A Heptagonal Fuzzy Number in Solving Fuzzy Sequencing Problem

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Abstract- **In this paper, we consider the fuzzy job sequencing problem, where processing time is taken as heptagonal fuzzy numbers .Also, Pascal's triangular graded mean for heptagonal fuzzy ranking index method has been applied to transform heptagonal fuzzy sequencing problem into crisp ones, using linguistic variable and solved by Johnson's algorithm. Further suitable example is discussed.**

Index Terms- Fuzzy sequencing problem, heptagonal fuzzy number, Pascal's Triangular graded mean technique.

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [9] in 1965. Operations research is a problem solving and decision making Science. Modeling is the essence of operations research. A sequencing problem is to determine the optimal sequence in which 'n' jobs to be performed by 'm' machines and various optimality criteria like minimum elapsed time ,minimum idle time, minimum inventory cost with the given conditions 1) the order of the machine in which each job should be performed 2) the actual or expected time required by the jobs on each of the machines. In this paper the processing times are taken as heptagonal fuzzy numbers ,using Pascal's triangular graded mean method, the fuzzy sequencing problem , the processing time can be converted to a crisp valued ,and solved using Johnson's algorithm. The total elapsed time and idle time for each machine is obtained by solving corresponding crisp sequencing problem.

2. PRELIMINARIES

Definition 2.1 (Fuzzy set [9]). Let X be a set. A fuzzy set A on X is defined to be a function $A: X \rightarrow [0,1]$ or $\mu_A: X \rightarrow [0,1]$. Equivalently, a fuzzy set A is defined to be the class of objects having the following representation $A = \{(x, \mu_A x): x \in X\}$ where $\mu_A: X \rightarrow [0,1]$, is a function called the membership function of A.

Definition 2.2 (Fuzzy number [4]). The fuzzy number A is a fuzzy set whose membership function $\mu_A(x)$ satisfies the following conditions :

(1) $\mu_A(x)$ is piecewise continuous;

(2) A fuzzy set A of the universe of discourse X is convex;

(3) A fuzzy set of the universe of discourse X is called a normal fuzzy set if $\exists x_i \in X$, $\mu_A(x_i) = 1$.

2.3 Linguistic Variables [8]

A linguistic variable is a variable whose values are linguistic terms. The concept of linguistic variables is applied in dealing with situations which are too complex (or) too ill-defined to be reasonably described in conventional quantitative expressions. For example ; Weight –is a linguistic variable ,its values can be very very high, very high, high, medium, low, very low, very very low etc. These values can be represented by fuzzy numbers.

2.4 Processing of 'n' Jobs through 2 Machines [6]

Let there be 'n' jobs say $A_1, A_2, A_3, \dots \dots . . A_n$ be processed through '2' machines say M_1, M_2 in the order $M_1 M_2$. Let t_{ij} be the fuzzy processing times taken by i^{th} job to be completed by j^{th} Machine. The well-known Johnson method can be extended to this problem, then we find Optimal sequence, total elapsed time and Idle time on Machines. Job machine fuzzy time for 'n' jobs and 2 machines are gives below.

Jobs	Machine M_1	Machine M_2
A_1	$t_{11} = (a_{11}, b_{11}, c_{11}, d_{11}, e_{11}$	$t_{12} = (a_{12}, b_{12}, c_{12}, d_{12}, e_{12}, f$

A_2	$t_{21} = (a_{21}, b_{21}, c_{21}, d_{21}, e_2$	$t_{22} = (a_{22}, b_{22}, c_{22}, d_{22}, e_{22},$
A_3	$t_{31} = (a_{31}, b_{31}, c_{31}, d_{31}, e_3$	$t_{32} = (a_{32}, b_{32}, c_{32}, d_{32}, e_{32},$
A_4	$t_{41} = (a_{41}, b_{41}, c_{41}, d_{41}, e_4$	$t_{42} = (a_{42}, b_{42}, c_{42}, d_{42}, e_{42},$
A_5	$t_{51} = (a_{51}, b_{51}, c_{51}, d_{51}, e_5$	$t_{52} = (a_{52}, b_{52}, c_{52}, d_{52}, e_{52},$

2.5 Pascal's Triangular Graded Mean for Pentagonal Fuzzy Number [6]

Let $\bar{A}_h = \{a_1, a_2, a_3, a_4, a_5$ be a Pentaconal fuzzy numbers,We can take the coefficient of fuzzy numbers from Pascal's triangles and apply the simple probability approach we get the following formula

$$P(A) = \frac{a_1 + 4a_2 + 6a_3 + 4a_4 + a_5}{16}$$

The coefficient of a_1, a_2, a_3, a_4, a_5 are 1,4,6,4,1.This Procedure is simply taken from from the Pascal's triangles.Thess are useful to take the coefficients of fuzzy variables are Pascal triangular numbers and we just add and divided by the total of pascal numbers..

3. HEPTAGONAL FUZZY NUMBER

3.1 Processing of 'n' Jobs through 2 Machines

Let there be 'n' jobs say $A_1, A_2, A_3, \dots \dots .. A_n$ be processed through '2' machines say M_1, M_2 in the order $M_1 M_2$. Let t_{ij} be the fuzzy processing times taken by i^{th} job to be completed by j^{th} Machine. The well-known Johnson method can be extended to this problem,then we find Optimal sequence,total elapsed time and Idle time on Machines. Job machine fuzzy time for 'n' jobs and 2 machines are gives below.Here fuzzy times are considered as heptaconal fuzzy number.

Jobs	Machine M_1	Machine M_2
A_1	$t_{11} = (a_{11}, b_{11}, c_{11}, d_{11}, e_1$	$t_{12} = (a_{12}, b_{12}, c_{12}, d_{12}, e_{12}, f$
A_2	$t_{21} = (a_{21}, b_{21}, c_{21}, d_{21}, e_2$	$t_{22} = (a_{22}, b_{22}, c_{22}, d_{22}, e_{22}, f$
A_3	$t_{31} = (a_{31}, b_{31}, c_{31}, d_{31}, e_3$	$t_{32} = (a_{32}, b_{32}, c_{32}, d_{32}, e_{32}, f$
A_4	$t_{41} = (a_{41}, b_{41}, c_{41}, d_{41}, e_4$	$t_{42} = (a_{42}, b_{42}, c_{42}, d_{42}, e_{42}, f$
A_5	$t_{51} = (a_{51}, b_{51}, c_{51}, d_{51}, e_5$	$t_{52} = (a_{52}, b_{52}, c_{52}, d_{52}, e_{52}, f$
A_6	$t_{61} = (a_{61}, b_{61}, c_{61}, d_{61}, e_6$	$t_{62} = (a_{62}, b_{62}, c_{62}, d_{62}, e_{62}, f$
A_7	$t_{71} = (a_{71}, b_{71}, c_{71}, d_{71}, e_7$	$t_{72} = (a_{72}, b_{72}, c_{72}, d_{72}, e_{72}, f$

3.2 Pascal's Triangular Graded Mean for Heptagonal Fuzzy Number

Let $\bar{A}_h = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ be a heptaconal fuzzy numbers,We can take the coefficient of fuzzy numbers from Pascal's triangles and apply the simple probability approach we get the following formula

$$P(A) = \frac{a_1 + 6a_2 + 15a_3 + 20a_4 + 15a_5 + 6a_6 + a_7}{64}$$

The coefficient of $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are 1,6,15,20,15,6,1.This Procedure is simply taken from from the Pascal's triangles.Thess are useful to take the coefficients of fuzzy variables are Pascal triangular numbers and we just add and divided by the total of pascal numbers..

3.3 Definition[5]

A heptagonal Fuzzy Number of a fuzzy set A is defined as $\bar{A}_h = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$, where $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are real numbers and its membership function is given by

$$\mu_{\bar{A}_h}(x) = \begin{cases} \frac{(x - a_1)}{2(a_2 - a_1)}, & for\ a_1 \le x \le a_2; \\ \frac{1}{2}, & for\ a_2 \le x < a_3; \\ \frac{(x - a_4)}{2(a_4 - a_3)} + 1, & for\ a_3 \le x \le a_4; \\ \frac{(a_4 - x)}{2(a_5 - a_4)} + 1, & for\ a_4 \le x \le a_5; \\ \frac{1}{2}, & for\ a_5 \le x < a_6; \\ \frac{(a_7 - x)}{2(a_7 - a_6)}, & for\ a_6 \le x \le a_7; \\ 0, & otherwise. \end{cases}$$

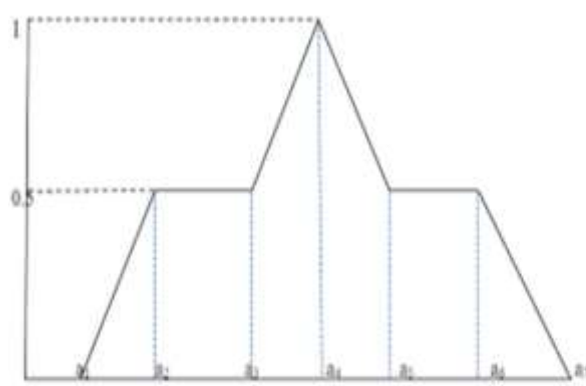

Fig 3.3 Graphical representation of Heptagonal fuzzy number

3.4 Conditions on Heptagonal Fuzzy Number

A Heptagonal Fuzzy number $\bar{A}_h$ should satisfy the following conditions:

1) $\mu_{\bar{A}_h}(x)$ is a Continuous function in the interval [0,1].
2) $\mu_{\bar{A}_h}(x)$ is strictly increasing and continuous function on $[a_1, a_2]$ and $[a_3, a_4]$.
3) $\mu_{\bar{A}_h}(x)$ is strictly decreasing and continuous function on $[a_4, a_5]$ and $[a_6, a_7]$.

3.5 Arithmetic operations on Heptagonal Fuzzy Number

If $\bar{A}_h = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ and $\bar{B}_h = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$ Then

Addition : $\bar{A}_h + \bar{B}_h = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7)$

Subtraction : $\bar{A}_h - \bar{B}_h = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6, a_7 - b_7)$

Multiplication: $\bar{A}_h \times \bar{B}_h = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4, a_5 \times b_5, a_6 \times b_6, a_7 \times b_7)$

3.6 Procedure for Solving Fuzzy Sequencing Problems

Step 1: Using Pascal's triangular graded mean approach, the fuzzy sequencing problem can be converted into crisp sequencing problem.

Step 2: The Optimal sequence for the crisp sequence problem is determined using crisp sequencing problem.

Step 3: After finding the Optimal sequence, Determine the total elapsed fuzzy time and also the fuzzy idle time on machines.

4. NUMERICAL EXAMPLE

Consider the fuzzy sequencing problem. Here the processing time of 7 jobs is given whose elements are fuzzy quantifiers which characterize the linguistic variables that are replaced by heptaconal fuzzy numbers. The problem is then solved by processing n jobs through two machines.

Jobs	Machine M_1	Machine M_2
J_1	Low	Medium
J_2	Medium	Very Low
J_3	Very Low	Very Very Low
J_4	Very Very Low	Good
J_5	Good	Very Good
J_6	Very Good	Very Very Good
J_7	Very Very Good	Low

Table 1.Quantitative data

The linguistic variables showing the qualitative data is converted into quantitative data using the full-table. As the processing time varies between 0 to 67 the minimum possible values is taken as 0 and the maximum possible value is taken as 67.

Very Very Low	(0,1,2,3,4,5,6)
Very Low	(8,10,12,14,16,18,20)
Low	(21,22,23,24,25,26,27)
Medium	(28,30,32,34,36,38,40)
Good	(41,42,43,44,45,46,47)
Very Good	(48,50,52,54,56,58,60)
Very Very Good	(61,62,63,64,65,66,67)

Table 2.Problem Table

Jobs	Machine M_1	Machine M_2
J_1	(21,22,23,24,25,26,27)	(28,30,32,34,36,38,40)
J_2	(28,30,32,34,36,38,40)	(8,10,12,14,16,18,20)
J_3	(8,10,12,14,16,18,20)	(0,1,2,3,4,5,6)
J_4	(0,1,2,3,4,5,6)	(41,42,43,44,45,46,47)
J_5	(41,42,43,44,45,46,47)	(48,50,52,54,56,58,60)
J_6	(48,50,52,54,56,58,60)	(61,62,63,64,65,66,67)
J_7	(61,62,63,64,65,66,67)	(21,22,23,24,25,26,27)

Table 3. Quantitative Table

Step 1: Apply Pascal's triangular graded mean for heptagonal fuzzy number, the fuzzy times can be Converted in to crisp times

$t_{11} = (21,22,23,24,25,26,27) = 24;$ $t_{21} = (28,30,32,34,36,38,40) = 34;$

$t_{31} = (8,10,12,14,16,18,20) = 14;$ $t_{41} = (0,1,2,3,4,5,6) = 3;$

$t_{51} = (41,42,43,44,45,46,47) = 44;$ $t_{61} = (48,50,52,54,56,58,60) = 54;$

$t_{71} = (61,62,63,64,65,66,67) = 54;$

$t_{72} = (21,22,23,24,25,26,27) = 24;$ $t_{12} = (28,30,32,34,36,38,40) = 34;$

$t_{22} = (8,10,12,14,16,18,20) = 14;$ $t_{32} = (0,1,2,3,4,5,6) = 3;$

$t_{42} = (41,42,43,44,45,46,47) = 44;$ $t_{52} = (48,50,52,54,56,58,60) = 54;$

$t_{62} = (61,62,63,64,65,66,67) = 54;$

Jobs	Machine M_1	Machine M_2
J_1	24	34
J_2	34	14
J_3	14	3
J_4	3	44
J_5	44	54
J_6	54	64
J_7	64	24

Optimum sequence

J_4	J_1	J_5	J_6	J_7	J_2	J_3

Jobs	Machine M_1		Machine M_2	
	Time in	Time out	Time in	Time out
J_4	0	3	3	47
J_1	3	27	47	81

J_5	27	71	81	135
J_6	71	125	135	199
J_7	125	189	199	223
J_2	189	223	223	237
J_3	223	237	237	240

Total elapsed time =240 Hrs

Idle time on machin M_1=3 Hrs

Idle time on machin M_2=3 Hrs.

5. CONCLUSION

In this paper, we have solved job sequencing problem with fuzzy processing times have considered heptagonal fuzzy numbers. A numerical example has been considered and solved to illustrate the proposed method. Fuzzy sequencing problem is easy to understand and helps to formulate uncertainity decision makers in real life situation.

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