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On Neutrosophic $\psi\alpha g$-Closed Sets

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***Abstract* - The aim of this paper is to introduce the concept of ψαg-closed sets in terms of neutrosophic topological spaces. We also study some of the properties of neutrosophic ψαg-closed sets. Further, we introduce continuity and contra continuity for the introduced set. The two functions and their relations are studied via a neutrosophic point set.**

***Index Terms* - neutrosophic topology; neutrosophic ψαg-closed set; neutrosophic ψαg-continuous function; neutrosophic contra ψαg -continuous mappings.**

1.INTRODUCTION

Zadeh [1] introduced and studied truth (t), the degree of membership, and defined the fuzzy set theory. The falsehood (f), the degree of no membership, was introduced by Atanassov [2–4] in an intuitionistic fuzzy set. Coker [5] developed intuitionistic fuzzy topology. Neutrality (i), the degree of indeterminacy, as an independent concept, was introduced by Smarandache [6,7] in 1998. He also defined the neutrosophic set on three components (t, f, i) = (truth, falsehood, indeterminacy). The Neutrosophic crisp set concept was converted to neutrosophic topological spaces by Salama et al. in [8]. This opened up a wide range of investigation in terms of neutosophic topology and its application in decision-making algorithms. Arokiarani et al. [9] introduced and studied α-open sets in neutrosophic topoloical spaces. Devi et al. [10–12] introduced $\alpha\psi$ -closed sets in general topology, fuzzy topology, and intutionistic fuzzy topology. In this article, the neutrosophic $\psi\alpha g$ -closed sets are introduced in neutrosophic topological space. Moreover, we introduce and investigate neutrosophic $\psi\alpha g$ -continuous and neutrosophic contra $\psi\alpha g$ -continuous mappings.

2.PRELIMINARIES

Let neutrosophic topological space (NTS) be(X,τ). Each neutrosophic set (NS) in (X,τ) is called a neutrosophic open set (NOS), and its complement is called a neutrosophic closed set (NCS). We provide some of the basic definitions in neutrosophic sets. These are very useful in the sequel.

Definition 1. [6] A neutrosophic set (NS) A is an object of the following form
$U = \{\langle x, \mu_U(x), \nu_U(a), \omega_U(x)\rangle : x \in X\}$ where the mappings $\mu_U : X \to I, \nu_U : X \to I$, and $\omega_U : X \to I$ denote the degree of membership (namely $\mu_U(x)$), the degree of indeterminacy (namely $\nu_U(x)$), and the degree of nonmembership (namely $\omega_U(x)$) for each element $x \in X$ to the set U, respectively, and $0 \le \mu_U(x) + \nu_U(x) + \omega_U(x) \le 3$ for each $a \in X$.

Definition 2. [6] Let U and V be NSs of the form $U = \{\langle a, \mu_U(x), \nu_U(x), \omega_U(x)\rangle : a \in X\}$ and $V = \{\langle x, \mu_V(x), \nu_V(x), \omega_V(x)\rangle : x \in X\}$. Then
(i) $U \subseteq V$ if and only if $\mu_U(x) \le \mu_V(x), \nu_U(x) \ge \nu_V(x)$ and $\omega_U(x) \ge \omega_V(x)$;
(ii) $\overline{U} = \{\langle x, \nu_U(x), \mu_U(x), \omega_U(x)\rangle : x \in X\}$;
(iii)$U \cap V = \{\langle x, \mu_U(x) \wedge \mu_V(x), \nu_U(x) \vee \nu_V(x), \omega_U(x) \vee \omega_V(x)\rangle : x \in X\}$;
(iv)$U \cup V = \{\langle x, \mu_U(x) \vee \mu_V(x), \nu_U(x) \wedge \nu_V(x), \omega_U(x) \wedge \omega_V(x)\rangle : x \in X\}$
We will use the notation $U = \langle x, \mu_U, \nu_V, \omega_U\rangle$ instead of $U = \{\langle x, \mu_U(x), \nu_V(x), \omega_U(x)\rangle : x \in X\}$. The NSs $0_\sim$ and $1_\sim$ are defined by $0_\sim = \{\langle x, \underline{0}, \underline{1}, \underline{1}\rangle : x \in X\}$ and $1_\sim = \{\langle x, \underline{1}, \underline{0}, \underline{0}\rangle : x \in X\}$.
Let f be a mapping from an ordinary set X into an ordinary set Y. If $V = \{\langle y, \mu_V(y), \nu_V(y), \omega_V(y)\rangle : y \in Y\}$ is an NS in Y, then the inverse image of V under f is an NS defined by
$f^{-1}(V) = \{\langle x, f^{-1}(\mu_V)(x), f^{-1}(\nu_V)(x), f^{-1}(\omega_V)(x)\rangle : x \in X\}$.

The image of NS $U = \{\langle y, f^{-1}(\mu_U)(y), f^{-1}(\nu_U)(y), f^{-1}(\omega_U)(y)\rangle : y \in Y\}$

under f is an NS defined by $f(U) = \{\langle y, f(\mu_U)(y), f(\nu_U)(y), f(\omega_U)(y)\rangle : y \in Y\}$ where

$$f(\mu_U)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_U(x), & if\ f^{-1}(V) \neq 0 \\ 0 & otherwise \end{cases}$$

$$f(\nu_U)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_U(x), & if\ f^{-1}(V) \neq 0 \\ 1 & otherwise \end{cases}$$

$$f(\omega_U)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \omega_U(x), & if\ f^{-1}(V) \neq 0 \\ 1 & otherwise \end{cases}$$

for each $y \in Y$.

Definition 3. [8] A neutrosophic topology (NT) in a nonempty set X is a family τ of NSs in X satisfying the following axioms:
(NT1) $0_\sim, 1_\sim \in \tau$;
(NT2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
(NT3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$

Definition 4. [8] Let U be an NS in NTS X. Then
$Nint(U) = \cup \{O : O$ is an NOS in X and $O \subseteq U\}$ is called a neutrosophic interior of U;
$Ncl(U) = \cap \{O : O$ is an NCS in X and $O \supseteq U\}$ is called a neutrosophic closure of U.

Definition 5. [9] A subset U of a neutrosophic space (X, τ) is called
1. a neutrosophic semi-open set if $U \subseteq Ncl(Nint(U))$, and a neutrosophic semi-closed set if $Nint(Ncl(U)) \subseteq U$,
2. a neutrosophic α-open set if $U \subseteq Nint(Ncl(Nint(U)))$, and a neutrosophic α-closed set if $Ncl(Nint(Ncl(U))) \subseteq U$.

The semi-closure and α-closure of a subset U of a neutrosophic space (X, τ) is the intersection of all semi-closed, α-closed) sets that contain U and is denoted by $Nscl(U)$ and $N\alpha cl(U)$).

Definition 6. A subset A of a neutrosophic topological space (X, τ) is called
1. a neutrosophic semi-generalized closed (briefly, Nsg-closed) set if $Nscl(U) \subseteq G$ whenever $U \subseteq G$ and G is neutrosophic semi-open in (X, τ);
2. a neutrosophic $N\psi$-closed set if $Nscl(U) \subseteq G$ whenever $U \subseteq G$ and G is Nsg-open in (X, τ).

3. ON NEUTROSOPHIC $\psi\alpha g$-CLOSED SETS

Definition 7. A neutrosophic $\psi\alpha g$-closed ($N\psi\alpha g$-closed) set is defined as if $N\psi cl(U) \subseteq G$ whenever $U \subseteq G$ and G is an $N\alpha g$-open set in (X, τ). Its complement is called a neutrosophic $\psi\alpha g$-open (N$\psi\alpha g$-open) set.

Definition 8. Let U be an NS in NTS X. Then
$N\psi\alpha g\ int(U) = \cup \{O : O$ is an $N\psi\alpha g\ OS$ in X and $O \subseteq U\}$ is said to be a neutrosophic $\psi\alpha g$ -interior of U;
$N\psi\alpha g\ cl(U) = \cap \{O : O$ is an $N\psi\alpha g\ CS$ in X and $O \subseteq U\}$ is said to be a neutrosophic $\psi\alpha g$ -closure of U.

Theorem 1. All $N\alpha$-closed sets and N-closed sets are $N\psi\alpha g$-closed sets.

Proof. Let U be an $N\alpha$-closed set, then $U = N\alpha cl(U)$. Let $U \subseteq G$, where G is Na-open. Since U is $N\alpha$-closed, $N\psi cl(U) \subseteq N\alpha cl(U) \subseteq G$. Thus, U is $N\psi\alpha g$ -closed.

Theorem 2. Every Nsemi-closed set in a neutrosophic set is an $N\psi\alpha g$-closed set.

Proof. Let U be an Nsemi-closed set in (X, τ), then $U = Nscl(U)$. Let $U \subseteq G$, where G is $N\alpha$-open in (X, τ). Since U is Nsemi-closed, $N\psi cl(U) \subseteq Nscl(U) \subseteq G$. This shows that U is $N\psi\alpha g$-closed set.
The converse of the above theorems are not true, as can be seen by the following counter example.

Example 1. Let $X = \{u, v, w\}$ and neutrosophic sets G_1, G_2, G_3, G_4 be defined by

$$G1 = \langle x, (\frac{u}{0.3}, \frac{v}{0.4}, \frac{w}{0.2}), (\frac{u}{0.5}, \frac{v}{0.1}, \frac{w}{0.2}), (\frac{u}{0.2}, \frac{v}{0.5}, \frac{w}{0.6})\rangle$$

$$G2 = \langle x, (\frac{u}{0.6}, \frac{v}{0.3}, \frac{w}{0.4}), (\frac{u}{0.1}, \frac{v}{0.5}, \frac{w}{0.1}), (\frac{u}{0.3}, \frac{v}{0.2}, \frac{w}{0.5})\rangle$$

$$G3 = \langle x, (\frac{u}{0.6}, \frac{v}{0.4}, \frac{w}{0.4}), (\frac{u}{0.1}, \frac{v}{0.1}, \frac{w}{0.1}), (\frac{u}{0.2}, \frac{v}{0.2}, \frac{w}{0.5})\rangle$$

$G4 = \langle x, (\frac{u}{0.3}, \frac{v}{0.3}, \frac{w}{0.2}), (\frac{u}{0.5}, \frac{v}{0.5}, \frac{w}{0.2}), (\frac{u}{0.3}, \frac{v}{0.5}, \frac{w}{0.6}) \rangle$
$G5 = \langle x, (\frac{u}{0.3}, \frac{v}{0.3}, \frac{w}{0.3}), (\frac{u}{0.5}, \frac{v}{0.5}, \frac{w}{0.4}), (\frac{u}{0.3}, \frac{v}{0.5}, \frac{w}{0.3}) \rangle$
$G6 = \langle x, (\frac{u}{0.6}, \frac{v}{0.4}, \frac{w}{0.5}), (\frac{u}{0.1}, \frac{v}{0.3}, \frac{w}{0.1}), (\frac{u}{0.3}, \frac{v}{0.3}, \frac{w}{0.4}) \rangle$
$G7 = \langle x, (\frac{u}{0.2}, \frac{v}{0.3}, \frac{w}{0.3}), (\frac{u}{0.5}, \frac{v}{0.5}, \frac{w}{0.2}), (\frac{u}{0.3}, \frac{v}{0.3}, \frac{w}{0.5}) \rangle$
Let $\tau = \{0_\sim, G_1, G_2, G_3, G_4, 1_\sim\}$. Here, G_6 is an $N\alpha$ open set, and $N\psi cl(G_5) \subseteq G_6$. Then G_5 is $N\psi\alpha g$-closed in (X,τ) but is not $N\alpha$-closed. Thus, it is not N-closed and G_7 is $N\psi\alpha g$ -closed in (X,τ), but not Nsemi-closed.

Theorem 3. Let (X,τ) be an NTS and let $U \in NS(X)$. If U is an $N\psi\alpha g$-closed set and $U \subseteq V \subseteq N\psi cl(U)$, then V is an N$\psi\alpha g$-closed set.

Proof. Let G be an $N\alpha$-open set such that $V \subseteq G$. Since $U \subseteq V$, then $U \subseteq G$. But U is $N\psi\alpha g$-closed, so $N\psi cl(U) \subseteq G$, since $V \subseteq N\psi cl(U)$ and $N\psi cl(V) \subseteq N\psi cl(U)$ and hence $N\psi cl(V) \subseteq G$. Therefore V is an $N\psi\alpha g$-closed set.

Theorem 4. Let U be an $N\psi\alpha g$-open set in X and $N\psi int(U) \subseteq V \subseteq U$, then V is $N\psi\alpha g$-open.

Proof. Suppose U is $N\psi\alpha g$-open in X and $N\psi int(U) \subseteq V \subseteq U$. Then $\overline{U}$ is $N\psi\alpha g$-closed and $\overline{U} \subseteq \overline{V} \subseteq N\psi cl(\overline{U})$. Then $\overline{U}$ is an $N\psi\alpha g$-closed set by Theorem 3. Hence, V is an $N\psi\alpha g$-open set in X.

Theorem 5. An NS U in an NTS (X,τ) is an $N\psi\alpha g$-open set if and only if $V \subseteq Nyint(U)$ whenever V is an $N\alpha$-closed set and $V \subseteq U$.

Proof. Let U be an $N\psi\alpha g$-open set and let V be an $N\alpha$-closed set such that $V \subseteq U$. Then $\overline{U} \subseteq \overline{V}$ and hence $N\psi cl(\overline{U}) \subseteq \overline{V}$, since $\overline{U}$ is $N\psi\alpha g$-closed. But $N\psi cl(\overline{U}) = \overline{N\psi\imath nt(U)}$, so $V \subseteq N\psi int(U)$.
Conversely, suppose that the condition is satisfied. Then $\overline{N\psi\imath nt(U)} \subseteq \overline{V}$ whenever $\overline{V}$ is an $N\alpha$-open set and $\overline{U} \subseteq \overline{V}$. This implies that $N\psi cl(\overline{U}) \subseteq \overline{V} = G$, where G is $N\alpha$-open and $\overline{U} \subseteq G$. Therefore, $\overline{U}$ is $N\psi\alpha g$-closed and hence $\overline{U}$ is $N\psi\alpha g$-open.

Theorem 6. Let U be an $N\psi\alpha g$-closed subset of (X,τ). Then $Nycl(U) - U$ does not contain any non-empty $N\psi\alpha g$-closed set.

Proof. Assume that U is an $N\psi\alpha g$-closed set. Let F be a non-empty $N\psi\alpha g$-closed set, such that $F \subseteq N\psi cl(U) - U = N\psi cl(U) \cap \overline{U}$. i.e., $F \subseteq N\psi cl(U)$ and $F \subseteq \overline{U}$. Therefore, $U \subseteq \overline{F}$. Since $\overline{F}$ is an $N\psi\alpha g$-open set, $N\psi cl(U) \subseteq \overline{F}\ ) \Rightarrow F \subseteq (N\psi cl(U) - U) \cap \overline{(N\psi cl(U))} \subseteq N\psi cl(U) \cap N\psi cl(U)$. i.e., $F \subseteq \phi$. Therefore, F is empty.

Corollary 1. Let U be an $N\psi\alpha g$-closed set of (X,τ). Then $N\psi cl(U)$-U does not contain any non-empty N-closed set.

Proof. The proof follows from the Theorem 3.9.

Theorem 7. If U is both $N\psi$-open and $N\psi\alpha g$-closed, then U is $N\psi$-closed.

Proof. Since U is both an $N\psi$-open and $N\psi\alpha g$-closed set in X, then $N\psi cl(U) \subseteq U$. We also have $U \subseteq N\psi cl(U)$. Thus, $N\psi cl(U) = U$. Therefore, U is an $N\psi$-closed set in X.

4. ON NEUTROSOPHIC $\psi\alpha g$-CONTINUITY AND NEUTROSOPHIC CONTRA $\psi\alpha g$-CONTINUITY

Definition 9. A function $f : X \to Y$ is said to be a neutrosophic $\psi\alpha g$-continuous (briefly, $N\psi\alpha g$-continuous) function if the inverse image of every open set in Y is an $N\psi\alpha g$-open set in X.

Theorem 8. Let $g : (X,\tau) \to (Y,\sigma)$ be a function. Then the following conditions are equivalent.
(i) g is $N\psi\alpha g$-continuous;
(ii) The inverse $f^{-1}(U)$ of each N-open set U in Y is $N\psi\alpha g$-open set in X.

Proof. The proof is obvious, since $g^{-1}(\overline{U}) = \overline{g^{-1}(U)}$ for each N-open set U of Y.

Theorem 9. If $g : (X,\tau) \to (Y,\sigma)$ is an $N\psi\alpha g$-continuous mapping, then the following statements hold:
(i)$g(N\psi\alpha gNcl(U)) \subseteq Ncl(g(U))$, for all neutrosophic sets U in X;

(ii)$N\psi\alpha gNcl(g^{-1}(V)) \subseteq g^{-1}(Ncl(V))$, for all neutrosophic sets V in Y.
Proof.
(i) Since $Ncl(g(U))$ is a neutrosophic closed set in Y and g is $N\psi\alpha g$-continuous, then $g^{-1}(Ncl(g(U)))$ is $N\psi\alpha g$-closed in X. Now, since $U \subseteq g^{-1}(Ncl(g(U)))$, $N\psi\alpha gcl(U) \subseteq g^{-1}(Ncl(g(U)))$. Therefore, $g(N\psi\alpha gNcl(U)) \subseteq Ncl(g(U))$.
(ii) By replacing U with V in (i), we obtain $g(N\psi\alpha gcl(g^{-1}(V))) \subseteq Ncl(g(g^{-1}(V))) \subseteq Ncl(V)$. Hence, $N\psi\alpha gcl(g^{-1}(V)) \subseteq g^{-1}(Ncl(V))$.

Definition 10. A function is said to be a neutrosophic contra $\psi\alpha g$-continuous function if the inverse image of each NOS V in Y is an $N\psi\alpha gC$ set in X.

Theorem 10. Let $g : (X,\tau) \to (Y,\sigma)$ be a function. Then the following assertions are equivalent:
(i) g is a neutrosophic contra $\psi\alpha g$-continuous function;
(ii) $g^{-1}(V)$ is an $N\psi\alpha gC$ set in X, for each NOS V in Y.

Proof. (i)) $\Rightarrow (ii)$ Let g be any neutrosophic contra $\psi\alpha g$ -continuous function and let V be any NOS in Y. Then $\bar{V}$ is an NCS in Y. Based on these assumptions, $g^{-1}(\bar{V})$ is an $N\psi\alpha gO$set in X. Hence, $g^{-1}(V)$ is an $N\psi\alpha gC$set in X.
The converse of the theorem can be proved in the same way.

Theorem 11. Let $g : (X,\tau) \to (Y,\sigma)$ be a bijective mapping from an NTS(X,τ) into an NTS(Y,σ). The mapping g is neutrosophic contra $\psi\alpha g$ -continuous, if $Ncl(g(U)) \subseteq g(N\psi\alpha gint(U))$, for each NS U in X.

Proof. Let V be any NCS in X. Then $Ncl(V) = V$, and g is onto, by assumption, which shows that $g(N\psi\alpha gint(g^{-1}(V))) \supseteq Ncl(g(g^{-1}(V))) = Ncl(V) = V$. Hence, $g^{-1}(g(N\psi\alpha gint(g^{-1}(V)))) \supseteq g^{-1}(V)$. Since g is an into mapping, we have $N\psi\alpha g\, int(g^{-1}(V)) = g^{-1}(V)(g(N\psi\alpha gint(g^{-1}(V)))) \supseteq g^{-1}(V)$. Therefore, $N\psi\alpha gint(g^{-1}(V)) = g^{-1}(V)$, so $g^{-1}(V)$ is an $N\psi\alpha gO$ set in X. Hence, g is a neutrosophic contra $\psi\alpha g$-continuous mapping.

Theorem 12. Let $g : (X_1,\tau) \to (Y_1,\sigma)$ be a function. If the graph $h: X_1 \to X_1 \times Y_1$ of g is neutrosophic contra $\psi\alpha g$-continuous, then g is neutrosophic contra $\psi\alpha g$-continuous.

Proof. For every NOS, V in Y_1 holds $g^{-1}(V) = 1 \wedge g^{-1}(V) = h^{-1}(1 \times V)$. Since h is a neutrosophic contra $\psi\alpha g$-continuous mapping and $1 \times V$ is an NOS in $X_1 \times Y_1, g^{-1}(V)$ is an $N\psi\alpha gC$ set in X_1, so g is a neutrosophic contra $\psi\alpha g$-continuous mapping.

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