Optimal Solution of Fuzzy Game Problem Using Heptagonal Fuzzy Numbers

M. Keerthana\(^1\), K. Mohana\(^2\), R. Jansi\(^3\)

\(^1\)PG Scholar, Department of Mathematics, Nirmala College for women, Coimbatore, Tamilnadu, India.
\(^2\)Assistant professor, Department of Mathematics, Nirmala college for women, Coimbatore, Tamilnadu, India.
\(^3\)Research scholar, Department of Mathematics, Nirmala college for women, Coimbatore, Tamilnadu, India.

Abstract- In this paper, we introduce a new concept of fuzzy game problem using heptagonal fuzzy numbers. Also, we convert the fuzzy valued game problem to crisp valued game problem using ranking method, which can be solved using row-minima and column maximum. Further, we discuss the solution of such fuzzy games with saddle point by Minimax - Maximin principle.

Index Terms- Fuzzy number, Membership function, Heptagonal fuzzy number, ranking of fuzzy number, saddle point.

I. INTRODUCTION

The mathematical treatement of the game theory was made available in 1944. When John von newmann and Oscar Morgenstem [7] published the famous article “theory of games and economic behaviour. Game theory has played an important role in the fields of decision making theory, economics, management etc. In 2016, Anandhi, studied an optimal solution for solving fuzzy pentagonal transportation problem. In 2017 Monisha and Sangeetha have discussed the solution of fuzzy game problem using pentagonal fuzzy number. The purpose of this paper is to introduced a new concept of fuzzy game problem using heptagonal fuzzy numbers. Also, we convert the fuzzy valued game problem to crisp valued game problem using ranking method, which can be solved using row-minima and column maximum. Further, we discuss the solution of fuzzy game with saddle point by minimax-maximin principle.

II. PRELIMINARIES

2.1. FUZZY SETS.[6]
A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval \([0, 1]\). (i.e.) \(A=\{x, \mu_A(x); x \in X\}\) here \(\mu_A: X \rightarrow [0,1]\) is a mapping called the degree of membership function of the fuzzy set A and \(\mu_A(x)\) is called the membership value of x in the fuzzy set A. These membership grades are often represented by real numbers ranging from \([0, 1]\).

2.2. FUZZY NUMBERS: [6]
A fuzzy set A defined on the set of real numbers \(\mathbb{R}\) is said to be a fuzzy number if its Membership function \(\mu_A: \mathbb{R} \rightarrow [0, 1]\) has the following characteristics.
(i) A is normal. It means that there exists an \(x \in \mathbb{R}\) such that \(\mu_A(x) = 1\).
(ii) A is convex. It means that for every \(x_1, x_2 \in \mathbb{R}\), \(\mu_A(\lambda x_1 + (1-\lambda) x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}\), \(\lambda \in [0,1]\)
(iii) \(\mu_A\) is upper semi-continuous. (iv) \(\text{Supp}(A)\) is bounded in \(\mathbb{R}\).

2.3. SADDLE POINT: [6]
A saddle point of a payoff matrix is that position in the payoff matrix. Where maximum of row minima, co-inside with the minimum of the column maxima. The pay off at the saddle point is called the value of the game denoted by \(\gamma\). The saddle point need not be unique. We denote the minimax value of the game by \(\gamma\) and the minimax value of the
game by Ŷ. The game is said to be fair. If γ = 0 = Ῡ.
The game is said to be strictly determinable. If γ = γ=

III. HEPTAGONAL FUZZY NUMBER

3.1. DEFINITION:
A heptagonal fuzzy number of a fuzzy set A is
defined as ={a1,a2,a3,a4,a5,a6,a7}, where
a1,a2,a3,a4,a5,a6,a7 are real numbers and its
membership function is given by

\[
\frac{1}{4} \left( \frac{x-a_1}{a_2-a_1}, \right. \quad \text{for } a_1 \leq x \leq a_2; \\
\frac{1}{4}, \quad \text{for } a_2 \leq x \leq a_3; \\
\frac{1}{4} + \frac{3}{4} \left( \frac{x-a_2}{a_3-a_2}, \right. \quad \text{for } a_3 \leq x \leq a_4; \\
\frac{1}{4} + \frac{3}{4} \left( \frac{a_4-x}{a_5-a_4}, \right. \quad \text{for } a_4 \leq x \leq a_5; \\
\frac{1}{4}, \quad \text{for } a_5 \leq x \leq a_6; \\
\frac{1}{4} \left( \frac{a_6-x}{a_7-a_6}, \right. \quad \text{for } a_6 \leq x \leq a_7; \\
0, \quad \text{otherwise;}
\]

Fig 3.2 Graphical representation of heptagonal fuzzy
numbers

IV. MATHEMATICAL FORMULATION OF A
FUZZY GAME PROBLEM [6]

Let player A have m strategies A1, A2...Am and
player B have n strategies B1, B2....Bn. Here, it is
assumed that each player has his choices from
amongst the pure strategies. Also it is assumed that
player A is always the gainer and player B is always

the loser. That is, all payoff are assumed in terms of
player A. Let aij be the payoff which player A gains
from player B if player A chooses strategy Ai and
player B chooses strategy Bj. Then the payoff b
matrix to player A.

4.1.PROCEDURE FOR SOLVING FUZZY GAME
PROBLEM
We shall present a solution to fuzzy game problem
involving strategies of the players using triangular
fuzzy numbers.
Step 1: Check whether a saddle point exists in the
problem. If it exists, the solution can be obtained
directly. If the saddle point does not exist, go to the
next step.
Step 2: Comparison of column strategies.
a) If elements of Column A ≤ elements of Column
B, Column A strategy dominates over column B
strategy. Hence delete column B strategy from
the pay off matrix.
b) Compare each column strategy with all possible
column strategies and delete inferior strategies as
for as possible.
Step 3: Comparison of row strategies.
a) If elements of Row A ≥ elements of Row B, Row
A strategy dominates over Row B strategy.
Hence delete Row B strategy from the pay off
matrix.
b) Compare each row strategy with all possible row
strategies and delete inferior strategies as for as possible.
c) The Game may reduce to a single cell giving
information about the value of the game and
optimal strategies of players. If not go to step 4.
d) Step 4: Dominance need not to be based on the
superiority of pure strategies only. A given
strategy can be dominated if it is inferior to an
average of two or more other pure strategies.

4.2.NUMERICAL EXAMPLE:
Consider the following fuzzy game problem

<table>
<thead>
<tr>
<th></th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>(5,7,9,11,13,15,17)</td>
<td>(1,3,5,7,9,11,13)</td>
<td>(3,5,7,9,11,13,15)</td>
<td>(8,10,12,14,16,18,20)</td>
<td>(1,4,7,10,13,16,19)</td>
</tr>
<tr>
<td>A2</td>
<td>(5,6,7,8,9,10,11)</td>
<td>(1,2,3,4,5,6,7)</td>
<td>(5,8,11,14,17,20,22)</td>
<td>(2,4,6,8,10,12,14)</td>
<td>(3,6,9,12,15,18,21)</td>
</tr>
<tr>
<td>A3</td>
<td>(6,8,10,12,14,16,18)</td>
<td>(2,3,4,5,6,7,8)</td>
<td>(4,6,8,10,12,14,16,18)</td>
<td>(4,5,6,7,8,9,10)</td>
<td>(0,1,2,3,4,5,6)</td>
</tr>
<tr>
<td>A4</td>
<td>(5,6,7,8,9,10,11)</td>
<td>(0,2,4,6,8,10,12)</td>
<td>(1,3,5,7,9,11,13)</td>
<td>(6,7,8,9,10,11,12)</td>
<td>(3,4,5,6,7,8,9)</td>
</tr>
</tbody>
</table>
SOLUTION:

Using ranking function

\[ P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 \]

\[ R(\bar{A}) = \frac{5 + 7 + 9 + 11 + 13 + 15 + 17}{7} = \frac{77}{7} = 11 \]

\[ R(5,7,9,11,13,15,17) = \frac{1 + 3 + 5 + 7 + 9 + 11 + 13}{7} = \frac{49}{7} = 7 \]

\[ R(1,3,5,7,9,11,13) = \frac{3 + 5 + 7 + 9 + 11 + 13 + 15}{7} = \frac{63}{7} = 9 \]

\[ R(3,5,7,9,11,13,15) = \frac{8 + 10 + 12 + 14 + 16 + 18 + 20}{7} = \frac{98}{7} = 14 \]

\[ R(1,4,7,10,13,16,19) = \frac{1 + 4 + 7 + 10 + 13 + 16 + 19}{7} = \frac{70}{7} = 10 \]

\[ R(5,6,7,8,9,10,11) = \frac{1 + 2 + 3 + 4 + 5 + 6 + 7}{7} = \frac{28}{7} = 4 \]

\[ R(1,2,3,4,5,6,7) = \frac{5 + 8 + 11 + 14 + 17 + 20 + 23}{7} = \frac{98}{7} = 14 \]

\[ R(5,8,11,14,17,20,23) = \frac{2 + 4 + 6 + 8 + 10 + 12 + 14}{7} = \frac{56}{7} = 8 \]

\[ R(2,4,6,8,10,12,14) = \frac{3 + 6 + 9 + 12 + 15 + 18 + 21}{7} = \frac{84}{7} = 12 \]

\[ R(3,6,9,12,15,18,21) = \frac{6 + 8 + 10 + 12 + 14 + 16 + 18}{7} = \frac{84}{7} = 12 \]

\[ R(6,8,10,12,14,16,18) = \frac{2 + 3 + 4 + 5 + 6 + 7 + 8}{7} = \frac{35}{7} = 5 \]

\[ R(2,3,4,5,6,7,8) = \frac{4 + 6 + 8 + 10 + 12 + 14 + 16}{7} = \frac{70}{7} = 10 \]

\[ R(4,6,8,10,12,14,16) = \frac{4 + 5 + 6 + 7 + 8 + 9 + 10}{7} = \frac{49}{7} = 7 \]

\[ R(4,5,6,7,8,9,10) = \frac{0 + 1 + 2 + 3 + 4 + 5 + 6}{7} = \frac{21}{7} = 3 \]

\[ R(0,1,2,3,4,5,6) = \frac{5 + 6 + 7 + 8 + 9 + 10 + 11}{7} = \frac{56}{7} = 8 \]

\[ R(5,6,7,8,9,10,11) = \frac{5 + 6 + 7 + 8 + 9 + 10 + 11}{7} = \frac{56}{7} = 8 \]
\[
\begin{align*}
R(0,2,4,6,8,10,12) &= \frac{0 + 2 + 4 + 6 + 8 + 10 + 12}{7} = \frac{42}{7} = 6 \\
R(1,3,5,7,9,11,13) &= \frac{1 + 3 + 5 + 7 + 9 + 11 + 13}{7} = \frac{49}{7} = 7 \\
R(6,7,8,9,10,11,12) &= \frac{6 + 7 + 8 + 9 + 10 + 11 + 12}{7} = \frac{63}{7} = 9 \\
R(3,4,5,6,7,8,9) &= \frac{3 + 4 + 5 + 6 + 7 + 8 + 9}{7} = \frac{42}{7} = 6
\end{align*}
\]

Minimum=maximum=7
Saddle point is (7,7)
Saddle point ath the position
Value of the game

V. CONCLUSION

In this paper, we consider some operations of heptagonal fuzzy number. The solution of such fuzzy game with saddle point by minimax-maximin principle is discussed.

REFERENCE