

# Study of dynamic response characteristics of process using MATLAB

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**Abstract -** The Process Dynamics and Control subject is traditionally taught using mathematical equations, Laplace Transform, Z-transform techniques. Usually students do not enjoy learning the subject based on complicated mathematics. It is observed that students understand the basic concepts better if complex mathematical theory is supplemented by using software simulation tools such as MATLAB and SIMULINK.

The process control trainers available in the Process Control laboratory have fixed design parameters, which are practically impossible to change during practical class. The effect of change in process parameters can be studied using MATLAB simulations. This resulted in an enhanced interactive learning of the basic concepts of Process Dynamics and Control, rather than learning through complex mathematical equations. In this paper, the dynamic behavior of standard processes is analyzed using MATLAB. This process knowledge is useful for controlling the process response to desired specifications.

**Index Terms -** Control System Toolbox, FOPDT, LTI viewer, MATLAB, Process Dynamics, SOPDT.

## INTRODUCTION

Today Process Control has become the core subject for many engineering disciplines like Electrical, Electronics, Instrumentation, Chemical/Petrochemical, Mechanical, Automobile, and many more. In Process Dynamics and Control course, students gain insight into theoretical study of dynamic behavior of processes and methods of controlling the response of the process to desired specifications. In the laboratory course, students conduct experiments on control system trainers to correlate the theoretical results with experimental results [1]-[4].

Introduction to MATLAB:

MATLAB is a language of technical computing developed by MATHWORKS, USA having [5] It is an interactive program for numerical computation and data visualization with its own programming language. The applicability of MATLAB is enhanced by various toolboxes which are customized for specific applications. In this paper, the Control System Toolbox is used for analyzing dynamic behavior of standard processes.

Control System Toolbox:

*Control System Toolbox™* software provides tools for systematically analyzing, designing, and tuning linear control systems. The system to be analyzed is modeled in suitable form (such as transfer function, zero-pole-gain, state-space forms). These models can be imported in Control System Toolbox as an object (also known as *LTI object* i.e. *Linear Time-Invariant object*). Conversion between various model forms can be implemented. Once the plant to be controlled is modeled as an LTI object, it can be analyzed for dynamic behavior and to design a controller using Control System Toolbox.

LTI Viewer:

The time or frequency response of the system can be established for various types of inputs (step, impulse, ramp) using LTI Viewer. The LTI viewer allows us to compute and graph the time responses and frequency responses of several models at once, which can be compared in terms of the response characteristics such as rise time, maximum overshoot, and stability margins. The plot options available are step, impulse, Bode, Nichols, Nyquist, singular value, and pole/ zero plots. In addition, the plant output response can also be

simulated for user-defined inputs and initial conditions to further investigate system performance.

This paper is organized as follows. The extensive literature survey discusses the mathematical models and dynamic response characteristics of standard first- and second-order processes with and without dead time lag are presented in section-II. In section-III, the proposed methodology is explained in detail. The mathematical models of standard first- and second-order processes are developed in Section-IV. The MATLAB simulation results obtained are presented in section-V. Finally, conclusions are drawn in section-VI.

### PROPOSED METHODOLOGY

The dynamic behavior of standard first- and second-order process is studied using MATLAB. The effect of change in process parameters on dynamic response characteristics is studied quantitatively. The results so obtained are compared with relevant theory. The process models are expressed in the transfer function (tf) or zero-pole-gain (z-p-k) form as follows-

The transfer function model (tf)

On taking the Laplace transform of the ODE model one gets the transfer function model in the form

$$G(s) = \frac{L[y(t)]}{L[x(t)]} = \frac{\bar{y}(s)}{\bar{x}(s)}$$

$$= \frac{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \dots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n}$$

$$= \frac{num(s)}{den(s)}$$

Where, num(s), den(s) are polynomials in s.

The polynomials num(s) and den(s) can be factored to get the zero-pole-gain (i.e. zpk) model form:

$$G = k \frac{(s-z_1)(s-z_2)\dots(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_n)}$$

Where, K is gain;  $z_1, z_2, \dots, z_m$  are the zeros;  $p_1, p_2, \dots, p_n$  are the poles of G.

The transfer function model in MATLAB:

```
>>sys = tf([b0 b1 b2-----bm],[a0 a1 a2-----an])
```

The zero-pole-gain (z-p-k) model in MATLAB:

```
>>sys = zpk([z1 z2--zm],[p1 p2---pn],[k])
```

Process model representation in LTI Viewer-

The step response characteristics of the open-loop systems described by the above model forms can be

obtained using LTI Viewer by typing at the MATLAB prompt

```
>>ltiview(sys)
```

This gives the step response of the LTI object (G or sys). The *right-click menus* give different *plot types* and *response characteristics* such as rise time, settling time, ultimate response.

### MATHEMATICAL FORMULATION

Classification of processes based on their dynamic behavior:

The dynamic behavior of a process can be derived from its mathematical model which is essentially an input (cause) -output (effect) relationship in the form of an ordinary differential equation (ODE) with constant coefficients. Hence the processes can be classified based on the order of the representative ODE models which further determines the dynamic behavior characteristics. Majority of the systems encountered are described by first or second-order ODE models which show characteristic dynamic behavior as explained below.

*First-order system:*

The transfer function of a first-order process is in the form:

$$G(s) = k / (\tau s + 1),$$

Where, k= steady-state or static gain

$$= \Delta (\text{output}) / \Delta (\text{input}),$$

and  $\tau$  = time constant

The static gain term k represents the ultimate output response for given change in input. Time constant  $\tau$  represents the time required for achieving the output change of 63.2% of the ultimate response. Thus,  $\tau$  is the measure of speed of response of the system.

The transfer function of a first-order system has a single real negative pole; therefore, its step response approaches ultimate value exponentially. These processes are called as self-regulated processes.

*First-order plus dead time system (FOPDT):*

A first-order system may be associated with dead time or transportation lag (FOPDT), which delays the response by magnitude of dead time  $t_d$ . The physical examples of first-order system are simple liquid storage tank and mercury thermometer.

*Second-order system:*

The transfer function of a second-order process is in the form:

$$G(s) = k / (\tau^2 + 2\tau\zeta s + 1),$$

Where  $k$  = static gain,  $\tau$  = period of natural (undamped) oscillations, and  $\zeta$  = damping ratio or coefficient.

Static gain term has same reference as that for a first-order process, while the value of damping ratio determine the nature and speed of output response.

The transfer function of a second-order process has two poles. The nature of these poles depends on the whether the value of damping coefficient  $\zeta$  is  $<1$ ,  $>1$ , or  $=1$  that gives rise to different dynamic behaviors as follows:

For  $\zeta < 1$ , the poles are a conjugate pair of complex numbers with negative real part so that step response exhibit *underdamped* oscillatory behavior with decreasing amplitude.

For  $\zeta > 1$ , the poles are real and distinct so that step response exhibit *overdamped* non-oscillatory behavior.

For  $\zeta = 1$ , the poles are real and equal so that step response exhibit *critically damped* behavior which is similar but faster than overdamped response.

For  $\zeta = 0$ , the poles are pure imaginary numbers (with zero real part) so that step response shows *undamped* behavior in the form of free (natural) oscillations with constant amplitude (sustained oscillations)

*Second-order plus dead time system (SOPDT):*

A second-order system may be associated with dead time or transportation lag (SOPDT) which delays the response by magnitude of dead time  $t_d$ . The physical examples of second-order system are two liquid tanks system, fluid column manometer, and a first-order system with PI controller.

*Inverse response systems-*

When the initial direction of process system's step response is opposite to the direction of final steady-state response, it is said to possess inverse response characteristic. The response of water level in boiler drum for step change in cold water flow rate shows inverse response behavior. Such systems have transfer function in two parts  $G_1(s)$  (main system) and  $G_2(s)$  (opposition system), in the form

$$G(s) = G_1(s) - G_2(s) = \frac{k_1}{\tau_1 s + 1} - \frac{k_2}{\tau_2 s + 1}$$

For step change in input, the steady-state output of system is  $(k_1 - k_2)$ .

IMPLEMENTATION AND SIMULATION

1. Step response characteristics of a first-order process

Transfer function:  $G(s) = 1/(s+1)$  ----(sys1)

*MATLAB Program*

```
>> sys1=tf([0 1],[1 1])
>> ltiview(sys1)
```

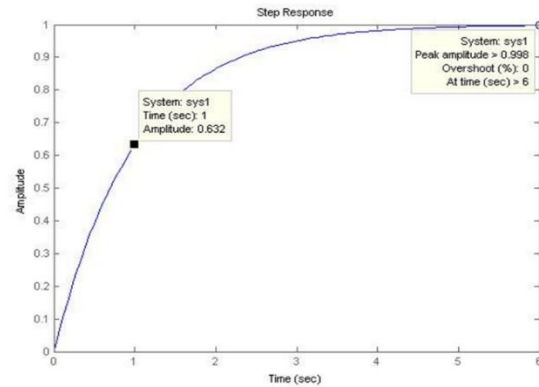


Fig. 1: Unit step response of a first-order process

2. Effect of change in static gain of a first-order process from 1 to 2, at constant time constant 1s on dynamic response characteristics of a first-order process

Transfer functions:

$G_1(s) = 1/(s+1)$  ----(sys1),  $G_2(s) = 2/(s+1)$  ----(sys2)

*MATLAB Program*

```
>> sys1=tf([0 1],[1 1])
>> sys2=tf([0 2],[1 1])
>> ltiview(sys1,sys2)
```

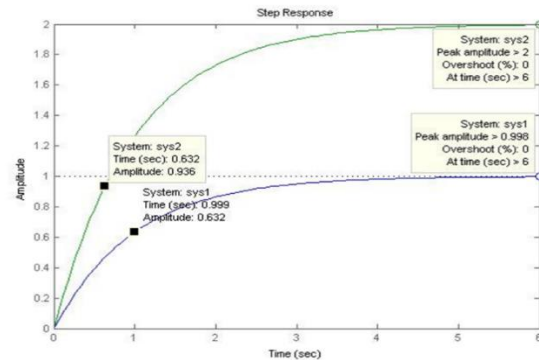


Fig. 2: Effect of change in static gain on step response of a first-order process

3. Effect of change in time constant from 1s to 2s at a constant gain 1-

Transfer functions:

$$G_1(s) = 1/(s+1) \text{-----(sys1)}, G_2(s) = 1/(2s+1) \text{-----(sys3)}$$

MATLAB Program

```
>> sys1=tf([0 1],[1 1])
>> sys3=tf([0 1],[2 1])
>> ltiview(sys1, sys3)
```

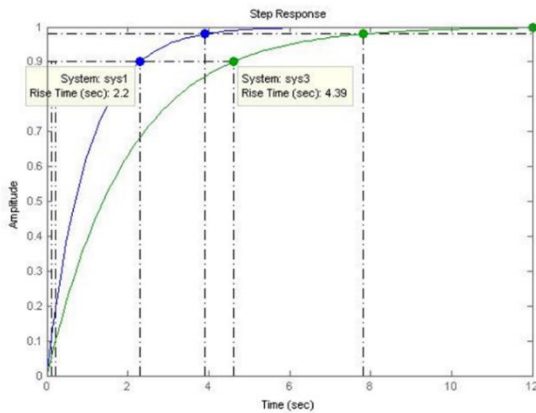


Figure 3-Effect of change in time constant at constant gain on step response of a first-order process

4. Effect of dead time lag on step response characteristics of a first-order process-

Transfer functions:

$$G_1(s) = 1/(s+1) \text{-----(sys1)}, G_4(s) = e^{-s}/(s+1) \text{-----(sys4)}$$

MATLAB Program

```
>> sys1=tf([0 1],[1 1])
>> sys4=tf([0 1],[1 1], 'iodelay', 1)
>> ltiview(sys1, sys4)
```

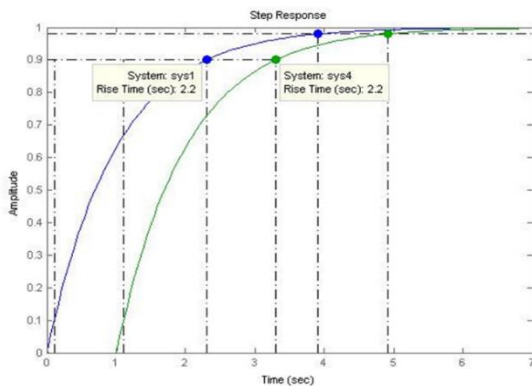


Fig. 4: Effect of dead time lag on step response characteristics of a first-order process

5. Step response characteristics of a second-order underdamped ( $\zeta < 1$ ), overdamped ( $\zeta = 1$ ), and critically damped ( $\zeta > 1$ ) systems-

Transfer functions

$$G_1(s) = 1/(s^2 + 4s + 1) \text{ Overdamped } (\zeta = 2) \text{-----(sys1)}$$

$$G_2(s) = 1/(4s^2 + 4s + 1) \text{ Critically damped } (\zeta = 1.0) \text{-(sys2)}$$

$$G_3(s) = 1/(s^2 + s + 1) \text{ Underdamped } (\zeta = 0.5) \text{-----(sys3)}$$

MATLAB Program

```
>> sys1=tf([0 0 1],[1 4 1])
>> sys2=tf([0 0 1],[4 4 1])
>> sys3=tf([0 0 1],[1 1 1])
>> ltiview(sys1, sys2, sys3)
```

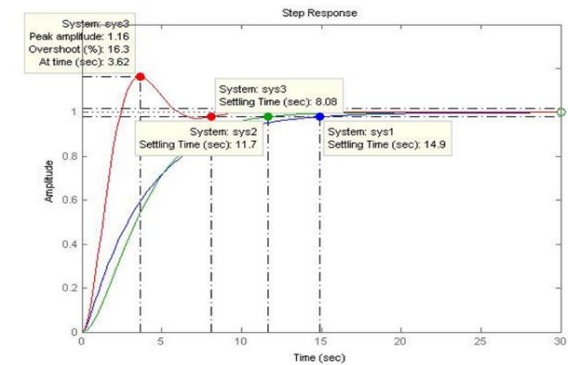


Fig.5: Unit step response characteristics of a second-order underdamped ( $\zeta < 1$ ), overdamped ( $\zeta = 1$ ), and critically damped ( $\zeta > 1$ ) systems

6. Effect of damping ratio on dynamic behavior of an underdamped system-

Transfer functions

$$G_1(s) = 1/s^2 + s + 1 \text{ -----}(\zeta = 0.5) \text{ (sys1)}$$

$$G_2(s) = 1/s^2 + 0.5s + 1 \text{ -----}(\zeta = 0.25) \text{ (sys2)}$$

$$G_3(s) = 1/s^2 + 0.2s + 1 \text{ -----}(\zeta = 0.1) \text{ (sys3)}$$

MATLAB Programme

```
>> sys1=tf([0 0 1],[1 1 1])
>> sys2=tf([0 0 1],[1 0.5 1])
>> sys3=tf([0 0 1],[1 0.2 1])
>> ltiview(sys1, sys2, sys3)
```

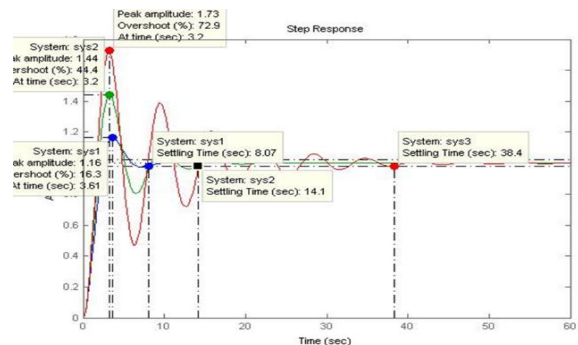


Fig.6: Effect of damping ratio on dynamic behavior of an underdamped system

7. Effect of damping ratio on dynamic behavior of an overdamped system-

Transfer functions

$$G_1(s) = 1/s^2 + 4s + 1 \text{-----} (\zeta=2) \text{ (sys1)}$$

$$G_2(s) = 1/s^2 + 5s + 1 \text{-----} (\zeta=2.5) \text{ (sys2)}$$

$$G_3(s) = 1/s^2 + 6s + 1 \text{-----} (\zeta=3) \text{ (sys3)}$$

MATLAB Programme

```
>>sys1=tf([0 0 1],[1 4 1])
```

```
>>sys2=tf([0 0 1],[1 5 1])
```

```
>>sys3=tf([0 0 1],[1 6 1])
```

```
>>ltiview(sys1, sys2,sys3)
```

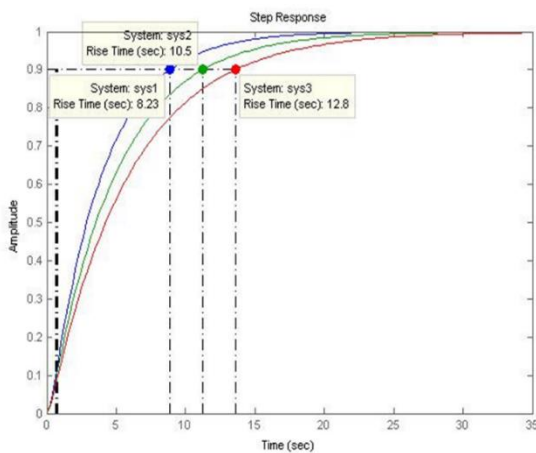


Fig. 7: Effect of damping ratio on dynamic behavior of an overdamped system

8. Step response characteristics of an inverse response systems:

Case-1:  $K_1 > K_2$  (both are positive) and  $\frac{K_2}{\tau_2} > \frac{K_1}{\tau_1}$

Transfer function  $G(s) = \frac{3}{2s+1} - \frac{2}{s+1}$  ----(sys1)

MATLAB Programme

```
>>sys1=tf([0 -1 1],[2 3 1])
```

```
>>ltiview(sys1)
```

Case 2:  $|K_1| > |K_2|$  (both are negative), and  $|K_2/\tau_2| > |K_1/\tau_1|$

Transfer function  $G(s) = \frac{-3}{s+1} - \frac{-2}{0.5s+1}$  ----(sys2)

MATLAB Program:

```
>>sys2=tf([0 1 -1],[0.5 1.5 1])
```

```
>>ltiview(sys2)
```

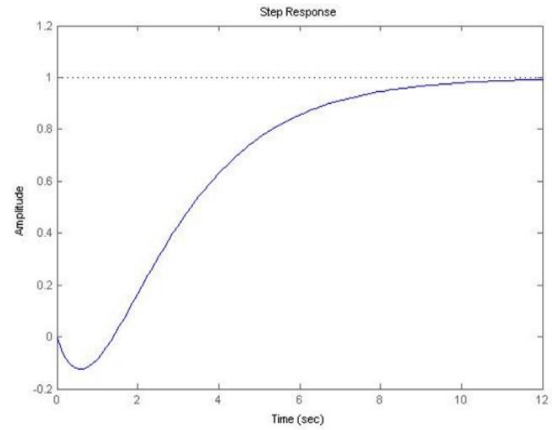


Fig. 8 (a): Unit step response of an inverse response system, Case 1  $K_1 > K_2$  (both are positive) and  $K_2/\tau_2 > K_1/\tau_1$

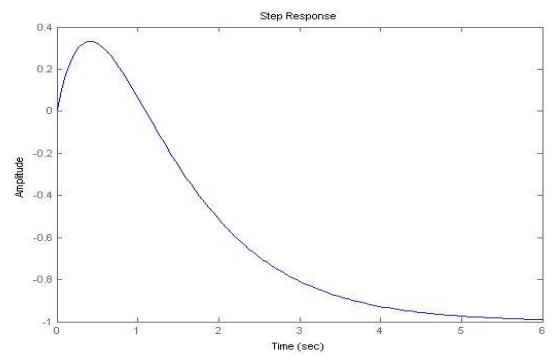


Fig. 8 (b): Unit step response of an inverse response system, Case-2-  $|K_1| > |K_2|$  (both are negative) and  $|K_2/\tau_2| > |K_1/\tau_1|$

RESULTS

1. Dynamic response of first-order system (Fig. 1)

Table I- Unit step response of a first-order system

Transfer function	Time constant	Settling time (sec)	Final response
$G(s) = 1/(s+1)$	1 s	~ 6 s	~ 1

2. Effect of change in static gain of a first-order process from 1 to 2, at constant time constant 1s on dynamic response characteristics of a first-order process. (Fig. 2)

Table II- Effect of change in static gain of a first-order process at constant time constant

System	Static gain	Time constant	Settling time (sec)	Final response

$G_1(s) = 1/(s+1)$ .	1	1s	~ 6s	~ 1
$G_2(s) = 2/(s+1)$ .	2	1s	~ 6s	~ 2

As the static gain is increased from 1s to 2s with same time constant, the speed of response remains same, but final steady-state response achieved increases.

3. Effect of change in time constant at constant gain (Fig. 3)

Table III- Effect of change in time constant at constant gain on response characteristics of a first-order process

System	Static gain	Time constant	Settling time (sec)	Final response
$G_1(s) = 1/(s+1)$ .	1	1s	~ 6s	~ 1
$G_3(s) = 1/(2s+1)$ .	1	2s	~12s	~ 1

As the time constant increases from 1 sec. to 2 sec., the ultimate response remains same but speed of response decreases.

4. Effect of dead time lag on step response characteristics of a first-order process (Fig. 4)

Table IV- Effect of dead time lag on step response characteristics of a first-order process

System	Static gain	Time constant	Time constant	Settling time (sec)	Final response
$G_1(s) = 1/(s+1)$ .	1	1s	0s	~ 6s	~ 1
$G_4(s) = e^{-s}/(s+1)$ .	1	1s	1s	~ 7s	~ 1

If process has dead time 1s, then the response just gets delayed by 1 sec. without any effect on time constant and ultimate response. But the settling time increases due to dead time.

5. Unit step response characteristics of a second-order process (Fig. 5)

Table V- Unit step response characteristics of second-order underdamped, overdamped, and critically damped processes

System	Rise time (sec)	Settling time (sec)	Final response
Overdamped (sys1)	8.23	14.9	1.00
Critically damped (sys2)	6.72	11.7	1.00

Underdamped (sys3)	1.64	8.08	1.00
--------------------	------	------	------

1. The underdamped system quickly reaches the final value first but then oscillates around it.
2. The over- and critically damped responses are non-oscillatory but sluggish than underdamped system.
3. The overdamped response is sluggish than critically damped response.

6. Effect of changing damping ratio of second-order underdamped process on step response characteristics (Fig. 6)

Table VI- Effect of changing damping ratio of second-order underdamped process on step response characteristics

System	Peak amplitude	%OS	Rise time (sec)	Settling time (sec)
Sys1 ( $\zeta=0.5$ )	1.16	16.3	1.65	8.07
Sys2 ( $\zeta=0.25$ )	1.44	44.4	1.27	14.1
Sys3 ( $\zeta=0.1$ )	1.73	72.9	1.11	38.4

The above results conclude that as damping ratio ' $\zeta$ ' of an underdamped system ( $<1$ ) increases, the step response becomes less oscillatory and sluggish with decreasing peak amplitude, %overshoot, rise time and decreasing settling time.

7. Effect of changing damping ratio of second-order overdamped process on step response characteristics (Fig. 7)

Table VII- Effect of changing damping ratio of second-order overdamped process on step response characteristics

System	Rise time (sec)	Settling time (sec)
Sys1 ( $\zeta=2$ )	8.23	14.9
Sys2 ( $\zeta=2.5$ )	10.5	19
Sys3 ( $\zeta=3$ )	12.8	23

The above results conclude that as damping ratio ' $\zeta$ ' of overdamped system ( $>1$ ) increases, the step response is non-oscillatory and sluggish with increasing rise time and settling time.

8. Dynamic response of an inverse response system (Fig. 8)

Case 1-

The unit step response shows inverse response characteristics in which initially response decreases (due to decreasing slope  $\frac{K_1}{\tau_1} - \frac{K_2}{\tau_2}$  at origin) and then increases to a positive value  $K=K_1-K_2=1$ .

Case 2-

The unit step response shows inverse response characteristics in which initially response increases (due to increasing slope  $\frac{K_1}{\tau_1} - \frac{K_2}{\tau_2}$  at origin) and then decreases to a negative value  $K=K_1-K_2=-1$ .

### CONCLUSION

In this work, the dynamic behavior of standard processes is studied. Also, the effect of variation in process parameters on dynamic response characteristics is explained in detail. It is observed that the dynamic response characteristics of process depend on its order and the types of poles and zeros of the respective transfer function. The process having one pole and one zero may show inverse response characteristics as explained above. This paper will be useful to understand the dynamic response characteristics of simple chemical processes. Readers will enjoy the MATLAB simulation exercises, rather than solving complex mathematical equations. Of course, one should have sound theory background knowledge before carrying out simulation work.

### REFERENCES

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