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IFgsr-closed sets in Intuitionistic Fuzzy Topological Spaces

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Abstract- **In this paper, a new class of sets, called Intuitionistic fuzzy gsr-closed sets and Intuitionistic fuzzy gsr-open sets are introduced and some of its properties were studied.**

Index Terms- **Intuitionistic fuzzy topology, Intuitionistic fuzzy gsr-closed sets, Intuitionistic fuzzy gsr-open sets.**

1. INTRODUCTION

The fuzzy concept was introduced by Zadeh [14] in 1965 and the theory of fuzzy topology was introduced and developed by C.L. Chang [2]. Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets.

S.S.Thakur and Rekka Chaturvedi [11] introduced and investigated the notions of RG-Closed sets in intuitionistic fuzzy topological spaces. Many fuzzy topological concepts such as semi closed , α closed, semi pre closed have been generalized for intuitionistic fuzzy topological spaces.

In this paper, we introduce the concepts of intuitionistic fuzzy gsr-closed sets, intuitionistic fuzzy gsr-open sets.The relation between intuitionistic fuzzy gsr-closed sets and other generalization of intuitionistic fuzzy closed sets are discussed.

2. PRELIMINARIES

Definition 2.1. [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$ where the functions $\mu_A(x) : X \rightarrow [0,1]$ and $\nu_A(x) : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy set in X.

Definition 2.2. [1] $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > / x \in X\}$. Then

- $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- $A^c = \{< x, \nu_A(x) , \mu_A(x) > / x \in X\}$
- $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) > / x \in X\}$
- $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu B(x) > / x \in X\}$

For the sake of simplicity, Let us use the notation $A = <x, \mu_A, \nu_A >$ instead of $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = < x, (\mu_A, \mu_B) , (\nu_A, \nu_B)>$ instead of $A = <x, (A/\mu_A , B/\mu_B) , (A/(\nu_A, B/\nu_B)>$.

The intuitionistic fuzzy sets $0\sim = \{ < x, 0, 1> / x \in X\}$ and $1\sim = \{ < x, 1, 0 >/ x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3. [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

- $0\sim, 1\sim \in \tau$
- $G_1 \cap G_2 \in \tau$, for any $G_1 , G_2 \in \tau$
- $\cup G_i \in \tau$ for any family $\{ G_i / i \in J\} \subseteq \tau$.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short) in X.

The complement (A^c) of an IFOS A in an IFTS(X,τ) is called an intuitionistic fuzzy closed set(IFCS in short) in X.

Definition 2.4. [3] Let (X,τ) be an IFTS and $A = <x, \mu_A, \nu_A >$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

- int(A) = ∪ {G / G is an IFOS in X and G ⊆ A},
- cl(A) = ∩ {K / K is an IFCS in X and A ⊆ K}.

Proposition 2.5. [3] For any IFSs A and B in (X,τ), we have
(1) int(A) ⊆ A,
(2) A ⊆ cl(A),
(3) A is an IFCS in X ⇔ cl(A) = A,
(4) A is an IFOS in X ⇔ int(A) = A,
(5) A ⊆ B ⇒ int(A) ⊆ int(B) and cl(A) ⊆ cl(B),
(6) int(int(A)) = int(A),
(7) cl(cl(A)) = cl(A),
(8) cl(A ∪ B) = cl(A) ∪ cl(B),
(9) int(A ∩ B) = int(A) ∩ int(B).

Proposition 2.6. [3] For any IFS A in (X,τ), we have
(1) int(0~) = 0~ and cl(0~) = 0~,
(2) int(1~) = 1~ and cl(1~) = 1~,
(3) $(\text{int}(A))^c = \text{cl}(A^c)$,
(4) $(\text{cl}(A))^c = \text{int}(A^c)$

Proposition 2.7. [3] If A is an IFCS in (X,τ) then cl(A) = A and if A is an IFOS in (X,τ) then int(A) =A. The arbitrary union of IFCSs is an IFCS in (X,τ).

Definition 2.8. [8] An IFS A = <x, μ_A, ν_A > in an IFTS(X,τ) is said to be an
(i) intuitionistic fuzzy regular open set(IFROS in short) if A = int(cl(A)),
(ii) intuitionistic fuzzy regular closed set(IFRCS in short) if A = cl(int(A)),
(iii) intuitionistic fuzzy semi open set(IFSOS in short) if A ⊆ cl(int(A)),
(iv) intuitionistic fuzzy semi closed set(IFSCS in short) if int(cl(A)) ⊆ A,
(v) intuitionistic fuzzy pre-open set(IFPOS in short) if A ⊆ int(cl(A)),
(vi) intuitionistic fuzzy pre-closed set(IFPCS in short) if cl(int(A)) ⊆ A,
(vii) intuitionistic fuzzy α- open set(IFαOS in short) if A ⊆ int(cl(int(A)),
(viii) intuitionistic fuzzy α-closed set(IFαCS in short) if cl(int(cl(A)) ⊆ A.

Definition 2.9. [8] Let an IFS A of an IFTS (X,τ).
Then the semi closure of A (scl(A) in short) is defined as scl(A) = ∩ {K / K is an IFSCS in X and A ⊆ K}.
Then the semi interior of A (sint(A) in short) is defined as sint(A) = ∪ {G / G is an IFSOS in X and G ⊆ A}

Result 2.10. [8] Let A be an IFS in (X,τ), then
(i) scl(A) = A ∪ int(cl(A))
(ii) sint(A) = A ∩ cl(int(A))

Definition 2.11. An IFS A in an IFTS (X,τ) is said to an
(i) intuitionistic fuzzy generalized closed (IFGCS) [12] if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X,
(ii) intuitionistic fuzzy regular generalized closed (IFRGCS) [11] if cl(A) ⊆ U whenever A ⊆ U and U is an IFROS in X,
(iii) intuitionistic fuzzy α - generalized closed (IFαGCS) [6] if αcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X,
(iv) intuitionistic fuzzy generalized α - closed (IFGαCS) [10] if αcl(A) ⊆ U whenever A ⊆ U and U is an IFαOS in X,
(v) intuitionistic fuzzy generalized pre - closed (IFGPCS) [7] if pcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X,

3. INTUITIONISTIC FUZZY GSR-CLOSED SETS

Definition 3.1. An IFS A in an IFTS (X,τ) is said to be an Intuitionistic fuzzy gsr-closed sets (IFGSRCS in short) if scl(A) ⊆ U and U is an IFROS in (X, τ).

Example 3.2. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.3,0.4), (0.7,0.5)>.Then the IFS A = <x, (0.3,0.5),(0.6,0.4)> is an IFGSRCS in (X,τ).

Theorem 3.3. Every IFCS is an IFGSRCS but not conversely.

Proof: Let A ⊆ U and U is an IFROS in (X,τ). Let A be an IFCS in X. Since scl(A) ⊆ cl(A) and A is a IFCS in (X,τ), scl(A) ⊆ cl(A) = A ⊆ U. Therefore A is an IFGSRCS in X.

Example 3.4. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.1,0.3), (0.6,0.5)>. Then the IFS A = <x, (0.5,0.4),(0.2,0.3)> is an IFGSRCS but not IFCS in (X,τ).

Theorem 3.5. Every IFGCS is an IFGSRCS but not conversely.

Proof: Let A ⊆ U and U is an IFROS in (X,τ). Let A be an IFGCS in (X,τ). Since every IFROS is an IFOS and scl(A) ⊆ cl(A), we have by hypothesis, scl(A) ⊆ cl(A) ⊆ U and hence A is an IFGSRCS.

Example 3.6. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.3,0.2), (0.7,0.8)>. Then the IFS A = <x, (0.3,0.2),(0.7,0.8)> is an IFGSRCS but not IFGCS in (X,τ).

Theorem 3.7. Every IFRGCS is an IFGSRCS but not conversely.
Proof: Let A ⊆ U where U is an IFROS in (X,τ). Let A be an IFRGCS in (X,τ). Since scl(A) ⊆ cl(A), we have by hypothesis, scl(A) ⊆ cl(A) ⊆ U and hence A is an IFGSRCS.

Example 3.8. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.3,0.2), (0.7,0.8)>. Then the IFS A = <x, (0.3,0.2),(0.7,0.8)> is an IFGSRCS, but not an IFRGCS (X,τ).

Theorem 3.9. Every IFαCS is an IFGSRCS but not conversely.
Proof: Let A ⊆ U where U is an IFROS in (X,τ) and Let A be an IFαCS in (X,τ). Since every α-closed set is semi-closed and scl(A) ⊆ α cl(A) ⊆ U. Thus A is an IFGSRCS.

Example 3.10. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.1,0.3), (0.6,0.5)> Then the IFS A = <x, (0.5,0.4),(0.2,0.3)> is an IFGSRCS but not IFαCS in (X,τ).

Theorem 3.11. Every IFGαCS is an IFGSRCS but not conversely.
Proof: Let A be an IFGαCS in (X,τ) and let A ⊆ U and U be an IFROs in (X,τ). Since every IFROS is an IFαOS and by hypothesis, we have scl(A) ⊆ α cl(A) ⊆ U. Therefore A is an IFGSRCS.

Example 3.12. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.3,0.4), (0.7,0.5)>. Then the IFS A = <x, (0.3,0.5),(0.6,0.4)> is an IFGSRCS but not IFGαCS in (X,τ).

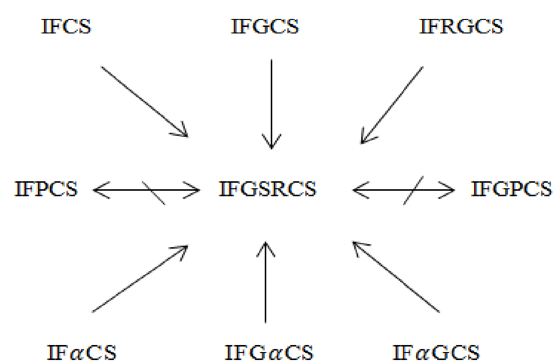
Theorem 3.13. Every IFαGCS is an IFGSRCS but not conversely.
Proof: Let A ⊆ U and U be an IFROS in (X,τ). Since every IFROS is an IFOS and A is IFαGCS, we have scl(A) ⊆ α cl(A) ⊆ U. Hence A is an IFGSRCS.

Example 3.14. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.3,0.4), (0.7,0.5)>. Then the IFS A = <x, (0.3,0.5),(0.6,0.4)> is an IFGSRCS but not IFαGCS in (X,τ).

Remark 3.15. The following examples show that IFGSRCS is independent of IFPCS, IFGPCS.

Example 3.16. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.2,0.3), (0.7,0.6)> Then IFS A = <x, (0.6,0.5), (0.3,0.4)> is an IFGSRCS but not IFPCS and IFGPCS.

Example 3.17. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.5,0.4), (0.5,0.6)> Then the IFS A = <x, (0.4,0.2),(0.6,0.7)> is an IFPCS and IFGPCS but not an IFGSRCS in (X,τ).
The following implications are true, none of them is reversible.



In this diagram by " A⟶ B" means A implies B but not conversely and " A↮ B" means A and B are independent of each other.

Theorem 3.18. Let (X,τ) be an IFTS. Then for every A ∈ IFGSRCS and for every B ∈ IFS in an IFTS (X,τ), A ⊆ B ⊆ scl(A) ⇒ B ∈ IFGSRCS in (X,τ).
Proof: Let B ⊆ U and U be an IFROS. Since A ⊆ B, A ⊆ U and A is an IFGSRCS, scl(A) ⊆ U, whenerver A ⊆ U, By hypothesis, B ⊆ scl(A), scl(B) ⊆ scl(scl(A)) = scl(A) ⊆ U. Therefore B is an IFGSRCS in (X,τ).

Theorem 3.19. If an IFS A is an IFRGCS such that A ⊆ B ⊆ cl(A), where B is an IFS in an IFTS (X,τ), then B is an IFGSRCS in (X,τ).
Proof: Let B ⊆ U and U be an IFROS. Then A ⊆ U. Since A is an IFRGCS and scl(A) ⊆ cl(A), we have

scl(A) ⊆ cl(A) ⊆ U. Now, scl(B) ⊆ cl(B) ⊆ cl(A) ⊆ U. Thus B is an IFGSRCS in (X,τ).

4.INTUITIONISTIC FUZZY GSR-OPEN SETS

Definition 4.1. An IFS A is said to be an intuitionistic fuzzy gsr-open set (IFGSROS in short) in (X,τ) if the complement A^c is an IFGSRCS in (X,τ)

Example 4.2. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.3,0.4), (0.7,0.5)>.Then the IFS A = <x, (0.6,0.4), (0.3,0.5)> is an IFGSROS in (X,τ).

Therorem 4.3. Every IFOS, IFGOS, IFRGOS, IFαOS, IFGαOS, IFαGOS is an IFGSROS in (X,τ). But the converse are not true in general.

Proof: Straight forward.

Example 4.4. Let X={a,b} and τ = {0~,G,1~} be an IFTS on X, where G = < x, (0.1,0.3), (0.6,0.5)> Then the IFS A = <x, (0.2,0.3), (0.5,0.4)>is an IFGSROS but not IFOS, IFαOS in (X,τ).

Example 4.5. Let X={a,b} and τ = {0~,G,1~} be an IFTS on X, where G = < x, (0.3,0.2), (0.7,0.8)>. Then the IFS A = <x, (0.7,0.8), (0.3,0.2)> is an IFGSROS but not an IFGOS, IFRGOS in (X,τ).

Example 4.6. Let X={a,b} and τ = {0~,G,1~} be an IFTS on (X,τ), where G = < x, (0.3,0.4), (0.7,0.5)>. Then the IFS A = A = <x, (0.6,0.4), (0.3,0.5)> is an IFGSROS but not IFGαOS, IFαGOS in (X,τ).

Theorem 4.7. Let (X,τ) be an IFTS. If A ∈ IFGSRO(X) then V ⊆ cl(int(A)) whenever V⊆ A and V is IFRCS in (X,τ).

Proof: Let us assume that A ∈ IFGSRO(X). Then A^c is an IFGSRCS in (X,τ). Therefore scl(A^c) ⊆ U whenever A^c ⊆ U and U is an IFROS in X. That is int(cl(A^c)) ⊆ U. This implies U^c ⊆ cl(int(A)) whenever U^c ⊆ A and U^c is IFRCS in X. Replacing U^c by V, we get V ⊆ cl(int(A)) whenever V ⊆ A and V is IFRCS.

Theorem 4.8. Let (X,τ) be an IFTS. Then for every A ∈ IFGSRO(X) and for every B ∈ IFS(X), sint(A) ⊆ B ⊆ A which implies B ∈ IFGSRO(X).

Proof: By hypothesis, A^c ⊆ B^c ⊆ $(\text{sint}(A))^c$. Let B^c ⊆ U and U be an IFROS. Since A^c ⊆ B^c, A^c ⊆ U. But A^c is an IFGSRCS scl(A^c) ⊆ U. Also B^c ⊆ $(\text{sint}(A))^c$ = scl(A^c). Therefore scl(B^c) ⊆ scl(scl(A^c)) = scl(A^c) ⊆ U. Hence B^c is an IFGSRCS which implies B is an IFGSRO in (X,τ)

Theorem 4.9. An IFS A of an IFTS (X,τ) is a IFGSROS if and only if U ⊆ sint(A) whenever U ⊆ A and U is an IFRCS.

Proof: Necessity: Suppose A is an IFGSROS in (X,τ). Let U be an IFRCS and U ⊆ A. Then U^c is an IFROS in (X,τ) such that A^c ⊆ U^c. Since A^c is an IFGSRCS, we have scl(A^c) ⊆ U^c. Hence $(\text{sint}(A))^c$ ⊆ U^c. Therefore U ⊆ sint(A)

Sufficiency: Let A be an IFS of (X,τ) and let U ⊆ sint(A) whenever U is an IFRCS. Then A^c ⊆ U^c and U^c is an IFROS. By hypothesis, $(\text{sint}(A))^c$ ⊆ U^c which implies scl(A^c) ⊆ U^c. Therefore A^c is an IFGSRCS in (X,τ). Hence A is an IFGSROS of (X,τ)

Corollary 4.10. An IFS A of an IFTS (X,τ) is an IFGSROS if and only if U ⊆ cl(int(A)) whenever U is an IFRCS and U ⊆ A.

Proof: Necessity: Suppose A is an IFGSROS in (X,τ). Let U be an IFRCS and U ⊆ A. Then U^c is an IFROS in (X,τ). Such that A^c ⊆ U^c. Since A^c is an IFGSRCS, we have scl(A^c) ⊆ U^c. Therefore int(cl(A^c)) ⊆ U^c. Hence $(\text{cl}(\text{int}(A))^c$ ⊆ U^c which implies U ⊆ cl(int(A)).

Sufficiency: Let A be an IFS of X and let U ⊆ cl(int(A)), whenever U is an IFRCS and U⊆ A. Then A^c ⊆ U^c and U^c is an IFROS in (X,τ). By hypothesis, $(\text{cl}(\text{int}(A))^c$ ⊆ U^c. Hence (cl(int(A^c)) ⊆ U^c , which implies A^c ⊆ U^c. Hence A is an IFGSROS of X.

Theorem 4.11. If an IFS A is an IFRGOS in (X,τ) such that int(A) ⊆ B ⊆ A, where B is an IFS in (X,τ), then B is an IFGSROS in (X,τ)

Proof: Let A be an IFRGOS and int(A) ⊆ B ⊆ A. Then A^c is an IFRGCS and A^c ⊆ B^c ⊆ cl(A^c). Then B^c is an IFGSRCS in (X,τ), by Theorem 3.19. Hence B is an IFGSRCS in (X,τ)

REFERENCES

[1] Atanassov.K.T, Intuitionistic fuzzy sets, Fuzzy sets and systems, 20(1986), 87-96.

[2] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24(1986), 182-190.
[3] Coker.D, An introduction to Intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 88(1997), 81-89.
[4] H. Gurcay, D. Coker and Es. A. Haydar, On fuzzy continuity in intuitionistic fuzzy topological spaces, The Journal of Fuzzy Mathematics, 5(1997), 365-378.
[5] D. Jayanthi and R. Santhi, Intuitionistic fuzzy generalized semi-pre closed sets, Tripura Math. Soc. (2009) 61–72.
[6] D. Kalamani, K. Sakthivel and C. S. Gowri, Generalized alpha closed sets in intuitionistic fuzzy topological spaces, Applied Mathematical Sciences,6(94)(2012), 4691- 4700.
[7] P. Rajarajeswari and L. Senthil Kumar,"Generalised Pre-Closed Sets in Intuitionistic Fuzzy Topologicical Spaces", International Journal of Fuzzy Mathematics and Systems, Vol. 1, no. 3(2011), pp. 253-262.
[8] Sakthivel, K., 2010, Intuitionistic Fuzzy Alpha Generalized Continuous Mappings and Intuitionistic Alpha Generalized Irresolute Mappings, Applied Mathematical Sciences., Vol. 4, no. 37., pp. 1831 – 1842.
[9] R. Santhi and K. Sakthivel, Intuitionistic fuzzy generalized semicontinuous mappings Advances in Theoretical and Applied Mathematics, 5(2009), 73-82.
[10] R. Santhi and K. Sakthivel, Intuitionistic fuzzy alpha generalized closed sets (Accepted in Mathematics Education).
[11] S. S. Thakur and Rekha Chaturvedi, Regular generalized closed set in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii Si Cercetari Stiintice,Seria:Matematica, 16(2006), 257-272.
[12] S. S. Thakur and Rekha Chaturvedi, Generalized closed set in intuitionistic fuzzy topology,The Journal of Fuzzy Mathematics, 16(3)(2008), 559-572
[13] SYoung Bae Jun and Seok-Zun Song, Intuitionistic fuzzy semipre open sets and Intuitionistic fuzzy semipre continuous mappings, Jour. of Appl. Math. And Computing,19(2005), 467-474.
[14] L.A. Zadeh, Fuzzy sets, Information and Control, 8(1965), 338-353.