Basic Mathematics to Robotic for Tracking Systems Used by Autonomous Solar Collectors

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***Abstract*- In this paper, we look through the basic mathematics to robotic applied for industrial robots. We then explain the different types of rotation combinations in robotic, the 4x4 matric homogeneous representation, rigid-motion matrix and velocity tensor. We then consider to propose applications to solar trackers used for common concentrators such as paraboloid, parabolic trough, hemispheric receiver, CPC etc. We finally propose their 4x4 matric homogeneous transformation and their rotation vectors according to the type of configuration of the tracker.**

***Index Terms*- robot, solar tracker, homogeneous matrix, velocity tensor, azimuth, altitude.**

I. INTRODUCTION

Most collectors only work with the unique condition sometimes sine qua none to have the incident solar rays coming to it exposition area in a perpendicular manner. We already know that the installation of collectors depends on the latitude angle of the area. The leaning of the collectors shall be such that the sun rays coming to the sheets be closer to the normal. This condition is absolute for concentrators, and is an increasing factor of efficiency for all collector, including photovoltaic panel. The ideal will be that collectors follow the sun to kip perpendicularity condition, but as usual, tracking the sun needs a powered motorized system and an alignment control. Any system as described above is a robot, and therefore, all the mathematics applied to robotic can be used to calculate a sun tracker. There are many writings on industrial robots, explaining how to program (Duysinx et al. 2004 [1]) and to calibrate those robots (Zhang et al., 2008 [2]). Merzouky et al. in 2011 [3] made an ostensive list of industrial robots. Hocine in 2003 [4] proposed an initiation to industrial robots, and Murray et al. in 1994 [5] explained how to apply mathematics to robotic. Yu et al. in 2012 [6] designed a mechanical configuration of a solar tracker. Afarulrazi et al. 2011 [7], Piotrowski et al. in 2014 [8] and Ferreira et al. in 2018 [9] use already robotic to solar trackers and explain how to program those trackers.

In this paper, we will give the basic mathematic tools for robotic that will be used to solar trackers. We will study more the use of homogeneous matrix in robotic and explain how some basic operations are done with them. We will apply to determine the homogeneous matrixes of some trackers in relation to positioning and velocity.

II.MATHEMATICS TO ROBOTIC FOR COMMON INDUSTRIAL ROBOTS

Tracking systems used by solar collectors are indeed robots. Therefore, all calculations linked to positioning, speed, inertia of those solar collectors use robotic mathematics. Common industrial robots consist on a bearer and a wrist (or gripper).

Robotic uses orientation matrix and translation vectors. The three basic orientation matrixes in accordance with the axe along which the rotation is considered are (Jlassi, 2013 [10]):

For a rotation around x axis (Figure 1):

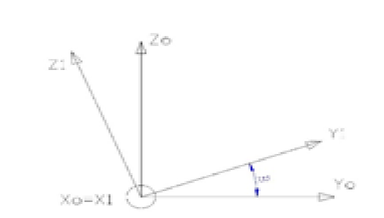

Figure 1: Rotation around x axis

$$\overline{\overline{R}}_x(\beta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \quad (01)$$

Around y axis (Figure 2):

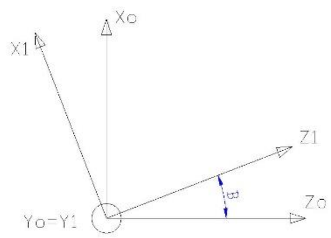

Figure 1: Rotation around y axis

$$\bar{\bar{R}}_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \quad (02)$$

and around z axis (Figure 3):

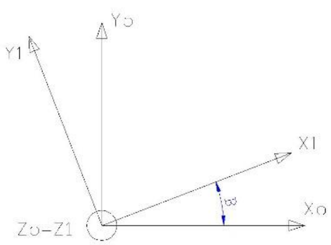

Figure 2: Rotation around z axis

$$\bar{\bar{R}}_z(\beta) = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (03)$$

We distinguish 3 usual types of angles combinations in robotic. The industrial robots use these angles combinations for the bearer (usually 3 angles) and the wrist (also 3 angles) (Briot *et al*. 2015 [11]).

- ZYZ EULER or "Tilt and Torsion-T&T" angles combination: $Rot_{z0}(\phi).Rot_{x1}(\theta).Rot_{z2}(\psi)$
- ZYX BRYANT or "Yaw-Pitch-Roll" angles combination: $Rot_{z0}(\phi).Rot_{y1}(\theta).Rot_{x2}(\psi)$
- ZXX LEBIAN angles combination: $Rot_{z0}(\phi).Rot_{x1}(\theta).Rot_{x2}(\psi)$

The 4x4 matrix homogeneous representation to calculate robots is commonly used for rigid motion, and is described as follow.

$$\bar{\bar{M}}_i^0 = \left[\begin{array}{c|c} \bar{\bar{Rot}}_i^0 & \vec{Tr}_i^0 \\ \hline 000 & 1 \end{array}\right] \quad (04)$$

The 4x4 matrix homogeneous transformation is a very commonly used computational matrix in robotics. It allows to calculate the rotation and the simultaneous displacement of a solid in space, also described as rigid body motion or transformation. In its expression, $\bar{\bar{Rot}}_i^0$ is the 3x3 rotation resulting matrix and $\bar{\bar{Tr}}_i^0$ is the resulting displacement 3D vector of the end landmark to the beginning one, the three zeros are the perspective correction or projection vector coordinates and 1 is the scaling factor. These two last entities have their values always unchanged in robotic (rigid body transformation).

Therefore, a 4x4 homogeneous Transposed matrix of rotation of an angle β along the $0\vec{x}i$ axis for example is written as follows (Kay, 2005 [12]):

$$\left[Rot\left(\vec{x}_i = \vec{x}_{i+1}\right), \beta\right]^T = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

(05)

Likewise, a 4x4 homogeneous matrix of translation to a distance L_j along the $0\vec{x}i$ axis for example is also written as follows:

$$\left[Tr\,\vec{x}_i, L_j\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & L_j \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad (06)$$

The matrixes are jointed from the first rotation including translations to the last, and calculated two by two starting from the last. If might be necessary to invert the matrixes according to the fact that we want to express the result at the endpoint or in the first landscape. Each part of the robot, the bearer and the wrist, is calculated separately, which leads to two final matrixes.

The velocity tensor is defined as follow

$$\upsilon_i^0 = \begin{Bmatrix} \vec{\Omega}(i/0) \\ \vec{V}(C \in S_i / 0) \end{Bmatrix}_{R_0} \quad (07)$$

For a rigid body, an associated 4x4 velocity homogeneous matrix can also be defined as follow:

$$\overline{\overline{T}}_i^{\,0} = \left[\begin{array}{c|c} \Omega_i^0 & \vec{V}_i^0 \\ \hline 000 & 0 \end{array}\right] \quad (08)$$

With $\begin{cases} \Omega_i^0 = -\overset{\circ}{\overline{\overline{R}}}ot_i^0 \,\overline{\overline{R}}ot_i^{T\,0} \\ \vec{V}_i^0 = \vec{V}(C \in S_i / 0) = -\Omega_i^0 \vec{T}_i^0 + \overset{\circ}{\vec{T}}{}_i^0 \end{cases}$ (09)

And where Ω_i^0 is an antisymmetric matrix that gives the vector $\vec{\Omega}(i/0)$ by identification, meanwhile, if $\vec{\Omega}(i/0) = \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\theta} \end{bmatrix}$,

Then $\Omega_i^0 = \begin{bmatrix} 0 & -\dot{\theta} & \dot{\beta} \\ \dot{\theta} & 0 & -\dot{\alpha} \\ -\dot{\beta} & \dot{\alpha} & 0 \end{bmatrix}$ a

III.POSITIONING OF SOLAR COLLECTORS WITH TRACKING ROBOTS

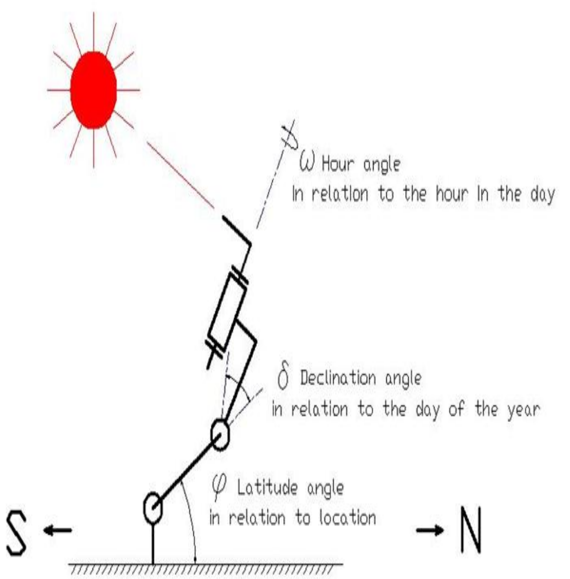

Figure 4: Kinematic chain for solar trajectory

Some solar collectors will only work by using direct sunlight, which make then dependent to trackers.

The sun position can therefore be defined directly by the latitude angle ϕ of the location, the declination angle δ and the hour angle ω.

The movement of the sun can be represented by a kinematic chain as shown on the figure 4 for an area located in the northern hemisphere.

The movement of the sun can also be defined by conventional parameters such as altitude angle h and azimuth angle a (Duffie *et al.* 1980 [13]). these two angles h and a are linked to the first ones through the following formulas, modified to fit tropical areas with no time-shift during seasons:

$$\begin{aligned} &for \quad \varphi \geq \delta, \quad \sin(h) = \cos(\varphi - \delta) \\ &for \quad \varphi < \delta, \quad \sin(180 - h) = \cos(\varphi - \delta) \end{aligned} \quad (11)$$

$$\sin(a) = \frac{\cos\delta \cos\omega}{\cos(h)} \quad (12)$$

The modification of the equation (11) takes into account the fact that the sun moves from north to south sometimes in the year went the declination angle becomes superior to the latitude angle.

These formulas use the declination angle that can be reasonably calculated using Cooper equation:

$$\delta = 23.45 \sin\left(360 \frac{284+n}{365}\right) \quad (13)$$

IV. ROBOTS TO TRACK THE SUN

The systems to track the sun will also be represented by kinematic chains and thus will correspond to robot models. We generally have 4 configurations positioning collectors:

1- The configuration (υ, γ) called Altazimuth which tracks the altitude and azimuth angles h and a. The tracker kinematic chain can be drawn as follow (Figure 5).

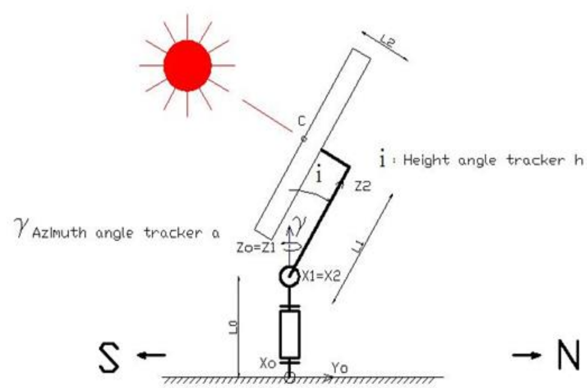

Figure 3: The Altazimuth tracker

We clearly notice the 2 rotations axis identical to EULER or LEBIAN first two angles robots. This robot is the heaviest one to calibrate, since the two

$$\bar{\bar{M}}(Alt)_2^0 = \begin{bmatrix} \cos\gamma & \sin\gamma\cos\upsilon & \sin\gamma\sin\upsilon & \sin\gamma(L_1\sin\upsilon + L_2\cos\upsilon) \\ -\sin\gamma & \cos\gamma\cos\upsilon & \cos\gamma\sin\upsilon & \cos\gamma(L_1\sin\upsilon + L_2\cos\upsilon) \\ 0 & -\sin\upsilon & \cos\upsilon & L_0 + L_1\cos\upsilon - L_2\sin\upsilon \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

The rotation angle vector solution is as follow:

$$\vec{\Omega}_{Alt}(2/0) = \begin{bmatrix} \dot{\upsilon} \\ -\dot{\gamma}\sin\upsilon \\ \dot{\gamma}\cos\upsilon \end{bmatrix} \quad (16)$$

2- The configuration (I, ε) called Equatorial which tracks the declination and hour angles δ and ω.

The declination angle moves quite slowly (1° each 2-3 days) and can even be manually adjusted. The hour angle in the other hand has a constant speed value, which makes the Equatorial robot very simple to command and program. Manually adjusting the declination angle makes it even simpler by only having a single angle to track.

The tracker kinematic chain can be drawn as follow (Figure 6).

angles (altitude and azimuth) have varying speeds and accelerations. Nevertheless, EULER or LEBIAN angles robots are so commonly used in factory productions that their motion control and programs are well known. In this case of solar tracking, the altazimuth robot tracks altitude and azimuth angles that are also calculated, which can generate some misalignment due to mistakes in calculations. To finish, in the case of the sun moving from North to South due to the declination angle superior to the latitude angle, this type of robot might be obliged to turn 180° from North to South to continue tracking the sun at a certain period of time in the year.

The Altazimuth robot homogenous matrix can be constructed as follow:

$$\bar{\bar{M}}(Alt)_2^0 = [Tr\vec{z}_0, L_0].\left[Rot\left(\vec{z}_0 = \vec{z}_1\right), \gamma\right]^T$$
$$[Tr\vec{z}_1, L_1].\left[Rot\left(\vec{x}_1 = \vec{x}_2\right), \upsilon\right]^T.[Tr\vec{y}_2, L_2] \quad (14)$$

The solution is represented by the following matrix:

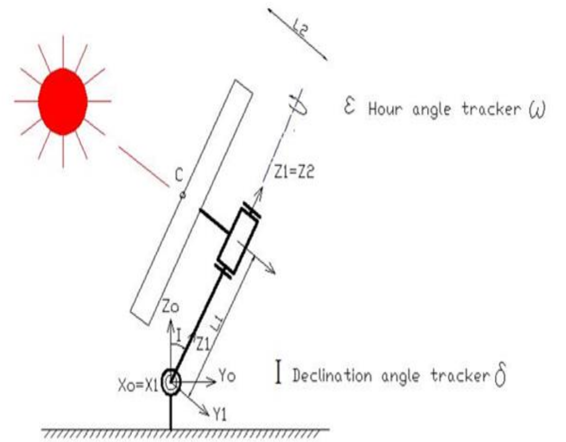

Figure 6: The Equatorial tracker

Its homogenous matrix can be constructed as follow:

$$\bar{\bar{M}}(Equ)_2^0 = [Tr\vec{z}_0, L_0].\left[Rot\left(\vec{x}_0 = \vec{x}_1\right), I\right]^T$$
$$[Tr\vec{z}_1, L_1].\left[Rot\left(\vec{z}_1 = \vec{z}_2\right), \varepsilon\right]^T.[Tr\vec{y}_2, L_2] \quad (17)$$

$$\bar{\bar{M}}(Equ)_2^0 = \begin{bmatrix} \cos\varepsilon & \sin\varepsilon & 0 & L_2\sin\varepsilon \\ -\cos I\sin\varepsilon & \cos I\cos\varepsilon & \sin I & L_1\sin I + L_2\cos I\cos\varepsilon \\ \sin I\sin\varepsilon & -\sin I\cos\varepsilon & \cos I & L_0 + L_1\cos I - L_2\sin I\cos\varepsilon \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

The rotation angle vector solution is as follow:

$$\vec{\Omega}_{Equ}(2/0) = \begin{bmatrix} \dot{I}\cos\varepsilon \\ \dot{I}\sin\varepsilon \\ \dot{\varepsilon} \end{bmatrix} \quad (19)$$

3- The configuration also called quasi-steady, which only tracks the declination angle. The fact that this particular angle moves quite slowly can be manually adjusted as stated above, and in this case, there will not be any need of tracking robot.

The tracker kinematic chains can be drawn as follow (Figure 7).

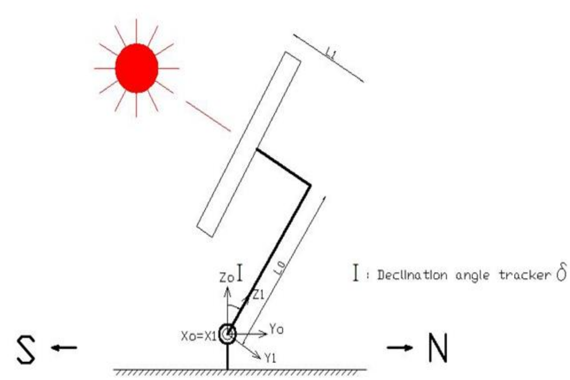

Figure 7: the Quasi-steady tracker

Its homogenous matrix can be constructed as follow:

$$\bar{\bar{M}}(QuF)_1^0 = \left[Tr\vec{z}_0, L_0\right]\left[Rot\left(\vec{x}_0 = \vec{x}_1\right), I\right]^T$$
$$\left[Tr\vec{z}_1, L_1\right].\left[Tr\vec{y}_1, L_2\right] \quad (20)$$

The solution is represented by the following matrix:

$$\bar{\bar{M}}(QuF)_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos I & \sin I & L_1 \sin I + L_2 \cos I \\ 0 & -\sin I & \cos I & L_0 + L_1 \cos I - L_2 \sin I \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (21)$$

The rotation angle vector solution is as follow:

$$\vec{\Omega}_{QuF}(1/0) = \begin{bmatrix} \dot{I} \\ 0 \\ 0 \end{bmatrix} \quad (22)$$

4- The configuration is the steady position which does not require any tracker. The collectors are permanently adjusted with the sun in relation to the local latitude angle (Figure 8).

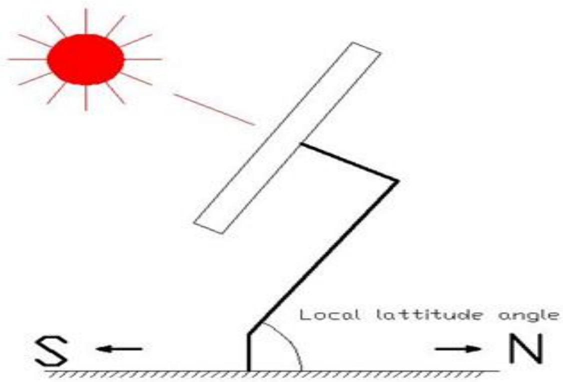

Figure 8: the Steady configuration

V.CONCLUSION

All the other calculation operations using 4x4 homogeneous matrixes such as exponential coordinates, motions, velocities, kinematics, dynamics and controls are also possible for these solar robots 4x4 homogeneous matrixes, with less complexity:

- The maximum rotation matrixes on a solar tracking robot is two, which makes it already less complicated than an industrial robot.
- Motions are slow. Sometimes, the rotation speed is constant, which also simplifies the dynamic mode calculations.

The only concern is the weight of the panels in motion, especially if there are some sparks of wind, which is very complicated to estimate. Nevertheless, a security position of the structure can be found.

All those examples taken are reduced to simple systems tracker for concentrators such as paraboloid, parabolic trough, hemispheric receiver, CPC and even photovoltaic panel. We excluded Fresnel SLATS and heliostats for central receiver. Nevertheless, even though those basic mathematics for robotic still apply to the excluded concentrators, their trackers are more complicated to monitor.

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