$\ddot{g}$ Open Sets in Intuitionistic Fuzzy Topological Spaces

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***Abstract*- In this paper, we introduce and study the notions of g¨ open sets in intuitionistic fuzzy topological spaces and study some their properties in intuitionistic fuzzy topological spaces.**

Index Terms- Intuitionistic fuzzy topology, Intuitionistic fuzzy g¨ closed sets , Intuitionistic fuzzy g¨ open sets.

I. INTRODUCTION

In 1965, Zadeh [9] introduced fuzzy sets and in 1968, Chang[2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of this notions. The notion of intuitionistic fuzzy sets was introduced by Atanassov[1] as a generalization of fuzzy sets. In 1997, Coker[3] introduced the concept of intuitionistic fuzzy topological spaces. In this paper , we introduce the notions of $\ddot{g}$ closed sets and g¨ open sets in intuitionistic fuzzy topological spaces and study some their properties in intuitionistic fuzzy topological spaces.

II.PRELIMINARIES

Throughout this paper,(X, τ)or simply X denote the intuitionistic fuzzy topological spaces (IFTS in short).For a subset A of X , the closure, the interior and the complement of A are denoted by cl(A) , int(A) and A^C respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1.

[1] Let X be a non-empty set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{<x, \mu_A(x), \nu_A(x)> / x \in X\}$ where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote the degree of membership(namely $\mu_A(x)$) and the degree of non-membership(namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x)+\nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2.

[1] Let A and B be IFSs of the form $A =\{<x, \mu_A(x), \nu_A(x) > / x \in X\}$ and $B = \{<x, \mu_B(x), \nu_B(x) > / x \in X\}$. Then

1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
2. $A = B$ iff $A \subseteq B$ and $B \subseteq A$,
3. $A^C = \{<x, \nu_A(x), \mu_A(x)> / x \in X\}$,
4. $A \cap B = \{<x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)> / x \in X\}$,
5. $A \cup B = \{<x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)> / x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu A, \nu A\rangle$ instead of $A = \{ \langle x, \mu A(x), \nu A(x)\rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu A, \mu B), (\nu A, \nu B)\rangle$ instead of $A = \langle x, (A/\mu A, B/\mu B), (A/\nu A, B/\nu B)\rangle$.The intuitionistic fuzzy sets $0\sim = \{<x, 0, 1> / x \in X\}$ and $1\sim = \{<x, 1, 0> / x \in X\}$
are respectively the empty set and the whole set in X.

Definition 2.3.

[3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms

1. $0\sim, 1\sim \in \tau$,
2. $G1 \cap G2 \in \tau$ for any $G1, G2 \in \tau$,
3. $\cup Gi \in \tau$ for any family $\{Gi / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^C of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4.

[3] Let (X, τ) be an IFTS and A=<x, μ_A, ν_A>be an IFS in X. Then

1. int(A) = ∪{G / G is an IFOS in X and G ⊆ A},
2. cl(A) = ∩{K / K is an IFCS in X and A ⊆ K},
3. $cl(A^C) = (int(A))^C$,
4. $int(A^C) = (cl(A))^C$.

Result 2.5.

[5] Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological spaces (X, τ) . Then cl(A ∪ B)=cl(A) ∪ cl(B).

Definition 2.6.

[4] An IFS A = <x, μ_A, ν_A> in an IFTS (X, τ) is said to be an

1. intuitionistic fuzzy semi-closed set (IFSCS in short) if int(cl(A)) ⊆ A,
2. intuitionistic fuzzy semi-open set (IFSOS in short) if A ⊆ cl(int(A)),
3. intuitionistic fuzzy α -closed set (IF α CS in short) if cl(int(cl(A))) ⊆ A,
4. intuitionistic fuzzy α -open set (IF α OS in short) if A ⊆ int(cl(int(A))),
5. intuitionistic fuzzy regular closed set (IFRCS in short) if cl(int(A)) = A,
6. intuitionistic fuzzy regular open set (IFROS in short) if A = int(cl(A)).
7. intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A⊆U and U is an IFOS in X .The complement of an IFGCS is an IFGOS [8],
8. intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFROS in X . The complement of an IFRGCS is an IFRGOS [7],
9. intuitionistic fuzzy generalized α -closed set (IFG α CS in short) if αcl(A) ⊆ U whenever A ⊆ U and U is an IF α OS in X . The complement of an IFG α CS is an IFG α OS [6],
10. intuitionistic fuzzy α –generalized closed set (IFαGCS) [6] if αcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X,
11. intuitionistic fuzzy W-closed set (IFWCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in X . The complement of an IFWCS is an IFWOS [5],

III. INTUITIONISTIC FUZZY $\ddot{g}$ CLOSED SETS

Definition 3.1.

An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy $\ddot{g}$ closed set(briefly IF$\ddot{g}$CS) if cl(A) ⊆ U whenever A ⊆ U and U is an IFSGOS in (X, τ).

The family of all IF$\ddot{g}$CSs of an IFTS (X, τ) is denoted by IF$\ddot{g}$CS (X). The complement of the IF$\ddot{g}$CS is an IF$\ddot{g}$OS in (X, τ) .

Theorem 3.2.

Every IFCS in (X, τ) is an IF$\ddot{g}$CS, but not conversely.

Proof:

Let A be an IFCS and A ⊆ U where U is an IFSGOS in (X, τ) . Then cl(A) = A ⊆ U , by hypothesis. Hence A is an IF $\ddot{g}$CS in (X, τ) .

Example 3.3.

Let X = {a, b} and G = <x, (0.5, 0.6), (0.5, 0.4)> . Then τ ={0~, G, 1~} is an IFT on X and the IFS A = <x, (0.4, 0.4), (0.6, 0.6) > is an IF$\ddot{g}$ CS, but not an IFCS in (X, τ).

Theorem 3.4.

Every IFRCS in (X, τ) is an IF$\ddot{g}$CS, but not conversely.

Proof:

Since every IFRCS is an IFCS and by Theorem 3.3., A is an IF$\ddot{g}$CS in X.

Example 3.5.

Let X = {a, b} and G = <x, (0.5, 0.6), (0.5, 0.4)> . Then τ ={0~, G, 1~} is an IFT on X and the IFS A = <x, (0.4, 0.4), (0.6, 0.6)> is an IF$\ddot{g}$ CS but not an IFRCS in (X, τ).

Theorem 3.6.

Every IF$\ddot{g}$CS in (X, τ) is an IFGCS, but not conversely.

Proof:

Let A ⊆ U and U be an IFOS in (X, τ) . Since every IFOS is an IFSGOS and by hypothesis, we have cl(A) ⊆ U. Hence A is an IFGCS in (X, τ) .

Example 3.7.

Let X = {a, b} and G = <x, (0.8, 0.8), (0.2, 0.1)> . Then τ ={0~, G, 1~} is an IFT on X and the IFS A = <x, (0.8, 0.7), (0.1, 0.3)> is an IFGCS but not an IF$\ddot{g}$CS in (X, τ) .

Theorem 3.8.
Every IF$\ddot{g}$ CS in (X, τ) is an IFRGCS, but not conversely.
Proof:
Let A ⊆ U and U be an IFROS in (X, τ) . Since every IFROS is an IFSGOS and by hypothesis, we have cl(A) ⊆ U. Hence A is an IFRGCS in (X, τ) .

Example 3.9.
Let X = {a, b} and G = <x, (0.8, 0.8), (0.2, 0.1)> . Then τ ={0~, G, 1~} is an IFT on X and the IFS A = <x, (0.9, 0.7), (0.1, 0.3)> is an IFRGCS but not an IF$\ddot{g}$CS in (X, τ) .

Theorem 3.10.
Every IF$\ddot{g}$CS in (X, τ) is an IFG α CS, but not conversely.
Proof:
Let A ⊆ U and U be an IF α OS in (X, τ) . Since every IF α OS is an IFSGOS and by hypothesis, we have αcl(A) ⊆ cl(A) ⊆ U . Hence A is an IFG α CS in (X, τ) .

Example 3.11.
Let X = {a, b} and G = <x, (0.3, 0.2), (0.7, 0.7)> . Then τ ={0~, G, 1~} is an IFT on X and the IFS A = <x, (0.6, 0.6), (0.3, 0.4)> is an IFG α CS but not an IF$\ddot{g}$ CS in (X, τ) .

Theorem 3.12.
Every IF$\ddot{g}$ CS in (X, τ) is an IFWCS, but not conversely.
Proof:
Let A ⊆ U and U be an IFSOS in (X, τ) . Since every IFSOS is an IFSGOS and by hypothesis, we have cl(A) ⊆ U. Hence A is an IFWCS in (X, τ).

Example 3.13.
Let X = {a, b} and G = <x, (0.7, 0.7), (0.3, 0.3)> . Then τ ={0~, G, 1~} is an IFT on X and the IFS A = <x, (0.4, 0.4), (0.6, 0.6)> is an IFWCS but not an IF$\ddot{g}$ CS in (X, τ) .

In the following diagram, we have provided the relation among various types of intuitionistic fuzzy closed sets with IF$\ddot{g}$ CS.

IFWCS

↗

IFRCS → IFCS → IF$\ddot{g}$ CS → IFGCS

IFRGCS

↘

IFGαCS

In this diagram “A → B”, we mean A implies B but not conversely.

Theorem 3.14.
Let A and B be two IF$\ddot{g}$ CSs in an IFTS (X, τ) then A ∪ B is an IF$\ddot{g}$ CS in X .
Proof:
Let U be an IFSGOS in (X, τ) such that A ∪ B ⊆ U . Since A and B are IF $\ddot{g}$ CSs, we have cl(A) ⊆ U and cl(B) ⊆ U. Therefore by the Result 1.2.6, cl(A) ∪ cl(B) = cl(A ∪ B) ⊆ U. Hence A ∪ B is an IF $\ddot{g}$ CS in (X, τ).

Theorem 3.15.
If A is an IF$\ddot{g}$ CS and A ⊆ B ⊆ cl(A), then B is an IF$\ddot{g}$ CS.
Proof:
Let U be an IFSGOS such that B ⊆ U . Since A is an IF$\ddot{g}$ CS,
we have cl(A) ⊆ U . By hypothesis cl(B) ⊆ cl(A) ⊆ U . Hence B is an IF$\ddot{g}$ CS

IV.INTUITIONISTIC FUZZY $\ddot{g}$ OPEN SETS

Definition 4.1.
An IFS A is said to be an intuitionistic fuzzy $\ddot{g}$ open set(IF$\ddot{g}$ OS in short) if A^C is an IF$\ddot{g}$ CS in X .

Theorem 4.2.
For any IFTS (X, τ) , we have the following:
1. Every IFOS is an IF$\ddot{g}$ OS,
2. Every IFROS is an IF$\ddot{g}$ OS.
Proof: Obvious.

Remark 4.3.

The converses of the above theorem need not be true in general as seen from the following examples.

Example 4.4.
Let $X = \{a, b\}$ and $G = <x, (0.5, 0.6), (0.5, 0.4)>$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X and the IFS $A = <x, (0.6, 0.6), (0.4, 0.4)>$ is an IF$\ddot{g}$ OS but not an IFOS and IFROS in (X, τ).

Theorem 4.5.
An IFS A of an IFTS (X, τ) is an IF$\ddot{g}$ OS if and only if $U \subseteq int(A)$ whenever $U \subseteq A$ and U is an IFSGCS.
Proof:
Assume that A is an IF$\ddot{g}$ OS in X. Let U be an IFSGCS such that $U \subseteq A$. Then U^C is an IFSGOS and $A^C \subseteq U^C$. Then by assumption A^C is an IF$\ddot{g}$ CS in X. Therefore we have $cl(A^C) \subseteq U^C$. Hence $U \subseteq int(A)$. Conversely, let U be an IFSGOS in X such that $A^C \subseteq U$. Then $U^C \subseteq A$ and U^C is an IFSGCS. Therefore $U^C \subseteq int(A)$. Since $U^C \subseteq int(A)$, we have $(int(A))^C \subseteq U$ that is $cl(A^C) \subseteq U$. Thus A^C is an IF$\ddot{g}$ CS. Hence A is an IF$\ddot{g}$ OS in X.

Theorem 4.6.
If A is an IF$\ddot{g}$ OS and $int(A) \subseteq B \subseteq A$, then B is an IF$\ddot{g}$ OS.
Proof:
If $int(A) \subseteq B \subseteq A$, then $A^C \subseteq B^C \subseteq (int(A))^C = cl(A^C)$. Since A^C is an IF$\ddot{g}$ CS then B^C is an IF$\ddot{g}$ CS. Therefore B is an IF$\ddot{g}$OS.

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