), one moving over the other forming translational shells of negative Gaussian curvature. The equation to the concave parabola is of the form $z = kx^2$.

The fact that hyperbolic paraboloids are doubly-ruled means that they are easy to construct using a series of straight structural members. As a consequence they are commonly used to construct thin ‘shell’ roofs. These can either be formed using timber or steel sections, that are then clad, or they can be constructed using concrete. The use of hyperbolic paraboloids as a form of thin shell construction was pioneered in the post-war era, as a hybrid of modern architecture and structural engineering. By being both lightweight and efficient, the form was used as a means of minimizing materials and increasing structural performance while also being capable of achieving impressive and seemingly complex designs.

Rather than derive their strength from mass, like many conventional roofs, thin shell roofs gain strength through their shape. The curvature of the shape reduces its tendency to buckle in compression (as a flat plane would) and means that they can achieve exceptional stiffness. By being braced in two directions they experience no bending and are able to withstand unequal loading, whether dead loads (such as equipment hung from the ceiling), or live loads (such as wind).

**II. DESIGN OF HYPAR SHELL ROOF**

\[
a = 9.595\text{m} \\
b = 14.5\text{m} \\
h = \frac{b}{5} = \frac{14.5}{5} = 2.9\text{m} \\
t = 100\text{mm}
\]

**STEP 1: LOADS**

- Dead load = 0.1 x $24^2$ = 2.4 kN/m$^2$
- Insulation = 0.2 kN/m$^2$
- Live load = 1.0 kN/m$^2$
- Total load = 3.6 kN/m$^2$

**STEP 2: SHEAR FORCE**

Shear force = $\frac{wab}{2h}$

\[
= \frac{3.6 \times 9.595 \times 6.15}{2 \times 1.23} = 86.355 \text{ kN/m}
\]

Stress = $\frac{86.355}{100} = 0.86355 \text{ N/mm}^2$

This will produce equal tension in the shell. As tension is greater than 0.5 N/mm$^2$ and even otherwise, we provide full steel for tension.

Area of steel, $A_S = \frac{86.355 \times 10^3}{230} = 375.46 \text{ mm}^2$/m

10mm dia at 200mm centre to centre.

**STEP 3: DESIGN OF SLOPING EDGE MEMBER 1:**

Edge beams can be formed by thickening of shells.

Length = $\sqrt{9.595^2 + 2.90^2} = 10.02\text{m}$

Compression = $86.355 \times 10.02 = 865.2771 \text{ kN}$

As compression varies and as member is constrained, slenderness need not to be considered.

For compression member, provide 1% steel.

Assume $f_c = 4 \text{ N/mm}^2$

\[
4A_C + 130 \left( \frac{A_C}{100} \right) = 865.2771 \times 10^3
\]

\[
A_C = 163259.83 \text{ mm}^2
\]

Depth should be > 4t, say 500mm

Breadth should be > 3t, say 350mm

Area is > 163259.83 mm$^2$

Steel area = $\frac{500 \times 350}{100} = 1750 \text{ mm}^2$

20mm dia at 150mm centre to centre.

**STEP 4: DESIGN OF EDGE SLOPING MEMBER 2:**
Length = $\sqrt{14.5^2 + 2.9^2}$ = 14.78m
Compressio = $86.355 \times 14.78$
= 1276.32 kN

As compression varies and as member is constrained, slenderness need not to be considered.
For compression member, provide 1% steel.
Assume $f_c$ = 4 N/mm$^2$

$4A_C + 130 \left(\frac{A_C}{100}\right) = 1276.32 \times 10^3$

$A_C = 240815.09 \text{ mm}^2$

Depth should be > 4t, say 500mm
Breadth should be > 3t, say 350mm
Area is > 240815.09 mm
Steel area $\frac{500 \times 350}{100} = 1750 \text{ mm}^2$

20mm dia at 150mm center to center.

**STEP 5: DESIGN OF RIDGE MEMBER 1:**
Total compression of ridge beam = $86.355 \times 10.02$
= 865.59 kN
Area of steel,

$A_{st} = \frac{865.59 \times 10^3}{230}$
= 3763.43 mm$^2$/m

Permissible stress for M30 grade concrete is 3.6 N/mm$^2$

$\frac{865.59 \times 10^3}{A_C + (9-1)3763.43}$ = 3.6 N/mm$^2$

$A_C = 210334.18 \text{ mm}^2$

Assume depth to be 450mm

$\frac{210334.18}{300} = 500\text{mm}$

Therefore the ridge beam 1 of size 500mmx450mm

**STEP 6: DESIGN OF RIDGE MEMBER 2:**
Total compression of ridge beam = 86.355X 14.78
= 1276.32 kN
Area of steel,

$A_{st} = \frac{1276.32 \times 10^3}{230} = 5549.21 \text{ mm}^2$/m

Permissible stress for M30 grade concrete is 3.6 N/mm$^2$

$\frac{1276.32 \times 10^3}{A_C + (9-1)5549.21}$ = 3.6 N/mm$^2$

$A_C = 310139.35 \text{ mm}^2$

Assume the depth to be 500mm

$\frac{131567.30}{300} = 650\text{mm}$

Therefore the ridge beam 2 of size 650mmx500mm.

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**III. CONCLUSION**

There was a change in architectural preferences (towards the post-modern) which made expressive structural forms less desirable from a stylistic point of view. The emergence and popularity of the hyperbolic paraboloid form in the post-war era was emblematic of the drive for construction innovation, and the desire to push the structural and economic performance of buildings to new levels. Despite a popular view that hyperbolic paraboloid forms were expensive and only suited to temperate climates.

**REFERENCES**

[1] Advanced reinforced concrete design by N.Krishnaraju

[2] Behaviour of hyperbolic paraboloid shell footings under point loading - Syed jalaludeen shah, Thahzin sageer, Anu sidharthan

[3] Design of 24-m span hyperbolic shell slab and its application in the construction of grain depots – by Feng Tian

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Fig-3: Reinforcement details of a hyperbolic paraboloid shell
[5] Gabled hyperbolic paraboloid roofs without edge beams - By Tamara Jadik, Associate Member, ASCE, and David P. Billington, Fellow, ASCE
[10] Wind Pressure Distribution on a Multiple Hyperbolic Paraboloid Shell Roof Building