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Computation of the Moment of Inertia of a Rectangular Lamina by Routh's Method

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***Abstract*- This paper deal of the moment of inertia of a rectangular lamina about the centriodal axes (X-X and Y-Y axes) to the research focused on determining the procedure by Routh's method. The work describes proposed computational values presenting the measurement of numerical problems. This paper deals the moment of inertia can also be defined as the modus operandi by Routh's method instead of integration method for the simulation and computation.**

***Index terms*- Rectangular Lamina, Moment of Inertia, Simulation and Computation**

INTRODUCTION

The property of matter by virtue of which it offers resistance in any change in its state of rest or of uniform motion is termed as inertia. The moment of a force about any point is the product of the force and the perpendicular distance between them. This is called as first moment of force. This first moment of force is again multiplied by the perpendicular distance between them, then the moment of moment of force or second moment of force or moment of inertia can be formed. If instead of force, an area of a body is considered, then the termed is known as area moment of inertia. If instead of a force, a mass of a body is considered, then the termed known as mass moment of inertia.

Moment of inertia is just a mathematical concept. It has no physical significance. Mass moment of inertia of a rigid body is considered as the measure of resistance of the body to rotation whereas area moment of inertia of an area is considered as the measure of resistance to bending. This concept is very useful to study the subjects of Strength of Materials, Machine Design and Structural Design.

1. Moment of Inertia of a Simple Area-

Consider a simple area (plane area) whose moment of inertia required to be found out. Split up the whole area into a number of small elements.

Consider the areas,

$A_1, A_2, A_3, A_4, \ldots\ldots, A_n$ = Areas of small elements;

Where 'n' be the number of areas

$r_1, r_2, r_3, r_4, \ldots\ldots, r_n$ = Corresponding distances of the elements from the line about which the moment of inertia is required to be found out.

Now according to the concept of Moment of Inertia of the simple area, i.e.,

$I = A_1r_1 + A_2r_2 + A_3r_3 + A_4r_4 + \ldots\ldots + A_nr_n$

$I = \sum Ar^2$

As a matter of fact, the units of moment of inertia of a simple plane area depends upon the units of area and length, i.e., the table chart shown here-

System of Units	Physical Quantity	Units
SI	Moment of Inertia	m^4
CGS	Moment of Inertia	cm^4
MKS	Moment of Inertia	m^4
FPS	Moment of Inertia	ft^4

Corollary: - If area in mm^2 and the length is also mm, then the Moment of Inertia can be expressed as mm^4.

2. Moment of Inertia by Routh's Method-

Routh's method is used for finding the moment of inertia of a simple plane area or a body of uniform thickness. Routh's method states that "If a body is symmetrical about three mutually perpendicular axes, then the moment of inertia about any one axis passing through its centre of gravity" is given by-

$I = \frac{A\,(or\,M) \times S}{3}$; for the condition of Square or Rectangular Lamina, where 'A' is the area of simple plane area, 'M' is the mass of the body and 'S' is the sum of the squares of the two semi-axes, other than the axis, about which the M.I is required to be found out.

$I = \frac{A\,(or\,M) \times S}{4}$; for the condition of Circular or Elliptical Lamina, where 'A' is the area of simple plane area, 'M' is the mass of the body and 'S' is the sum of the squares of the two semi-axes, other than the axis, about which the M.I is required to be found out.

$I = \frac{A\,(or\,M) \times S}{5}$; for the condition of Spherical Lamina, where 'A' is the area of simple plane area, 'M' is the mass of the body and 'S' is the sum of the squares of the two semi-axes, other than the axis, about which the M.I is required to be found out.

3. Moment of Inertia of a Rectangular Section by Routh's Method-

We know that the moment of inertia for the whole length of Rectangular Lamina about X-X and Y-Y axes is defined as $I_{XX} = \frac{bd^3}{12}$ and $I_{YY} = \frac{db^3}{12}$ respectively followed by the Integration method. But as per the concept of Routh's method, Moment of Inertia for the whole length of Rectangular Lamina about X-X and Y-Y axes are discussed below-

$I_{XX} = \frac{AS}{3}$;

where A = (b × d) and sum of the squares of semi-axes Y-Y and Z-Z, $S = (\frac{d}{2})^2 + 0 = \frac{d^2}{4}$.

Therefore, $I_{XX} = \frac{(b \times d) \times d^2}{4 \times 3}$

$I_{XX} = \frac{bd^3}{12}$; Similarly, $I_{YY} = \frac{db^3}{12}$

Illustration 1: Find the moment of inertia of a rectangular lamina 60 mm × 40 mm about its centre of gravity with the help of Routh's method.

Computation 1: Given, Width of the lamina be (b) = 60 mm and Depth of the lamina be (d) = 40 mm respectively.

We know that the Area of the lamina (A) = b × d= 60 × 40 = 2400 mm^2

Sum of the squares of Semi axes Y - Y and Z - Z,

$S = (\frac{40}{2})^2 + 0 = 400$ mm;

Therefore, $I_{XX} = \frac{AS}{3} = \frac{(2400 \times 400)}{3} = 32 \times 10^4\ mm^4$

Similarly, Sum of the squares of Semi axes X-X and Z-Z,

$S = (\frac{60}{2})^2 + 0 = 900$ mm;

Therefore, $I_{YY} = \frac{AS}{3} = \frac{(2400 \times 900)}{3} = 72 \times 10^4\ mm^4$.

4. Moment of Inertia of a Hollow Rectangular Section by Routh's Method-

We know that the moment of inertia for the whole length of outer Rectangular Lamina about X-X and Y-Y axes is defined as $I_{XX}(1) = \frac{bd^3}{12}$ and $I_{YY}(1) = \frac{db^3}{12}$ and inner rectangular lamina about X-X and Y-Y axes is defined as $I_{XX}(2) = \frac{b1d1^3}{12}$ and $I_{YY}(2) = \frac{d1b1^3}{12}$ respectively followed by the Integration method. But as per the concept of Routh's method, Moment of Inertia for the whole length of outer Rectangular Lamina about X-X and Y-Y axes as discussed below-

$I_{XX}(1) = \frac{AS}{3}$;

where A = (b × d) and sum of the squares of semi-axes Y-Y and Z-Z, $S = (\frac{d}{2})^2 + 0 = \frac{d^2}{4}$.

Therefore, $I_{XX}(1) = \frac{(b \times d) \times d^2}{4 \times 3}$

$I_{XX}(1) = \frac{bd^3}{12}$; Similarly, $I_{YY}(1) = \frac{db^3}{12}$

Similarly, Moment of Inertia for the whole length of inner Rectangular Lamina about X-X and Y-Y axes as discussed below-

$I_{XX}(2) = \frac{A1S1}{3}$;

where A = (b × d) and sum of the squares of semi-axes Y-Y and Z-Z, $S1 = (\frac{d}{2})^2 + 0 = \frac{d^2}{4}$.

Therefore, $I_{XX}(2) = \frac{(b1 \times d1) \times d1^2}{4 \times 3}$

$I_{XX}(2) = \frac{b1d1^3}{12}$; Similarly, $I_{YY}(2) = \frac{d1b1^3}{12}$

Therefore, M.I of the hollow Rectangular Section about X-X axis,

I_{XX} = M.I of the Outer Rectangle – M.I of the Inner Rectangle.

$I_{XX} = \frac{AS}{3} - \frac{A1S1}{3}$

Similarly, $I_{YY} = \frac{AS}{3} - \frac{A1S1}{3}$.

Illustration 2: Find the moment of inertia of a hollow rectangular section about its centre of gravity, if the external dimensions are 40 mm × 30 mm and internal dimensions are 25 mm × 15 mm respectively.

Computation 2: Given, width of the outer lamina be (b) = 40 mm and depth of the outer lamina be (d) = 30 mm respectively.

We know that the Area of the lamina (A) = b × d= 40 × 30 = 1200 mm^2

Sum of the squares of Semi axes Y - Y and Z - Z,

$S = (\frac{30}{2})^2 + 0 = 225$ mm;

Therefore, $I_{XX}(1) = \frac{AS}{3} = \frac{(1200 \times 225)}{3} = 9 \times 10^4$ mm^4

Similarly, Sum of the squares of Semi axes X-X and Z- Z,

$S = (\frac{40}{2})^2 + 0 = 400$ mm;

Therefore, $I_{YY}(1) = \frac{AS}{3} = \frac{(1200 \times 400)}{3} = 16 \times 10^4$ mm^4.

Again given, width of the inner lamina be (b1) = 25 mm and depth of the inner lamina be (d1) = 15 mm respectively.

We know that the Area of the lamina (A1) = b × d= 25 × 15 = 375 mm^2

Sum of the squares of Semi axes Y - Y and Z - Z,

$S1 = (\frac{15}{2})^2 + 0 = 56.25$ mm;

Therefore, $I_{XX}(2) = \frac{A1S1}{3} = \frac{(375 \times 56{,}25)}{3} = 7.03$ mm^4

Similarly, Sum of the squares of Semi axes X-X and Z- Z,

$S1 = (\frac{25}{2})^2 + 0 = 156.25$ mm;

Therefore, $I_{YY}(2) = \frac{A1S1}{3} = \frac{(375 \times 156.25)}{3} = 19.53$ mm^4.

Now, M.I of the hollow Rectangular Section about X-X axis,

I_{XX} = M.I of the Outer Rectangle – M.I of the Inner Rectangle.

$I_{XX} = \frac{AS}{3} - \frac{A1S1}{3} = (9 \times 10^4 - 7.03)$ mm^4 = 89.99 mm^4

Similarly, $I_{YY} = \frac{AS}{3} - \frac{A1S1}{3} = (16 \times 10^4 - 19.53)$ mm^4 = 159.99 mm^4.

CONCLUSION

The moment of inertia is an important properties to be considered for designing the strength of the materials, machine design, theory of structure etc. In this paper, the research focused about the M.I of the Rectangular Lamina and Hollow Rectangular Lamina to set up by finding out the whole section of the moment of inertia about the centriodal axes as per the computations with the aided of Routh's method. So, about the conclusion of the article, it is evident that instead of integration method, we can easily find out the simulation of M.I about the whole section of rectangular lamina.

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