IFgsr-Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

Anitha S[1], Mohana K[2]

[1]*Research Scholar, Department of Mathematics, Nirmala College for Women*

[2]*Assistant Professor, Department of Mathematics, Nirmala College for Women*

Abstract- **The aim of this paper is to establish intuitionistic fuzzy gsr-continuous mappings and to study some of their properties. Further we introduce intuitionistic fuzzy gsr irresolute mappings and investigate their characterizations.**

Index Terms- Intuitionistic fuzzy topology, Intuitionistic fuzzy continuous mappings, Intuitionistic fuzzy gsr continuous mappings.

1. INTRODUCTION

Zadeh [14] introduced the fuzzy concept in 1965 and the theory of fuzzy topology was introduced and developed by C.L. Chang [3]. Atanassov [2] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. In 1997 Coker[4] defined intuitionistic fuzzy topological spaces. This approach provided a wide field for investigation in the area of fuzzy topology and its applications. One of the directions is related to the properties of intuitionistic fuzzy sets introduced by Gurcay [5] in 1997.

We define the notion of intuitionistic fuzzy gsr-continuous mappings and intuitionistic fuzzy gsr-irresolute mappings. We discuss characterizations of intuitionistic fuzzy gsr- continuous mappings and irresolute mappings. We also established their properties and relationships with other classes of early defined forms of intuitionistic continuous mappings.

2. PRELIMINARIES

Definition 2.1. [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{< x, \mu A(x), \nu A(x) > / x \in X\}$ where the functions $\mu A(x) : X \to [0,1]$ and $\nu A(x) : X \to [0,1]$ denote the degree of membership (namely $\mu A(x)$) and the degree of non-membership (namely $\nu A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu A(x) + \nu A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy set in X.

Definition 2.2. [1] $A = \{< x, \mu A(x), \nu A(x) > / x \in X\}$ and $B = \{< x, \mu B(x), \nu B(x) > / x \in X\}$. Then

$A \subseteq B$ if and only if $\mu A(x) \leq \mu B(x)$ and $\nu A(x) \geq \nu B(x)$ for all $x \in X$

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

$Ac = \{< x, \nu A(x) , \mu A(x) > / x \in X\}$

$A \cap B = \{< x, \mu A(x) \wedge \mu B(x), \nu A(x) \vee \nu B(x) > / x \in X\}$

$A \cup B = \{< x, \mu A(x) \vee \mu B(x), \nu A(x) \wedge \nu B(x) > / x \in X\}$

For the sake of simplicity, Let us use the notation $A = <x, \mu A, \nu A >$ instead of $A = \{< x, \mu A(x), \nu A(x) > / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = < x, (\mu A, \mu B) , (\nu A, \nu B)>$ instead of $A = <x, (A/\mu A , B/\mu B) , (A/(\nu A, B/\nu B)>$.

The intuitionistic fuzzy sets $0\sim = \{ < x, 0, 1> / x \in X\}$ and $1\sim = \{ < x, 1, 0 >/ x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3. [4] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

$0\sim, 1\sim \in \tau$

$G1 \cap G2 \in \tau$, for any $G1 , G2 \in \tau$

$\cup Gi \in \tau$ for any family $\{ Gi / i \in J\} \subseteq \tau$.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short) in X.

The complement (Ac) of an IFOS A in an IFTS(X,τ) is called an intuitionistic fuzzy closed set(IFCS in short) in X.

Definition 2.4. [2] Let X and Y be two nonempty sets and $f : X \rightarrow Y$ be a function. Then
If $B = \{< y, \mu\_B(y), \nu\_B(y) > / y \in Y\}$ is an IFS in Y , then the preimage of B under f denoted by $f^{-1}(B)$ is the IFS in X defined by $f^{-1}(B) = \{< x, f^{-1}(\mu\_B)(x), f^{-1}(\nu\_B)(x) > / x \in X\}$

Definition 2.5. [4] Let (X,τ) be an IFTS and $A = <x, \mu A, \nu A>$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by
$int(A) = \cup \{G / G$ is an IFOS in X and $G \subseteq A\}$,
$cl(A) = \cap \{K / K$ is an IFCS in X and $A \subseteq K\}$.

Definition 2.6. [10] An IFS $A = <x, \mu A, \nu A>$ in an IFTS(X,τ) is said to be an
(ii) intuitionistic fuzzy regular open set(IFROS in short) if $A = int(cl(A))$,
(iii) intuitionistic fuzzy regular closed set(IFRCS in short) if $A = cl(int(A))$,
(iiii) intuitionistic fuzzy semi open set(IFSOS in short) if $A \subseteq cl(int(A))$,
(iiv) intuitionistic fuzzy semi closed set(IFSCS in short) if $int(cl(A)) \subseteq A$,
(iv) intuitionistic fuzzy pre-open set(IFPOS in short) if $A \subseteq int(cl(A))$,
(ivi) intuitionistic fuzzy pre-closed set(IFPCS in short) if $cl(int(A)) \subseteq A$,
(ivii) intuitionistic fuzzy α- open set(IFαOS in short) if $A \subseteq int(cl(int(A))$,
(iviii) intuitionistic fuzzy α-closed set(IFαCS in short) if $cl(int(cl(A)) \subseteq A$.

Definition 2.7. [10] Let an IFS A of an IFTS (X,τ).
Then the semi closure of A (scl(A) in short) is defined as $scl(A) = \cap \{K / K$ is an IFSCS in X and $A \subseteq K\}$.
Then the semi interior of A (sint(A) in short) is defined as $sint(A) = \cup \{G / G$ is an IFSOS in X and $G \subseteq A\}$

Definition 2.8. An IFS A in an IFTS (X,τ) is said to an
(i) intuitionistic fuzzy generalized closed (IFGCS) [13] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X,
(ii) intuitionistic fuzzy α - generalized closed (IFαGCS) [9] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X,
(iii) intuitionistic fuzzy generalized α - closed (IFGαCS) [9] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFαOS in X,

Definition 2.9.[1] An IFS A in an IFTS (X,τ) is said to be an Intuitionistic fuzzy gsr-closed sets (IFGSRCS in short) if $scl(A) \subseteq U$ and U is an IFROS in (X, τ).
An IFS A is said to be an intuitionistic fuzzy gsr-open set (IFGSROS in short) in (X,τ) if the A^c is an IFGSRCS in (X,τ).

Definition 2.10. Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ).
Then f is said to be an
(1) intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in IFOS(X)$ for every $B \in \sigma$ [5],
(2) intuitionistic fuzzy α -continuous (IFα continuous in short) if $f^{-1}(B) \in IF\alpha OS(X)$ for every $B \in \sigma$ [8],
(3) intuitionistic fuzzy pre continuous (IFP continuous in short) if $f^{-1}(B) \in IFPOS(X)$ for every $B \in \sigma$ [8].
(4) intuitionistic fuzzy generalized continuous(IFG continuous in short) if $f^{-1}(B)$ is an IFGCS for every IFCS B of (Y, σ)[13],
(5) intuitionistic fuzzy generalized semi continuous(IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS for every IFCS B of (Y, σ)[11],
(6) intuitionistic fuzzy α -generalized continuous(IFαG continuous in short) if $f^{-1}(B)$ is an IFαGCS for every IFCS B of (Y, σ)[10],
(7) intuitionistic fuzzy generalized α continuous(IFGα continuous in short) if $f^{-1}(B)$ is an IFGαCS for every IFCS B of (Y, σ)[9].

Defnition 2.11. [11]Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ).
Then f is said to be an intuitionistic fuzzy irresolute (IF irresolute in short) if $f^{-1}(B) \in IFCS(X)$ for every IFCS B in Y.

Lemma 2.12. [6] Let $g : X \rightarrow X \times Y$ be the graph of a function $f : X \rightarrow Y$. If A is an IFS of X and B is an IFS of Y , then $g^{-1}(A\times B)(x) = (A\cap f^{-1}(B))(x)$.

3. INTUITIONISTIC FUZZY GSR-CONTINUOUS MAPPINGS

Definition3.1. Let A be an IFS in an IFTS (X, τ). Then the intuitionistic fuzzy gsr interior and intuitionistic fuzzy gsr closure of A are defined as follows.
IFgsrint(A) = ∪{G |G is an IFGSROS in X and G ⊆ A},
IFgsrcl(A) = ∩{K | K is an IFGSRCS in X and A ⊆ K}.

Proposition 3.2. Let A be an IFS in X, then A ⊆IFgsrcl(A) ⊆ scl(A) ⊆cl(A).

Proof: Obvious.
Theorem 3.3. Let A be an IFGSRCS in X, then IFgsrcl(A) = A.

Proof: Let A be an IFGSRCS, IFgsrcl(A) is the smallest IFGSRCS which contains A, which is nothing but A. Thus IFgsrcl(A) = A.

Theorem 3.4. Let A be an IFGSROS in X, then IFgsrint(A) = A.

Proof: Similar to 3.3.
Definition 3.5. A mapping f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy gsr-continuous (IFGSR continuous in short) if f -1(B) is an IFGSRCS in (X,τ) for every IFCS B of (Y, σ).

Example 3.6. Let X = {a, b}, Y = {u, v} and Let G1= ⟨x,(0.3,0.4),(0.7,0.5)⟩ and G2 = ⟨y,(0.5,0.5),(0.4,0.5)⟩. Then τ = {0~, 1~, G1 } and σ = {0~, 1~, G2} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then f -1(B) is an IFGSRCS in (X, τ) for every IFCS B of (Y, σ). Therefore f is an IFGSR-continuous.

Example 3.7. Let X = {a, b}, Y = {u, v}.Let G1=⟨x,(0.3,0.4),(0.7,0.5)⟩ and G2 = ⟨y,(0.5,0.6),(0.4,0.3)⟩. Then τ = {0~, 1~, G1 } and σ = {0~, 1~, G2} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then for IFCS B = ⟨y,(0.4,0.3),(0.5,0.6)⟩ of (Y, σ), f -1(B) is not an IFGSRCS in (X, τ). Therefore f is not an IFGSR-continuous.

Theorem 3.8. Every IF continuous mapping is an IFGSR -continuous mapping.

Proof: Suppose f : (X, τ) → (Y, σ) is an IF continuous mapping and Let B be an IFCS in Y. By assumption, f is an IF continuous mapping, f -1(B) is an IFCS in X. Since every IFCS is an IFGSRCS, f -1(B) is an IFGSRCS in X. Therefore f is an IFGSR -continuous mapping.
The converse of the above theorem need not be true as seen from the following example.

Example 3.9. Let X = {a, b}, Y = {u, v}. Let G1 = ⟨x,(0.1,0.3),(0.6,0.5)⟩ and G2 =⟨x,(0.2,0.3),(0.5,0.4)⟩. Then τ = {0~, 1~, G1 } and σ = {0~, 1~, G2} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then for IFCS B = ⟨y,(0.5,0.4),(0.2,0.3)⟩ of (Y, σ), f -1(B) is not an IFCS in (X, τ). Thus, f is not an IF continuous. But f is an IFGSR- continuous.

Theorem 3.10. Every IFG- continuous mapping is an IFGSR -continuous mapping.

Proof: Consider f : (X, τ) → (Y, σ) be an IFG-continuous mapping. Suppose B is an IFCS in Y. By the assumption f is an IFG- continuous mapping, f -1(B) is an IFGCS in X. Since every IFGCS is an IFGSRCS, f -1(B) is an IFGSRCS in X. It follows that f is an IFGSR -continuous mapping.

Example 3.11. Let X = {a, b}, Y = {u, v}. Let G1=⟨x,(0.3,0.2),(0.7,0.8)⟩ and G2= ⟨x,(0.7,0.8),(0.3,0.2)⟩. Then τ = {0~, 1~, G1 } and σ = {0~, 1~, G2} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then for IFCS B = ⟨y,(0.3,0.2),(0.7,0.8)⟩ of (Y, σ), f -1(B) is not an IFGCS in (X, τ). Therefore f is not an IFG -continuous. But f is an IFGSR continuous.

Theorem 3.12. Every IFα -continuous mapping is an IFGSR- continuous mapping.

Proof: Suppose f : (X, τ) → (Y, σ) be an IFα -continuous mapping. If B be an IFCS in Y. By the assumption we have, f as an IFα -continuous mapping, f -1(B) is an IFαCS in X. Since every IFαCS is an IFGSRCS, f -1(B) is an IFGSRCS in X. Therefore f is an IFGSR -continuous mapping.

Example 3.13. Let X = {a, b}, Y = {u, v}. Let G1=⟨x,(0.2,0.3),(0.6,0.5)⟩ and G2 = ⟨y,(0.3,0.3),(0.6,0.4)⟩. Then τ = {0~, 1~, G1 } and σ = {0~, 1~, G2} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then for IFCS B = ⟨y,(0.6,0.4),(0.3,0.3)⟩ of (Y, σ), f -1(B) is not an IFαCS in (X, τ). Therefore f is not an IFα -continuous. But f is an IFGSR -continuous.

Theorem 3.14. Every IFαG- continuous mapping is an IFGSR- continuous mapping.
Proof: Suppose f : (X, τ) → (Y, σ) be an IFαG -continuous mapping. And if B is an IFCS in Y. By the assumption, f is an IFαG-continuous mapping, f -1(B) is an IFαGCS in X. Since every IFαGCS is an IFGSRCS, f -1(B) is an IFGSRCS in X. Thus f is an IFGSR- continuous mapping.

Example 3.15. Let X = {a, b}, Y = {u, v}. Let G1=⟨x,(0.3,0.4),(0.7,0.5)⟩ and G2 = ⟨y,(0.6,0.4),(0.3,0.5)⟩. Then τ = {0~, 1~, G1 } and σ = {0~, 1~, G2} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then for IFCS B = ⟨y,(0.3,0.5),(0.6,0.4)⟩ of (Y, σ), f -1(B) is not an IFαGCS in (X, τ). Therefore f is not an IFαG -continuous. But f is an IFGSR- continuous.

Theorem 3.16. Every IFGα -continuous mapping is an IFGSR -continuous mapping.
Proof: Consider f : (X, τ) → (Y, σ) be an IFGα -continuous mapping and B be an IFCS in Y. By the assumption, f is an IFGα -continuous mapping, f -1(B) is an IFGαCS in X. Since every IFGαCS is an IFGSRCS, f -1(B) is an IFGSRCS in X. Therefore f is an IFGSR -continuous mapping.

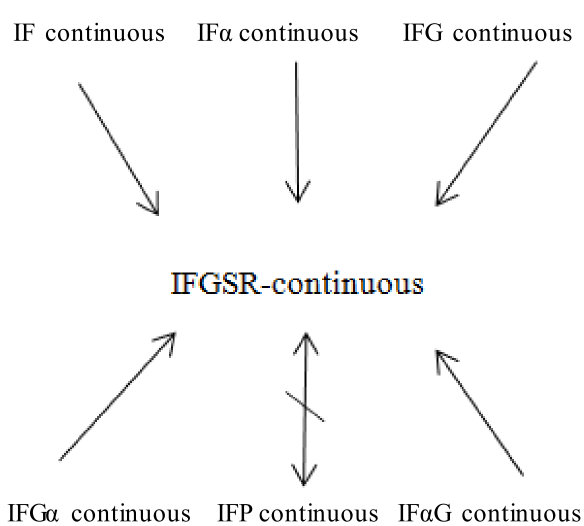
Example 3.17. Let X = {a, b}, Y = {u, v}. Let G1=⟨x,(0.3,0.4),(0.7,0.5)⟩ and G2 = ⟨y,(0.6,0.4),(0.3,0.5)⟩. Then τ = {0~, 1~, G1 } and σ = {0~, 1~, G2} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then for IFCS B = ⟨y,(0.3,0.5),(0.6,0.4)⟩ of (Y, σ), f -1(B) is not an IFαGCS in (X, τ). Therefore f is not an IFGα -continuous. But f is an IFGSR -continuous.

Remark 3.18. An IFGSR -continuous mapping is independent of IFP -continuous mapping.

Example 3.19. Let X = {a, b}, Y = {u, v}. Let G1=⟨x,(0.2,0.3),(0.7,0.6)⟩ and G2 = ⟨y,(0.3,0.4),(0.6,0.5)⟩. Then τ = {0~, 1~, G1 } and σ = {0~, 1~, G2} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then for IFCS B = ⟨y,(0.6,0.5),(0.3,0.4)⟩ of (Y, σ), f -1(B) is not an IFPCS in (X, τ). Therefore f is not an IFP -continuous. But f is an IFGSR continuous.

Example 3.20. Let X = {a, b}, Y = {u, v}. Let G1=⟨x,(0.5,0.4),(0.5,0.6)⟩ and G2 = ⟨y,(0.6,0.7),(0.4,0.2)⟩. Then τ = {0~, 1~, G1 } and σ = {0~, 1~, G2} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then for IFCS B = ⟨y,(0.4,0.2),(0.6,0.7)⟩ of (Y, σ), f -1(B) is not an IFGSRCS in (X, τ). Therefore f is not an IFGSR -continuous. But f is an IFP -continuous.

Remark 3.21. We obtain the following diagram from the results we discussed above.



Theorem 3.22. A mapping f : (X, τ) → (Y, σ) is an IFGSR-continuous if and only if the inverse image of every IFOS in Y is an IFGSROS in X.

Proof: Necessary Part: Let A be an IFOS in Y. This implies Ac is an IFCS in Y. Since f is an IFGSR-continuous, f -1(Ac) is an IFGSRCS in X. Since f -1(Ac) = (f -1(A))c , f -1(B)) is an IFGSROS in X. Hence the inverse image of an IFOS in Y is an IFGSROS in X.

Sufficient Part: Let A be an IFCS in Y. Then Ac is an IFOS in Y. By hypothesis, f -1(Ac) is an IFGSROS in X. Since f -1(Ac) = (f -1(A))c, f -1(A) is an IFGSRCS in X. Hence f is an IFGSR-continuous function.

Theorem 3.23. Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy gsr-continuous mapping. Then the following statements hold.

(i) f (IFgsrc1(A)) ⊆c1(f (A)), for every intuitionistic fuzzy set A in X.

(ii) IFgsrc1(f−1(B)) ⊆ f−1(c1(B)) for every intuitionistic fuzzy set B in Y .

Proof: (i) Let A ⊆X. Then c1(f (A)) is an intuitionistic fuzzy closed set in Y. Since f is intuitionistic fuzzy gsr-continuous, f −1(c1(f (A))) is intuitionistic fuzzy gsr-closed in X. Since A ⊆ f −1(f (A)) ⊆ f −1(c1(f (A))) and f −1(c1(f (A))) is intuitionistic fuzzy gsr-closed, implies IFgsrc1(A) ⊆ f −1(c1(f (A))). Hence f(IFgsrc1(A)) ⊆c1(f (A)).

(ii) Replacing A by f −1(B) in (i), we get f (IFgsrcl(f −1(B))) ⊆ c1(f (f −1(B))) = c1(B)

f (IFgsrcl(f −1(B))) ⊆ c1(B). Hence IFgsrc1(f−1(B)) ⊆ f −1(c1(B)).

Theorem 3.24. Let f : X → Y be a mapping and g : X → X ×Y the graph of the mapping f .Then f is intuitionistic fuzzy gsr-continuous if g is so.

Proof. Let B be an IFOS in Y . Then by Lemma 2.12, f −1(B) = f −1(1∼ ×B) = 1∼ ∩ f −1(B) =g−1(1∼×B). Since B is an IFOS Y , 1∼×B is an IFOS in X ×Y . Also since g is intuitionistic fuzzy gsr-continuous implies that g−1(1∼ ×B) is an IFGSROS in X. Therefore f −1(B) is an IFGSROS in X. Hence f is intuitionistic fuzzy gsr-continuous mapping.

Theorem 3.25. If f : X →Y is intuitionistic fuzzy gsr-continuous and g : Y →Z is intuitionistic fuzzy continuous, then g ◦ f : X →Z is intuitionistic fuzzy gsr-continuous.

Proof. Let B be any intuitionistic fuzzy closed set in Z. Since g is intuitionistic fuzzy continuous, g−1(B) is intuitionistic fuzzy closed set in Y . Since f is intuitionistic fuzzy gsr-continuous mapping f −1[g−1(B)] is an intuitionistic fuzzy gsr-closed set in X. (g ◦ f)−1(B) = f −1(g−1(B)) is intuitionistic fuzzy gsr-closed set, for every intuitionistic fuzzy closed B in Z. Hence g ◦ f is intuitionistic fuzzy gsr-continuous mapping.

Defnition 3.26. An IFTS (X, τ) is said to be an

(1) intuitionistic fuzzy gsr T*1⁄2 (in short IFgsr T*1⁄2)space if every IFGSRCS in X is an IFCS in X,

(2) intuitionistic fuzzy gsr T**1⁄2 (in short IFgsr T**1⁄2)space if every IFGSRCS in X is an IFGCS in X.

Theorem 3.27. Let f: (X, τ) → (Y, σ) be an IFGSRS continuous mapping, then f is an IF continuous mapping if X is an IFgsr T*1⁄2 space.

Proof: Let A be an IFCS in Y. Then f-1(A) is an IFGSRCS in X, by hypothesis. Since X is an IFgsr T*1⁄2, f-1(A) is an IFCS in X. Hence f is an IF continuous mapping.

Theorem 3.28. Let f: (X,τ) → (Y,σ) be an IFGSR continuous mapping, then f is an IFG continuous mapping if X is an IFgsr T**1⁄2 space.

Proof: Let A be an IFCS in Y. Then f -1(A) is an IFGSRCS in X, by hypothesis. Since X is an IFgsr T**1⁄2, f -1(A) is an IFGCS in X. Hence f is an IFG continuous mapping.

Theorem 3.29. Let f: (X, τ) → (Y, σ) is a mapping from an IFTS X into an IFTS Y . If X is IFgsr T*1⁄2 space, then f is intuitionistic fuzzy gsr-continuous iff it is intuitionistic fuzzy continuous.

Proof: Let f be intuitionistic fuzzy gsr-continuous mapping and let A be an intuitionistic fuzzy closed set in Y . Then by definition of intuitionistic fuzzy

gsr continuous f-1(A) is intuitionistic fuzzy gsr-closed in X. Since X is IFgsr T*1⁄2 space, f −1(A) is intuitionistic fuzzy closed set. Hence f is intuitionistic fuzzy continuous.
Conversely, assume that f is intuitionistic fuzzy continuous. Then by f is intuitionistic fuzzy gsr-continuous mapping.

Theorem 3.30. Let X, X1, X2 are IFTS's and pi : X1 × X2 → Xi (i = 1,2) are projections of X1 ×X2 onto Xi. If f : X → X1 × X2 is intuitionistic fuzzy gsr-continuous, then pi ◦ f (i = 1,2) is intuitionistic fuzzy gsr-continuous mapping.

Proof: The proof follows from the facts that projections are intuitionistic fuzzy continuous mappings.

4. INTUITIONISTIC FUZZY GSR IRRESOLUTE MAPPINGS

Definition 4.1. A mapping f : X →Y from an IFTS X into an IFTS Y is said to be intuitionistic fuzzy gsr irresolute (intuitionistic fuzzy gsr-irresolute) if f −1(B) is an IFGSRCS in X for every IFGSRCS B in Y .

Theorem 4.2. Let f : X →Y is a mapping from an IFTS X into an IFTS Y . Then every intuitionistic fuzzy gsr-irresolute mapping is intuitionistic fuzzy gsr-continuous.

Proof: Assume that f : X →Y is an intuitionistic fuzzy gsr-irresolute mapping and let B be an IFCS in Y . Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy gsr-closed. Therefore B is an IFGSRCS in Y . Since f is intuitionistic fuzzy gsr-irresolute, by definition f −1(B) is IFGSRCS in X. Hence f is intuitionistic fuzzy gsr-continuous.

Example 4.3. Let X = {a, b}, Y = {u, v}. Let G1 = ⟨x,(0.8,0.4,0.4),(0.1,0.6,0.6)⟩ and G2 = ⟨y,(1,0.4,0.4),(0,0.6,0.6)⟩. Then τ = {0∼, 1∼, G1 } and σ = {0∼, 1∼, G2} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then f is an IFGSR continuous. We have B = ⟨y,(0,0.4,0.2),(0,0.6,0.6)⟩ is an IFGSRCS in Y but f -1(B) is not an IFGSRCS in X. Therefore f is not an IFGSR irresolute mapping.

Theorem 4.4. Let f : X →Y be a mapping from a IFTS X into an IFTS Y. Then the following statements are equivalent.
(i) f is intuitionistic fuzzy gsr-irresolute mapping.
(ii) f −1(B) is an IFGSROS in X, for every IFGSROS B in X.

Proof: Similar to Theorem3.22.
Theorem 4.5. Let f : X →Y be a mapping from an IFTS X into an IFTS Y . Then the following statements are equivalent.
(i) f is an intuitionistic fuzzy gsr- irresolute mapping.
(ii) f −1(B) is an IFGSROS in X for each IFGSROS B in Y .
(iii) IFgsrcl(f −1(B)) ⊆ f −1(IFgsrcl(B)), for each IFS B of Y .
(iv) f −1(IFgsrintB) ⊆IFgsrint(f −1(B)), for each IFS B of Y .

Proof: (i)⇒(ii) It can be proved by using the complement and Definition 4.1.
(ii)⇒(iii) Let B be an IFS in Y . Since B ⊆IFgsrcl(B), f −1(B) = f −1(IFgsrcl(B)). Since IFgsrcl(B) is an IFSGCS in Y , by our assumption, f −1(IFgsrcl(B)) is an IFGSRCS in X. Therefore IFgsrcl(f −1(B)) ⊆ f −1(IFgsrcl(B)).
(iii)⇒(iv) By taking complement we get the result.
(iv)⇒(i) Let B be any IFGSROS in Y . Then IFgsrint(B) = B. By our assumption we have f −1(B) =
f −1(IFgsrint(B)) ⊆IFgsrint(f −1(B)), so f −1(B) is an IFGSROS in X. Hence f is intuitionistic fuzzy gsr-irresolute mapping.

Theorem 4.6. If a mapping f : X → Y is intuitionistic fuzzy gsr-irresolute mapping, then
f (IFgsrcl(B)) ⊆scl(f (B)) for every IFS B of X.
Proof. Let B be an IFS of X. Since scl(f (B)) is an IFGSRCS in Y , by our assumption f −1(scl (f (B))) is an IFGSRCS in X. Furthermore B ⊆ f −1(f (B)) ⊆f −1(scl(f (B))) and hence IFgsrcl(B) ⊆ f −1(scl(f (B))) and consequently f (IFgsrcl(B)) ⊆ f(f −1(scl(f (B)))) ⊆scl(f (B)).

Theorem 4.7. Let f : X →Y and g : Y → Z are intuitionistic fuzzy gsr-irresolute mappings, where X,Y ,Z are IFTS. Then g ◦ f is an intuitionistic fuzzy gsr-irresolute mapping.

Proof. Let B be an intuitionistic fuzzy gsr-closed set in Z. Since g is an intuitionistic fuzzy gsr irresolute mapping, $g^{-1}(B)$ is an intuitionistic fuzzy gsr-closed set in Y . Also since f is intuitionistic fuzzy gsr irresolute mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy gsr-closed set in X. $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$ for each B in Z. Hence $(g \circ f)^{-1}(B)$ is an intuitionistic fuzzy gsr-closed set in X. Therefore $g \circ f$ is an intuitionistic fuzzy gsr irresolute mapping.

Theorem 4.8. Let (X, τ), (Y, σ), (Z, δ) be any intuitionistic fuzzy topological spaces. Let $f :(X, \tau) \to (Y, \sigma)$ be intuitionistic fuzzy gsr irresolute and $g : (Y, \sigma) \to (Z, \delta)$ is intuitionistic fuzzy continuous, then $g \circ f$ is intuitionistic fuzzy gsr continuous.

Proof. Let B be any intuitionistic fuzzy closed set in Z. Since g is intuitionistic fuzzy continuous,
$g^{-1}(B)$ is IFCS in Y . Since every IFCS is an IFGSRCS. Therefore $f^{-1}(B)$ is an IFGSRCS in Y . But since f is an intuitionistic fuzzy gsr- irresolute mapping $f^{-1}(g^{-1}(B))$ is an IFGSRCS in X.
$(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$ is IFGSRCS in X. Hence $g \circ f$ is intuitionistic fuzzy gsr-continuous.

Theorem 4.9. Let f: X →Y be intuitionistic fuzzy gsr-irresolute mapping. Then f is intuitionistic fuzzy irresolute mapping if (X, τ) is intuitionistic fuzzy gsr $T^*1/2$ space.

Proof. Let B be an IFCS in Y . Then B is an IFGSRCS in Y . Since f is intuitionistic fuzzy gsr-irresolute, $f^{-1}(B)$ is an IFGSRCS in X. But (X, τ) is intuitionistic fuzzy gsr $T^*1/2$ space implies $f^{-1}(B)$ is an IFCS in X. Hence f is intuitionistic fuzzy irresolute.

REFERENCES

[1] Anitha.S, Mohana K., IFgsr-closed sets in intuitionistic fuzzy topological spaces, Internat.J. of Innovative research in technology, Volume 5, Issue 2(2018).

[2] Atanassov.K.T, Intuitionistic fuzzy sets, Fuzzy sets and systems, 20(1986), 87-96.

[3] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24(1986), 182-190.

[4] Coker.D, An introduction to Intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 88(1997), 81-89.

[5] Gurcay. H., Coker. D.,and Es. A. Haydar., On fuzzy continuity in intuitionistic fuzzy topological spaces, Jour. of Fuzzy Math., 5(1997), 365-378.

[6] I.M. Hanafy, Completely continuous functions in Intuitionistic fuzzy topological spaces, Czechoslovak Mathematical Journal, 53(2003), 793–803.

[7] M.Jeyaraman, A.Yuvarani andO.Ravi, Intuitionistic fuzzy _-generalized semi Continuous and irresolute mappings, International Journal of Analysis and Applications, 3(2)(2013), 93-103.

[8] Joung Kon Jeon, Young Bae Jun, and Jin Han Park, Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity, International Journal of Mathematics and Mathematical Sciences, 19(2005), 3091-3101.

[9] Kalamani. D, Sakthivel. K and Gowri. C. S., Generalized alpha closed sets in intuitionistic fuzzy topological spaces, Applied Mathematical Sciences, 6(2012), 4691-4700.

[10] Sakthivel, K., 2010, Intuitionistic Fuzzy Alpha Generalized Continuous Mappings and Intuitionistic Alpha Generalized Irresolute Mappings, Applied Mathematical Sciences. Vol. 4, no. 37. pp. 1831 – 1842.

[11] Santhi. R and Sakthivel. K., Intuitionistic fuzzy generalized semi continuous mappings, Advances in Theoretical and Applied Mathematics, 5(2009), 73-82.

[12] R. Santhi and K. Arun Prakash, Intuitionistic fuzzy semi-generalized irresolute mappings,Tamkang Journal of Mathematics 42(2) ,(2011) 119-134.

[13] Thakur. S. S and Rekha Chaturvedi., Generalized closed sets in intuitionistic fuzzy topology, The Journal of Fuzzy Mathematics, 16(2008), 559-572.

[14] L.A. Zadeh, Fuzzy sets, Information and Control, 8(1965), 338-353.