On Decompositions of Continuity in Topological Spaces

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***Abstract*- In this paper, we obtain two decompositions of continuity via strong semi closed sets in topological spaces.**

Index Terms- Decomposition, strong semi closed set.

I. INTRODUCTION AND PRELEMINARIES

A subset S of a topological space is said to be regular open in X if S = int(cl(S))

1.1. Definition
A subset S of X is called

a) Strong semiclosed set in X if $S = A\cap F$, where A is regular open and F is closed in X,
b) Generalized closed (g-closed), if $cl(S) \subseteq G$ whenever $S \subseteq G$ whenever $S \subseteq G$ and G is regular open in X,
c) Generalized continuos(g-continuous) if $f^{-1}(F)$ is g-closed in X for each closed set F in Y.

1.2. Definition
A map $f : X \rightarrow Y$ is called

a) generalized continuos (g-continuous) if $f^{-1}(F)$ is g-closed in X for each closed set F in Y,
b) regular generalized continuos (rg-continuous) if $f^{-1}(F)$ is rg-closed in X for each closed set F in Y,
c) Strong semicontinuos if $f^{-1}(F)$ is strong semiclosed in X for each closed set F in Y.

II.DECOMPOSITION

Two decompositions of continuous maps, two decompositions of closed maps and two decompositions of contra continuous maps in topological spaces are obtained in this section. Further, we consider Baker's[1]decomposition of continuity for a brief study.

2.1.Theorem
A subset S is closed in X if and only if it is both g-closed and strong semi closed in X.

Proof:
Necessity is trivial. We prove the sufficiency. Let S be both g-closed and strong semi-closed in X. Since, S is strong semi-closed in X,$S = A\cap F$, where A is regular open and F is closed in X. So, $S\subseteq A$ and $S\subseteq F$. Since S is g-closed, $cl(S)\subseteq A$.Also, we have, $cl(S)\subseteq cl(F) = F$. This implies, $cl(S)\subseteq A\cap F = S$. Hence cl(S)=S. Therefore Sis closed.

2.2.A Theorem
subset S is closed in X if and only if it is both rg-closed and strong semi closed in X.
Proof:
Necessity is trivial. We prove the sufficiency. Let S be both rg-closed and strong semi closed in X. Since, S is strong semi closed in X,$S=A\cap F$, where AisregularopenandFisclosedinX.So,$S\subseteq A$ and$S\subseteq F$. Since S is rg-closed, $cl(S)\subseteq A$. Also, wehave, $cl(S)\subseteq cl(F)= F$. This implies, $cl(S)\subseteq A\cap F=S$. Hence cl(S)=S.ThereforeSisclosed.

2.3. Corollary
Thefollowingareequivalentforamapf:$X\rightarrow Y$:
(i) fiscontinuous,
(ii)fis g-continuous and strong semi continuous,
(iii)fis rg-continuous and strong semi continuous.
Proof:
TheprooffollowsfromTheorem2.1.andTheorem2.2.

2.4. Definition
A map f:$X\rightarrow Y$ is said to be a strong semi closed map if f(F) is strong semi closed in Y for each closed set F in X.

2.5. Corollary
The following are equivalent for a map f:$X \rightarrow Y$:
(i) fis closed,
(ii) fis g-closed and strong semi closed,
(iii) fis rg-closed and strong semi closed.
Proof:
Follows from Theorem 2.1. and Theorem 2.2.

2.6. Corollary

The following are equivalent for a map f: X→Y :

1. f is contra continuous,
2. fis contrag-continuous and contra strong semi continuous,
3. fis contrarg-continuous and contra strong semi continuous.

Proof:

FollowsfromTheorem2.1. andTheorem 2.2.

Baker[1] obtained ade composition of continuity: A map f:X→ Y is continuous if and only if, for each open set V in Y, $f^{-1}(V)\subseteq$int (cl(f^{-1}(cl(V)))) and f^{-1}(V) is open in the sub space cl(f^{-1}(cl(V))).

For brevity, a map f : X→Y is called B_1-continuous if f^{-1}(V) ⊆int (cl(f^{-1}(cl(V)))) for each open set V in Y and a map f:X→ Y is called B_2-continuous if f^{-1}(V) isopeninthesubspacecl(f^{-1}(cl(V)))for each open set V in Y, in the present study.

Example 2.7.

LetX= {a,b,c}and□={□,{a},X}. Definef:(X,□)→(X,□) byf(a) = f(b)= a andf(c)= c. Then f is B_1-continuous ,g-continuous and rg-continuous but is neither B_2-continuous nor strong semi continuous.

Example 2.8.

Let X = {a, b, c} and τ ={ □, {a}, {b}, {a, b}, X}. Define f : (X, □) → (X, □) by f(b) = f(c) = a and f(a) = c. Then f is B_1-continuous, strong semi continuous but is neither B_2-continous nor g-continuous nor
rg-continuous.

Example 2.9.

Let X = {a, b, c, d, e}, Y = {x, y, z}, τ = {□□, {a}, {b}, {a, b}, X} and □ = {□□, {x}, {y, z}, Y}. Define f: (X, τ)→ (Y, □) by f(a) = f(b) = f(c) = x , f{d) = y and f(e) = z. Then f is g-continuous and rg-continuous but notB_1-continuous.

Example2.10.

LetX={a,b,c,d},Y={x,y,z},τ={□,{c}
,{d},{c,d},{a,c},{a,c,d},X}and
σ={□,{z},{x,y},Y}. Define f : (X,τ)→(Y,σ)byf(a)= f(b)= f(c) =x andf(d)= z.Thenf isstrong semicontinuousandB_2-continuousbut isneitherg-continuousnorrg-continuousnorB_1-continuous.

Example2.11.

Let (X, τ) and (Y, □) be as in Example 2.10. Define f :(X, τ) → (Y, □) by f(a) = f(b) = x , f(c) = z
and f(d) = y. Then f is B_2-continuous but not strong semi continuous.

III.REMARK

In this remark, we sum up the results.

(a)The decomposition of continuity into semi continuity and D(c, s)-continuity obtained in[2] is the same as the decomposition of continuity obtained by Ganster and Reilly[3], because the notion of semi continuity coincides with the notion of quasi continuity[4] and the notion of D(c, s)-continuitycoincides with the notion of ic-continuity.

(b) The decompositions of continuity obtained in Corollary 2.3.are different from other decompositions of continuity considered in this paper.

(c) The decompositions of continuity obtained in Corollary 2.3. are independent of other decompositions of continuity considered, including Levine's and Baker's, except those involving semi continuity and□-continuity.

(d) The second decomposition obtained, is an improvement over the first.

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