New Approach for Power Transformer Modeling Using State Space Analysis

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***Abstract*- The procedure for determining the state of the system is called state variable analysis. State-variable analysis, or state space analysis, is a procedure that can be applied both to linear and nonlinear circuits. It is also applicable for circuits time variant and time invariant parameters.**

A state variable based approach minimizes computation time and memory requirements. It allows the use of parameterized nonlinear device models, thus improving robustness.

To design state equation for an electric network the state variables specify the energy stored in a set of independent energy storage elements. In this thesis, a power transformer with two windings is modeled. Therefore, leakage inductors magnetizing inductors are analyzed and assumed as state variables.

Once obtaining the state variables, the internal fault like turn to turn and turn to ground of the power transformer can be identified.

The flexibility of proposed method by applying to the power transformer exists in our laboratory by using digital computer algorithm developed in MATLAB.

I. INTRODUCTION

To propose a new approach for modeling of power transformer using state space analysis. The equivalent circuit of a power transformer drawn from which the energy storing elements are taken as state variables. On obtaining the state equations, the controllability and observability tests are conducted in order to identify internal faults of the power transformer. It is also extended to check the stability of the given transformer by using Routh-Hurwitz Criterion. To develop a digital computer programming for the proposed method and to consider numerical examples to show the flexibility of the method.

II.STATE SPACE REPRESENTATION

The state of the system may be considered as the least amount of information that must be known about the given system at a given time to determine it's subsequent dynamics completely.

A suitable selection of the independent variables results in a set of first order differential equations those are linearly independent. These variables and equations are known as state variables and state equations respectively.

State model of a linear time invariant system is a special case of the general time invariant model.

Derivative of each state variable now becomes a linear combination of the system states and inputs, i.e,

$$\dot{x}_1=a_{11}x_1+a_{12}x_2+\ldots\ldots+a_{1n}x_n+b_{11}u_1+b_{12}u_2+\ldots\ldots+b_{1m}u_m$$
$$\dot{x}_2=a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n+b_{21}u_1+b_{22}u_2+\ldots+b_{2m}u_m \quad (2.1)$$
$$\dot{x}_n=a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n+b_{n1}u_1+b_{n2}u_2+\ldots\ldots\ldots+b_{nm}u_m$$

Where the coefficient a_{ij} and b_{ij} are constants. In the vector matrix form, equ(1) may be written as

$\dot{x}(t)=Ax(t)+Bu(t)$

Where x(t) is n×1 state vector, u(t) is m×1 input vector, A is n×n system matrix defined by

$$A=\begin{bmatrix} a_{11} & a_{12} & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & a_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & a_{nn} \end{bmatrix}$$

And B is n×n input matrix defined

$$B=\begin{bmatrix} b_{11} & b_{12} & . & . & b_{1n} \\ b_{21} & b_{22} & . & . & b_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ b_{n1} & b_{n2} & . & . & b_{nn} \end{bmatrix}$$

Similarly, the output variables at time 't' are linear combinations of the values of input and state variables at time 't',i.e,

$y_{1(t)=}\ c_{11}x_1(t)+\ldots\ldots..+c_{1n}x_n(t)+d_{11}u_1(t)+\ldots\ldots.+d_{1m}u_m(t)$

$y_{p(t)=}\ c_{pp}x_p(t)+\ldots\ldots..+c_{pn}x_n(t)+d_{pp}u_p(t)+\ldots\ldots.+d_{pm}u_m(t)$

Where the coefficients c_{ij} and d_{ij} are constants. This set of equations may be put in the vector matrix form.

y(t)=Cx(t)+Du(t)

Where y(t) is p×1 output vector, C is p×n output matrix defined by

$$C=\begin{bmatrix} c_{11} & c_{12} & . & . & c_{1n} \\ c_{21} & c_{22} & . & . & c_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ c_{p1} & c_{p2} & . & . & c_{pn} \end{bmatrix}$$

And D is p×m transmission matrix is defined by

$$D=\begin{bmatrix} d_{11} & d_{12} & . & . & d_{1m} \\ d_{21} & d_{22} & . & . & d_{2m} \\ . & . & . & . & . \\ . & . & . & . & . \\ d_{p1} & d_{p2} & . & . & d_{pm} \end{bmatrix}$$

The state model of linear time invariant system is thus given by the following equations

$\dot{x}(t)=Ax(t)+Bu(t)$; State Equation

y(t)=Cx(t)+Du(t); Output Equation.

Where,

x=state vector (n×1)

u=control vector (m×1)

y=output vector (p×1)

A=matrix (n×n)

B=matrix (n×m)

C=matrix (p×n)

D=matrix (p×m)

III. CONTROLLABILITY AND OBSERVABILITY

CONTROLLABILITY: As system is said to be completely state controllable if it is possible to transfer the system state from any initial state $x(t_0)$ to any desired state x(t) in specified finite time by a control vector u(t).

Consider a single input linear time invariant system

$\dot{x}$ =Ax+Bu

Where x is n- dimensional state vector; u is control signal; A is n×n matrix and B is n×m matrix.

$\dot{x}$=Ax+Bu is completely controllable if and only if the rank of the composite matrix is 'n'.

$Q= [B;AB;A^2B\ldots\ldots A^{N-1}B]$

OBSERVABILITY: A system is said to be completely observable, if every state $x(t_0)$ can be completely identified by measurements of the output y(t) over a finite time interval.

Considered the state model of an n^{th} order single input-output linear time invariant system,

$\dot{x}$=Ax+Bu

y=Cx

is completely observable if and only if the rank of composite matrix is n.

$Q= [C^T:A^TC^T:\ldots\ldots\ldots.(A^T)^{n-1}C^T]$

This condition is also referred as the pair (AC) being observable.

IV. ROUTH STABILITY CRITERION

The characteristic equation is

$q(s)=a_0s^n+a_1s^{n-1}+a_1s^{n-2}+\ldots\ldots\ldots+a_{n-1}s^1+a_ns^0=0$

The formation of routh array is as follows,

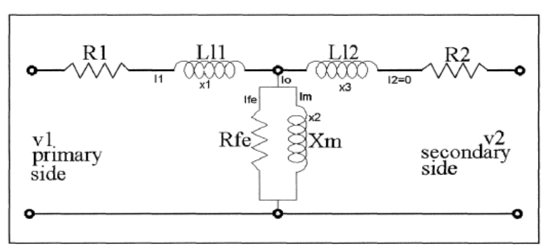
S^n	a_0	a_2	a_4	a_6
S^{n-1}	a_1	a_3	a_5	a_7
S^{n-2}	c_1	c_2	0	0
S^{n-3}	d_1	d_2	0	0
	*			
	*			
	*			
S	a_n			

If all the coefficients in the first column of the Routh array are positive then the given system is stable.

If any element in the first column of the Routh array is negative then the given system is unstable.

V. MODELLING OF POWER TRANSFORMER EQUIVALENT CIRCUIT TO STATE MODEL EQUATIONS

NO LOAD CONDITION:



Apply

i) KVL for primary side, then we get

$L_{11}dI_1/dt=V_1-I_1R_1-R_{fe}(I_1-I_m)$

$L_{11}\dot{X}_1=V_1-R_1X_1-R_{fe}(X_1-X_2)$

$L_{11}\dot{X}_1=V_1-(R_1+R_{fe})X_1+R_{fe}X_2$

$\dot{X}_1=V_1/L_{11}-(R_1+R_{fe})X_1+R_{fe}X_2/L_{11}$

ii) Apply KVL for magnetizing circuit,

$L_m dI_m/dt=R_{fe}(I_1-I_m)$

$L_m dI_m/dt=R_{fe}(X_1-X_2)$

$\dot{X}_2=R_{fe}/L_m\ (X_1-X_2)$

$\dot{X}_2=(R_{fe}*X_1/L_m)-(R_{fe}X_2/L_m)$

iii) Apply KCL at node-1,

$I_1=I_0=I_m+I_{fe}$

Output equation

$V_2=R_{fe}(I_1-I_m)$

$=R_{fe}X_1-R_{fe}X_2$

The state matrix is

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix}=\begin{bmatrix}-(R_1+R_{fe})/L_{11} & R_{fe}/L_{11}\\ R_{fe}/L_m & -R_{fe}/L_m\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}+\begin{bmatrix}1/L_{11}\\ 0\end{bmatrix}u$$

And also output state matrix

$$V_1=\begin{bmatrix}R_{fe} & -R_{fe}\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}+[0]$$

By comparing with state equations

$\dot{x}=Ax+Bu$

and

$y=Cx+Du$

We get the state matrices as,

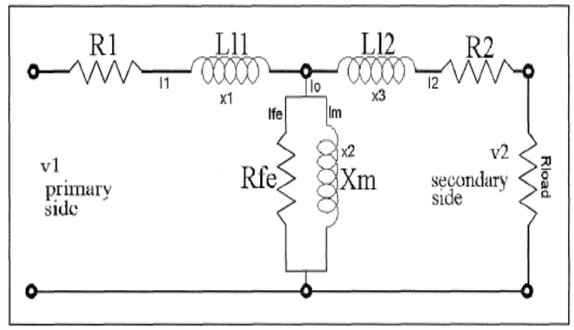
$$A=\begin{bmatrix}-(R_1+R_{fe})/L_{11} & R_{fe}/L_{11}\\ R_{fe}/L_m & -R_{fe}/L_m\end{bmatrix}$$

$$B=\begin{bmatrix}1/L_{11}\\ 0\end{bmatrix}$$

$$C=\begin{bmatrix}R_{fe} & -R_{fe}\end{bmatrix}$$

$D=[0]$

LOADED CONDITION:



i)Apply KVL for primary side, then we get

$L_{11}dI_1/dt=V_1-I_1R_1-R_{fe}(I_1-I_m-I_2)$

$L_{11}\dot{X}_1=V_1-R_1X_1-R_{fe}(X_1-X_2-X_3)$

$L_{11}\dot{X}_1=V_1-(R_1+R_{fe})X_1+R_{fe}X_2+R_{fe}X_3$

$\dot{X}_1=V_1/L_{11}-(R_1+R_{fe})X_1+R_{fe}X_2/L_{11}+R_{fe}X_3/L_{11}$

ii) Apply KVL for magnetizing circuit,

$L_m dI_m/dt=R_{fe}(I_1-I_m-I_2)$

$L_m dI_m/dt=R_{fe}(X_1-X_2-X_3)$

$\dot{X}_2=R_{fe}/L_m\ (X_1-X_2-X_3)$

$\dot{X}_2=R_{fe}/L_mX_1-R_{fe}/L_mX_2-R_{fe}/L_mX_3$

iii) Apply KVL for the secondary side,

$R_{fe}(I_1-I_m-I_2)=L_{12}dI_2/dt+R_2I_2+R_lI_2$

$L_{12}dI_2/dt=R_{fe}I_1-R_{fe}I_m-R_{fe}I_2-R_2I_2-R_1I_2$

$L_{12}\dot{X}_3=R_{fe}X_1-R_{fe}X_m-R_{fe}X_2-R_2X_2-R_1X_2$

$\dot{X}_3=R_{fe}X_1/L_{12}-R_{fe}X_m/L_{12}-R_{fe}X_2/L_{12}-R_2X_2/L_{12}-R_1X_2/L_{12}$

iv) For output equation,apply KVL at node-1

$I_1=I_0+I$

Out put equation is $V_2=I_2R_L$

And the state matrix is

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\end{bmatrix}=\begin{bmatrix}-(R_1+R_{fe})/L_{11} & R_{fe}/L_{11} & R_{fe}/L_{11}\\ R_{fe}/L_m & -R_{fe}/L_m & -R_{fe}/L_m\\ R_{fe}/L_{12} & -R_{fe}/L_{12} & -(R_{fe}+R_2+R_1)/L_{12}\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix}+\begin{bmatrix}1/L_{11}\\ 0\\ 0\end{bmatrix}u$$

And the output state matrix is $V_2=\begin{bmatrix}0 & 0 & R_L\end{bmatrix}\begin{bmatrix}X_1\\X_2\\X_3\end{bmatrix}+[0]$

By comparing with state equations

$\dot{x}=Ax+Bu$

y=Cx+Du

We get the state matrices as,

$$A=\begin{bmatrix}-(R_1+R_{fe})/L_{11} & R_{fe}/L_{11} & R_{fe}/L_{11}\\ R_{fe}/L_m & -R_{fe}/L_m & -R_{fe}/L_m\\ R_{fe}/L_{12} & -R_{fe}/L_{12} & -(R_{fe}+R_2+R_1)/L_{12}\end{bmatrix}$$

$$B=\begin{bmatrix}1/L_{11}\\0\\0\end{bmatrix}$$

$$C=\begin{bmatrix}0 & 0 & R_L\end{bmatrix}$$

D=0

VI. PROGRAM IN MATLAB NEW APPROACH FOR POWER TRANSFORMER MODELLING

No load and Full load condiation:

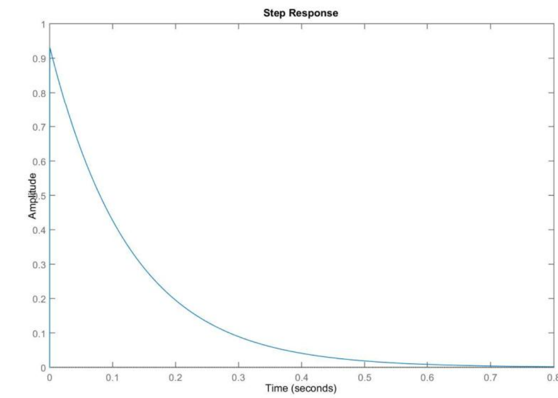
```
num=[Enter the numerator coefficients];
den=[Enter the denominator coefficients];
'g(s)'
g=tf(num,den);
ltiview
```

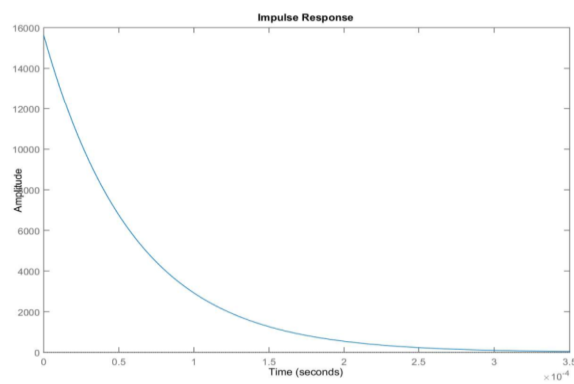
Controllability and observable:

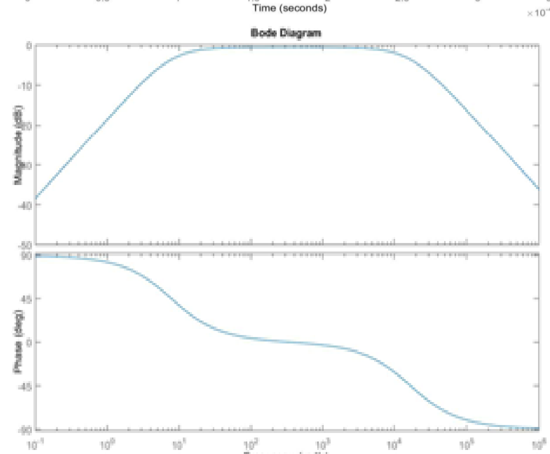
```
A=[Enter the values of A matrix];
B=[Enter the values of B matrix];
C=[values of C matrix];
D=[0];
K1=A*B;
K2=A*A*B;
K3=[B K1 K2];
QC=det(K3);
K1=A.'*C.';
K2=A.'*K1;
K3=[C.' K1 K2];
Q0=det(K3);
if(QC~=0)&(Q0~=0)
fprintf('\n sytem is completely CONTROLLABLE& completely OBSERVABLE.\n');
elseif (QC~=0)&(Q0~=0)
fprintf('system is completely CONTROLLABLE & not completely OBSERVABLE.');
elseif(QC~=0)&(Q0~=0)
fprintf('system is not completely CONTROLLABLE & completely OBSERVABLE.');
else
fprintf('system is not completely CONTROLLABLE & not completely OBSERVABLE.');
end
```

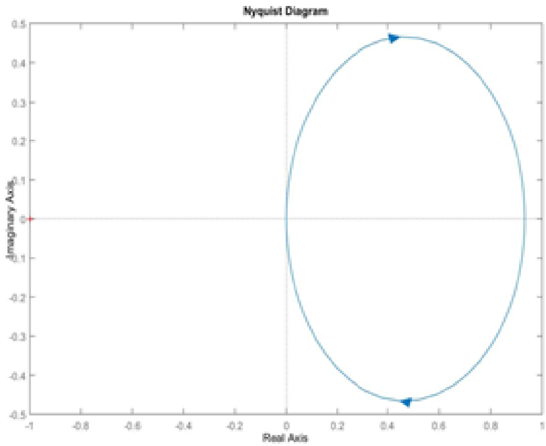
VII.RESULTS

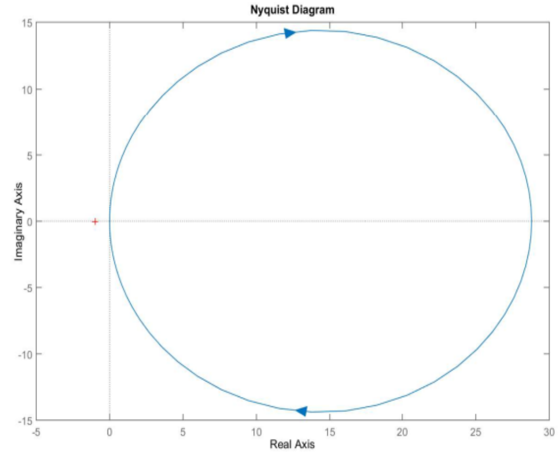
No load condition:



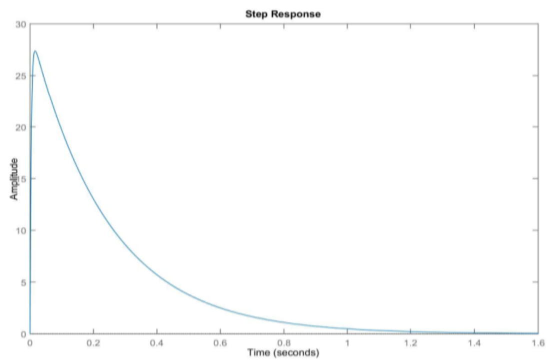


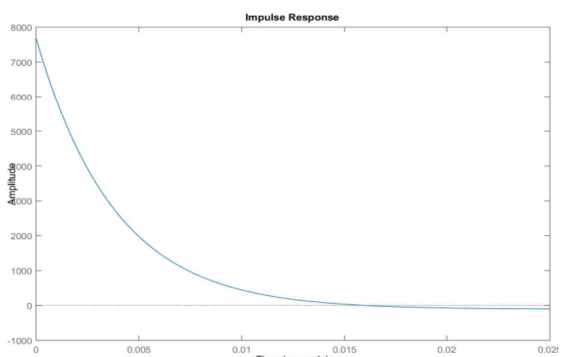


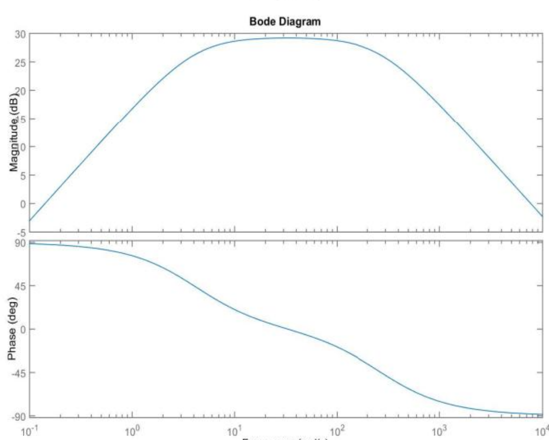




Full load condition:







VIII. CONCLUSION

In this thesis we addressed the definitions of state equations of two- winding transformer. As an example of the study, magnetizing inrush condition is analyzed and monitored in laboratory environment. It is clear that the whole system is controllable and observable during magnetizing inrush. If these conditions change, there is an internal fault during energizing.

It is also extended in finding the stability of the system. The superiority of the proposed method is shown by applying for the power transformer existing in our laboratory and simulated in digital computer using the computer algorithm.

REFERENCES

[1] An efficient method to compute transfer function of a transformer from its equivalent circuit, K.RAGAVAN and L.SATISH IEEE.

[2] Control system engineering fourth edition,” NORMAN S.NISE”.

[3] Control system engineering by” I.J NAGARATH, M. GOPAL”.

[4] Control system principles and design by- “M.GOPAL”.

[5] Control systems by A.NAGOOR KANI.

[6] Electrical machines by ”P.S.BIMBRA”.