A New Ranking Method for Solving Nanogonal Fuzzy Travelling Salesman Problem

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Abstract- **Travelling Salesman Problem is similar to Assignment problem. The objective of travelling salesman problem is to find the shortest possible route and minimizing the total travelling time. In this paper Travelling Salesman problem is considered with Nanogonal Fuzzy numbers. Nanogonal fuzzy numbers are defuzzified using new ranking method. Fuzzy Travelling Salesman problem is transformed into crisp travelling salesman problem and it is solved by Hungarian method. A numerical example is presented and the optimal solution is derived by using Hungarian method.**

Index terms- **Nanogonal Fuzzy Number, Ranking Method, Numerical Example**

1. INTRODUCTION

Travelling salesman problem is similar to assignment problem. In a travelling salesman problem, a salesman starts with a particular city and he visits each city exactly once and return to the home city. The objective of travelling salesman problem is to find the shortest possible route and the total travelling time is minimized. Travelling salesman problem has been applied in various fields like mathematics, computer science, chemistry, physics etc. L.A. Zadeh [15] was first introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Hadi Basirzadeh [7] approached a new technique for solving fuzzy transportation problem. S.Dhanasekar, S.Hariharan and P.Sekar [5] were solved the fuzzy travelling salesman problem using Yager's ranking method. Abusinghal and Priyaka pandey [1] were solved travelling salesman problems using dynamic programming method. Swetha Rana, Saurabh Ranjan Srivastava [13] solved the travelling salesman problem using improved genetic algorithm. P. Ghadle Kirtiwant, and M. Muley Yogesh [6] were studied the applications of assignment problem in travelling salesman problem. AmitKumar and Anila Gupta [4] were solved the assignment and travelling salesman problems with LR fuzzy parameters. S.Yahya Mohamed and M.Divya [14] discussed a fuzzy travelling salesman problem with octagonal fuzzy numbers using α-cut method. V.Mythili, M.Kaliappan, S.Hariharan and S.Dhanasekar [10] proposed a dynamic programming method were solving the travelling salesman problem with fuzzy numbers. Jagunath nayak, Sudharsan, Nanda and Srikumar Acharya [9] were applied the Hungarian method to solve travelling salesman problem with fuzzy cost. Amit kumar Rana [3] discussed a study on fuzzy travelling salesman problem using fuzzy number. AmitKumar and Anila Gupta [4] were first solved fuzzy assignment problem and fuzzy travelling salesman problem with different membership functions. D.Stephen Dinagar and K.Thiripurasundari [12] were finding the optimal solution of fuzzy travelling salesman problem. G.Nirmala and R.Anju [11] solved the travelling salesman problem using fuzzy quantifier. Hadi Basirzadeh [8] proposed a method called ones assignment method for solving travelling salesman problem.

2. PRELIMINARIES

Definition 2.1.A fuzzy set is characterized by a membership function mapping element of a domain space or the universe of discourse X to the unit interval {0,1}

(i.e) $A=\{x,\mu_A(x)\ ; x\in X\}$ .Here $\mu_A(x)=1$

Definition 2.2.A fuzzy set A of the universe of discourse X is called normal fuzzy set implying that there exist atleast one $x \in X$ Such that $\mu_A(x) = 1$

Definition 2.3.The support of fuzzy set in the Universal set X is the set that contains all the elements of X that have anon- zero membership grade in $\tilde{A}$.(i.e) $Supp(\tilde{A}) = \{x \in X / \mu_{\tilde{A}}(x) > 0\}$

Definition 2.4. Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$the α - cut, α_A is the crisp set $\alpha_A = \{x \in X / A(x) \geq \alpha, \alpha \in [0,1]$

Definition 2.5.A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be fuzzy number if its membership function $\mu_A(x): R \to [0,1]$ has the following properties

1 A must be a normal and convex fuzzy set

2 α_A must be a closed interval for every $\alpha \in (0,1]$

3 The support of $\tilde{A}$ must be bounded

Definition 2.6.A fuzzy number $\tilde{A}$ is called triangular function is denoted by $\tilde{A} = (a_1, a_2, a_3)$ whose membership function is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \leq x \leq a_3 \\ 0 & x > a_3 \end{cases}$$

3. NANOGONAL FUZZY NUMBER

A fuzzy number $\tilde{A}$ is a Nanogonal fuzzy number defined by $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$ are real numbers and its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ k_1\left(\dfrac{x - a_1}{a_1 - a_2}\right) & a_1 \leq x \leq a_2 \\ k_1 & a_2 \leq x \leq a_3 \\ k_1 + (1 - k_1)\left(\dfrac{x - a_3}{a_4 - a_3}\right) & a_3 \leq x \leq a_4 \\ k_1 + (1 - k_1)\left(\dfrac{x - a_4}{a_5 - a_4}\right) & a_4 \leq x \leq a_5 \\ k_1 + (1 - k_1)\left(\dfrac{a_6 - x}{a_6 - a_5}\right) & a_5 \leq x \leq a_6 \\ k_1 + (1 - k_1)\left(\dfrac{a_7 - x}{a_7 - a_6}\right) & a_6 \leq x \leq a_7 \\ k_1 & a_7 \leq x \leq a_8 \\ k_1\left(\dfrac{a_9 - x}{a_9 - a_8}\right) & a_8 \leq x \leq a_9 \\ 0 & x > a_9 \end{cases}$$

Where $0 < k_1 < 1$

3.1. ARITHMETIC OPERATIONS ON HEXADECAGONAL FUZZY NUMBER

Let $\tilde{A}_{NFN} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ & $B_{NFN} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9)$ be two Nanogonal fuzzy numbers then the addition, subtraction and scalar multiplication can be defined as

$$\tilde{A}_{NFN} + \tilde{B}_{NFN} = [a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8, a_9 + b_9]$$

$$\tilde{A}_{NFN} - \tilde{B}_{NFN} = [a_1 - b_9, a_2 - b_8, a_3 - b_7, a_4 - b_6, a_5 - b_5, a_6 - b_4, a_7 - b_3, a_8 - b_2, a_9 - b_1]$$

$$\lambda\tilde{A}_{NFN} = [\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5, \lambda a_6, \lambda a_7, \lambda a_8, \lambda a_9]$$

$$\lambda\tilde{B}_{NFN} = [\lambda b_1, \lambda b_2, \lambda b_3, \lambda b_4, \lambda b_5, \lambda b_6, \lambda b_7, \lambda b_8, \lambda b_9]$$

3.2. MEASURE OF FUZZY NUMBER

The measure of $\tilde{A}_\omega$ is a measure is a function $M_o : R_\omega(I) \to R^+$ which assign a non negative real numbers $M_o^{NFN}(\tilde{A}_\omega)$ that expresses the measure of

$$M_o^{NFN}(\tilde{A}_\omega) = \frac{1}{2}\int_\alpha^{k_1}\left(f_1(r) + \bar{f}_1(r)\right)dr + \frac{1}{2}\int_{k_1}^{\omega}\left(g_1(s) + \bar{g}_1(s)\right)ds$$

Where $0 \leq \alpha < 1$

4. PROPOSED RANKING METHOD

Let $\tilde{A}$ be a normal Nanogonal fuzzy number. The measure of $\tilde{A}$ is calculated as follows

$$M_o^{NFN}\left(\tilde{A}_\omega\right)=\frac{1}{2}\int_0^{k_1}\left(f_1(r)+\bar{f}_1(r)\right)dr+\frac{1}{2}\int_{k_1}^{1}\left(g_1(s)+\bar{g}_1(s)\right)ds$$

$$M_o^{NFN}(\tilde{A})=\frac{1}{4}\left\{\left(a_1+a_2+a_8+a_9\right)k_1+\left(a_3+a_4+2a_5+a_6+a_7\right)(1-k_1)\right\},\ where\ 0<k_1<1$$

5. NUMERICAL EXAMPLE

Consider the following Nanogonal Fuzzy Travelling Salesman Problem

	A	B	C	D
A	∞	(0,1,2,3,4, 5,6,7,8)	(1,3,5,7,9, 11,13,15, 17)	(1,2,4,5,6, 9,12,15, 20)
B	(2,4,6,8,10, 12,14,16, 18)	∞	(2,3,4,7,8, 10,12,13, 15)	(2,3,6,9,11 ,12,16, 20, 21)
C	(1,3,7,9,11, 15,17,19, 23)	(1,2,3,6,9, 12,15, 18, 21)	∞	(0,2,5,6,8, 9,10,12, 15)
D	(1,4,8,,9,11 ,12,13,14, 16)	(2,3,4,6,7, 8,10,15,18)	(0,4,8,9,12 ,16,20, 22, 24)	∞

By applying our proposed ranking method, the Nanogonal defuzzified and it is given in the table

	A	B	C	D
A	∞	5.4	12.2	10.2
B	13.5	∞	11.1	14.8
C	15.7	12.6	∞	10.5
D	13.8	10.2	11.5	∞

Applying the Hungarian method, we find the optimal assignment. The optimal Assignment is $A\rightarrow B\rightarrow C\rightarrow D\rightarrow A$. The optimal solution is 40.8

6. CONCLUSION

In this paper Travelling Salesman problem is considered with Nanogonal Fuzzy numbers. Nanogonal fuzzy numbers are defuzzified using new ranking method. Fuzzy Travelling Salesman problem is transformed into crisp travelling salesman problem and it is solved by Hungarian method. A numerical example is presented and the optimal solution is derived by using Hungarian method.

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