

Transportation Problem with Nanogonal Intuitionistic Fuzzy Numbers Solved Using Ranking Technique and Russell’s Method

Kaviya.R¹, Mohana.K², Jansi.R³

¹PG Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore

²Assistant Professor, Department of Mathematics, Nirmala College for Women, Coimbatore

³Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore

Abstract- We introduces nanogonal intuitionistic fuzzy number with its membership and non-membership functions. We define a solution methodology for transportation problem in an intuitionistic fuzzy environment in which cost are represented by nanogonal intuitionistic fuzzy numbers. Further, using a ranking technique called Accuracy function for nanogonal intuitionistic fuzzy numbers and Russell’s Method, an illustrative example is given.

I. INTRODUCTION

The theory of fuzzy set introduced by Zadeh[10] in 1965 has achieved good real life application in many fields. Atanassov[1,2] proposed the concept of intuitionistic fuzzy sets in 1986.He found that it is useful in vague concept. Degree of membership and the degree of non-membership of an element in the set has separated in intuitionistic fuzzy set. Klir[6] has proved theory based applications in fuzzy environment. Transportation problem is a particular class of linear programming, which is associated with day-to-day activities in our real life. Transportation problem was originally introduced by Hitchcock [5] in 1941. An intuitionistic fuzzy set is a powerful tool to deal with such vagueness. Felix.A ,Christopher.S and Victor Devadoss.[4]A developed ,“A Nanogonal Fuzzy Number and its arithmetic operation”.Ponnivalavan.K and Pathinathan.T [8]studied “Intuitionistic Pentagonal Fuzzy Number”.Pramila.K and Uthra.G[9] obtained “Optimal Solution of an Intuitionistic Fuzzy Transportation Problem”. The purpose of this paper is to introduce nanogonal intuitionistic fuzzy number in a more simplified way which is easy to handle and as a natural interpretation. Also, we find the optimal

solution for intuitionistic fuzzy transportation problem using nanogonal intuitionistic fuzzy number.

II. PRELIMINARIES

2.1. Definition:[3]

Intuitionistic Fuzzy Set:-

Let X be a universal set. An intuitionistic fuzzy Set A in X is defined as an object of the form $A' = \{(x, \mu_{A'}(x), \vartheta_{A'}(x)) : x \in X\}$ where the functions $\mu_{A'}: X \rightarrow [0,1]$, $\vartheta_{A'}: X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to the set A' respectively and for every $x \in X$ in A' , $0 \leq \mu_{A'}(x) + \vartheta_{A'}(x) \leq 1$ holds.

2.2. Definition:[4]

A nanogonal fuzzy number A' of an intuitionistic fuzzy set is defined as $a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1$ real numbers and its membership function $\mu_{A'}(x)$ is given by,

$$\mu_{A'}(x) = \begin{cases} \left(\frac{x - a_1}{4(b_1 - a_1)} \right) & , a_1 \leq x \leq b_1 \\ \frac{1}{4} + \left(\frac{x - b_1}{4(c_1 - b_1)} \right) & , b_1 \leq x \leq c_1 \\ \frac{1}{2} + \left(\frac{x - c_1}{4(d_1 - c_1)} \right) & , c_1 \leq x \leq d_1 \\ \frac{3}{4} + \left(\frac{x - d_1}{4(e_1 - d_1)} \right) & , d_1 \leq x \leq e_1 \\ 1 - \left(\frac{x - e_1}{4(f_1 - e_1)} \right) & , e_1 \leq x \leq f_1 \\ \frac{3}{4} - \left(\frac{x - f_1}{4(g_1 - f_1)} \right) & , f_1 \leq x \leq g_1 \\ \frac{1}{2} - \left(\frac{x - g_1}{4(h_1 - g_1)} \right) & , g_1 \leq x \leq h_1 \\ \left(\frac{i_1 - x}{4(i_1 - h_1)} \right) & , h_1 \leq x \leq i_1 \\ 0 & , \text{Otherwise} \end{cases}$$

2.3.Defintion: [3]

Intuitionistic fuzzy Number:-

An intuitionistic fuzzy subset $A' = \{(x_i, \mu_{A'}(x), \vartheta_{A'}(x) / x_i \in X\}$, of the real line R is called an intuitionistic fuzzy number if the following condition holds.

i) There exists $m \in R, \mu_{A'}(m) = 1$ and $\vartheta_{A'}(m) = 0$, (m is the mean value of A')

ii) $\mu_{A'}$ is a continuous mapping from R to the closed interval [0,1] for all $x \in R$, the relation $0 \leq \mu_{A'} + \vartheta_{A'} \leq 1$ holds.

The membership and the non-membership function of A' is of the following form.

$$\mu_{A'}(x) = \begin{cases} 0 & \text{for } -\alpha < x < m - \alpha \\ l_1(x) & \text{for } x \in [m - \alpha, m] \\ 1 & \text{for } x = m \\ h_1(x) & \text{for } x \in [m, m + \beta] \\ 0 & \text{for } m + \beta \leq x < \alpha \end{cases}$$

where $l_1(x)$ and $h_1(x)$ are strictly increasing and decreasing functions in $[m - \alpha, m]$ and $[m, m + \beta]$ respectively.

$$\vartheta_{A'}(x) = \begin{cases} 1 & \text{for } -\alpha < x < m - \alpha \\ l_2(x) & \text{for } x \in [m - \alpha', m]; 0 \leq f_1(x) + f_2(x) \leq 1 \\ 0 & \text{for } x = m \\ h_2(x) & \text{for } x \in [m, m + \beta']; 0 \leq h_1(x) + h_2(x) \leq 1 \\ 1 & \text{for } m + \beta' \leq x < \alpha \end{cases}$$

Here , m is the mean value of A' , α and β are called left and right spreads of membership and non-membership function of $\mu_{A'}(x)$ respectively. α' and β' are called left and right spreads of membership and non-membership function of $\vartheta_{A'}(x)$ respectively.

Symbolically, the intuitionistic fuzzy number is represented as $A' = (m; \alpha, \beta; \alpha', \beta')$.

2.4.Russell's Method:- [7]

There are many methods to find the basic feasible solution; one among them is Russell's Method for solving transportation problem. Russell's method is probably the better one since it generates a near optimal initial feasible solution. Here in this paper Russell's method is suitably modified and used for solving intuitionistic fuzzy transportation problem.

III. NANOGONAL INTUITIONISTIC FUZZY NUMBER[NIFN]

3.1.Definition:

A nanogonal intuitionistic fuzzy number A' of an intuitionistic fuzzy set is defined as

$$A' = \{(a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1), (a_2, b_2, c_2, d_2, e_2, f_2, g_2, h_2, i_2)\}$$

where all

$a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1, a_2, b_2, c_2, d_2, e_2, f_2, g_2, h_2, i_2$ are real numbers and its membership function $\mu_{A'}(x)$, non-membership function $\vartheta_{A'}(x)$ are given by ,

$$\mu_{A'}(x) = \begin{cases} \left(\frac{x - a_1}{4(a_2 - a_1)}\right) & , a_1 \leq x \leq a_2 \\ \frac{1}{4} + \left(\frac{x - a_2}{4(a_3 - a_2)}\right) & , a_2 \leq x \leq a_3 \\ \frac{1}{2} + \left(\frac{x - a_3}{4(a_4 - a_3)}\right) & , a_3 \leq x \leq a_4 \\ \frac{3}{4} + \left(\frac{x - a_4}{4(a_5 - a_4)}\right) & , a_4 \leq x \leq a_5 \\ 1 - \left(\frac{x - a_5}{4(a_6 - a_5)}\right) & , a_5 \leq x \leq a_6 \\ \frac{3}{4} - \left(\frac{x - a_6}{4(a_7 - a_6)}\right) & , a_6 \leq x \leq a_7 \\ \frac{1}{2} - \left(\frac{x - a_7}{4(a_8 - a_7)}\right) & , a_7 \leq x \leq a_8 \\ \left(\frac{a_9 - x}{4(a_9 - a_8)}\right) & , a_8 \leq x \leq a_9 \\ 0 & , \text{Otherwise} \end{cases}$$

$$\vartheta_{A'}(x) = \begin{cases} 1 - \left(\frac{x - b_1}{4(b_2 - b_1)}\right) & , b_1 \leq x \leq b_2 \\ \frac{3}{4} - \left(\frac{x - b_2}{4(b_3 - b_2)}\right) & , b_2 \leq x \leq b_3 \\ \frac{1}{2} - \left(\frac{x - b_3}{4(b_4 - b_3)}\right) & , b_3 \leq x \leq b_4 \\ \frac{1}{4} - \left(\frac{x - b_4}{4(b_5 - b_4)}\right) & , b_4 \leq x \leq b_5 \\ \left(\frac{x - b_5}{4(b_6 - b_5)}\right) & , b_5 \leq x \leq b_6 \\ \frac{1}{4} + \left(\frac{x - b_6}{4(b_7 - b_6)}\right) & , b_6 \leq x \leq b_7 \\ \frac{1}{2} + \left(\frac{x - b_7}{4(b_8 - b_7)}\right) & , b_7 \leq x \leq b_8 \\ 1 - \left(\frac{b_9 - x}{4(b_9 - b_8)}\right) & , b_8 \leq x \leq b_9 \\ 0 & , \text{Otherwise} \end{cases}$$

3.2. Arithmetic Operations of NIFN:-

Let

$$A' =$$

$$\{(a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1), (a_2, b_2, c_2, d_2, e_2, f_2, g_2, h_2, i_2)\}$$

and

$$B' =$$

$$\{(a_3, b_3, c_3, d_3, e_3, f_3, g_3, h_3, i_3), (a_4, b_4, c_4, d_4, e_4, f_4, g_4, h_4, i_4)\}$$

be two nanogonal intuitionistic fuzzy numbers , then the arithmetic operations are as follows:

Addition:-

$$A' + B' = \{(a_1 + a_3, b_1 + b_3, c_1 + c_3, d_1 + d_3, e_1 + e_3, f_1 + f_3, g_1 + g_3, h_1 + h_3, i_1 + i_3), (a_2 + a_4, b_2 + b_4, c_2 + c_4, d_2 + d_4, e_2 + e_4, f_2 + f_4, g_2 + g_4, h_2 + h_4, i_2 + i_4)\}$$

Subtraction:-

$$A' - B' = \{(a_1 - i_3, b_1 - h_3, c_1 - g_3, d_1 - f_3, e_1 - e_3, f_1 - d_3, g_1 - c_3, h_1 - b_3, i_1 - a_3), (a_2 - i_4, b_2 - h_4, c_2 - g_4, d_2 - f_4, e_2 - e_4, f_2 - d_4, g_2 - c_4, h_2 - b_4, i_2 - a_4)\}$$

3.2.Ranking of NIFN based on Accuracy Function:-

Accuracy function of a nanogonal intuitionistic fuzzy number

$$A' = \{(a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1, i_1), (a_2, b_2, c_2, d_2, e_2, f_2, g_2, h_2, i_2)\}$$

is defined as

$$H(A)' = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, e_1 + e_2, f_1 + f_2, g_1 + g_2, h_1 + h_2, i_1 + i_2)/9$$

IV. PROPOSED ALGORITHM

1. In intuitionistic fuzzy transportation problem, the quantities are reduced into an integer using the ranking method called Accuracy Function.
2. In the reduced IFTP, identify the row and column difference considering the least two numbers of the respective row and column.
3. Select the maximum among the difference (if more than one, then select any one) and allocate the respective demand / supply to the minimum value of the corresponding row or column.
4. We take a difference of the corresponding supply and demand of the allocated cell which leads either of one to zero, eliminating corresponding row or column (eliminate both row or column if both demand and supply is zero).
5. Repeat the steps 2,3 and 4 until all the rows and columns are eliminated.
6. Finally total minimum cost is calculated as sum of the product of the cost and the allocated value.

4.1.An Illustrative Example:-

Consider a 3x3 nanogonal intuitionistic fuzzy number,

	D ₁	D ₂	D ₃	Supply
S ₁	[[1,3,5,7,10,11,13,16,18], (0,3,4,6,11,13,14,17,19)]	[[2,4,5,9,11,12,13,15,18], (0,2,6,9,10,14,16,16,18)]	[[2,3,4,8,10,13,16,18,19], (1,3,8,10,12,13,14,16,19)]	30
S ₂	[[1,3,6,9,13,14,16,17,18], (1,3,9,12,13,14,15,16,18)]	[[2,3,5,7,10,11,12,13,16], (2,3,4,8,10,14,16,17,18)]	[[2,4,6,7,8,9,10,11,13], (1,3,6,8,12,13,15,16,18)]	35
S ₃	[[3,5,9,15,16,16,17,18,20], (3,8,10,15,16,16,17,19,20)]	[[3,4,7,9,12,15,17,18,20], (5,6,10,13,14,15,18,19,20)]	[[3,4,5,6,8,9,10,11,15], (2,4,5,7,9,11,13,15,16)]	45
Demand	40	50	20	110

Since $\sum Demand = \sum Supply$. The problem is a balanced transportation problem. Using the proposed algorithm, the solution of the problem is as follows:

Applying Accuracy function on nanogonal intuitionistic fuzzy number,

$$[(1,3,5,7,10,11,13,16,18),(0,3,4,6,11,13,14,17,19)]$$

we have,

$$H(A') = [1+0+3+3+5+4+7+6+10+11+11+13+13+14+16+17+18+19] / 9 = 19$$

$$H(A') = [2+0+4+2+5+6+9+9+11+10+12+14+13+16+15+16+18+18] / 9 = 20$$

$$H(A') = [2+1+3+3+4+8+8+10+10+12+13+13+16+14+18+16+19+19] / 9 = 21$$

$$H(A') = [1+1+3+3+6+9+9+12+13+13+14+14+16+15+17+16+18+18] / 9 = 22$$

$$H(A') = [2+2+3+3+5+4+7+8+10+10+11+14+12+16+13+17+16+18] / 9 = 19$$

$$H(A') = [2+1+4+3+6+6+7+8+8+12+9+13+10+15+11+16+13+18] / 9 = 18$$

$$H(A') = [3+3+5+8+9+10+15+15+16+16+16+16+17+17+18+19+20+20] / 9 = 27$$

$$H(A') = [3+5+4+6+7+10+9+13+12+14+15+15+17+1+8+18+19+20+20] / 9 = 25$$

$$H(A') = [3+2+4+4+5+5+6+7+8+9+9+11+10+13+11+15+15+16] / 9 = 17$$

REDUCED TEBLE:-

	D ₁	D ₂	D ₃	Supply
S ₁	19	20	21	30
S ₂	22	19	18	35
S ₃	27	25	17	45
Demand	40	50	20	110

Solution: Step 1:-

	D ₁	D ₂	D ₃	
S ₁	19	20	21	30 (1)
S ₂	22	19	18	35 (1)
S ₃	27	25	17	25, 45 (8)
	40 (3)	50 (1)	20 (1)	110

And finally,

	D ₁	D ₂	D ₃			
S ₁	30 (1)	19	20	21	30 (1)	
S ₂	22	35 (1)	19	18	35 (1)	
S ₃	10 (1)	27	15 (1)	25	17	45 (8)
	40 (3)	50 (1)	20 (1)		110	

The Optimal Solution is $(20 \times 17) + (30 \times 19) + (35 \times 19) + (10 \times 27) + (15 \times 25) = 2220$

V. CONCLUSION

A method for finding the optimal solution in an intuitionistic fuzzy environments has been proposed. We have used accuracy function ranking method and Russell’s method to find the optimal solution for nanogon intuitionistic transportation problem. Thus this method provides an applicable optimal solution which helps the decision maker while they are handling real life transportation problem having intuitionistic fuzzy parameters.

REFERENCES

[1] Atanassov.K and Fuzzy sets, more on Intuitionistic Fuzzy Sets, Fuzzy sets and Systems , 33,pp.37-46,1989.
 [2] Atanassov.K.T.” Intuitionistic Fuzzy Sets”, Fuzzy Sets and Systems, Vol.20(1),pp:87-96,(1983).
 [3] Dr.Annie Christi.M.S, Mrs.Kasthuri.B, “Transportation Problem with Pentagonal Intuitionistic Fuzzy Numbers Solved using Ranking Technique and Russell’s Method”, International Journal of Engineering Research

and Applications, Vol.6,ISSN:2248-9622,Issue-2,pp:82-86,(2016).
 [4] Felix.A, Christopher.S and Victor Devadoss.A , “ A Nanogon Fuzzy Number and its arithmetic operation, International Journal of Mathematics and its Applications,Vol.3,pp.185-195,2015.
 [5] Hitchcock.F.L, ”The distribution of a product from Several Sources to numerous localities”,Journal of Mathematics and Physics,Vol.20,pp.224-230,1941.
 [6] Klir.G.J, Fuzzy set theory foundations and applications; Prentice hall , Inc,1997.
 [7] Narayanamoorthy.S, Saranya.S and Maheshwari.S , “ A Method for solving Fuzzy Transportation Problem [FTP] using Fuzzy Russell’s Method”, International Journal of Intelligent Systems and Applications, Vol.5,No.2,IISN:2074-904,pp:71-75,(2013).
 [8] Ponnivalavan.K and Pathinathan.T. “Intuitionistic Pentagonal Fuzzy Number”, ARPN Journal of Engineering and Applied Sciences, Vol.10, No.12,ISSN 1819-6608,pp:5446-5450,(2015).
 [9] Pramila.K and Uthra.G . “Optimal Solution of an Intuitionistic Fuzzy Transportation Problem”, Annals of Pure and Applied Mathematics, Vol.8,No.2,ISSN:2279-087,pp:67-73,(2014).
 [10] Zadeh.L.A Fuzzy Sets, Information and Control ,338-353,8(1965).