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A Study on Two Edge Disjoint Hamiltonian circuits in Vehicle Routing Problem

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***Abstract* - Hamiltonian graph is one of the main concepts of graph theory. There are many types of graphs in graph theory. But in this paper, we discuss Hamiltonian graph and its edge disjoint Hamilton circuit. In Hamiltonian graph a path which traverse each vertices of a graph exactly once. It has been found that the intersection graph obtained from Euler graph is not a Hamiltonian graph. The graph G(3m+7,6m+14) for m≥6 which is also planner and regular of degree 4, and non-bipartite, has two edge disjoint Hamiltonian circuit. Hamiltonian graph in this case is applied to transportation vehicle routing problem.**

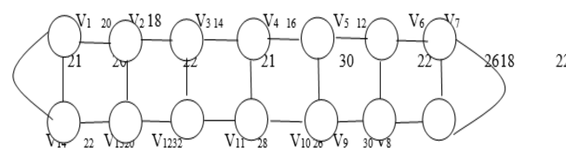
INTRODUCTION

Graph theory is about analysis of graphs in Mathematics. Graphs are one of the superior objects of study in discrete mathematics. In general, a graph is Constitute as a set of vertices i.e. nodes or points connected by edges i.e. arcs or line. Graphs are therefore mathematical structures used to model which gives matching between the objects. Now a days graph is found on Google Road maps, Celestial, when excogitate schemes and delineation. Graphs shore many computer programs that make modern elucidation and technological processes possible. They accord to the amelioration of thinking, both logical and abstract. A Consequence is made between undirected graphs, where edges link two vertices symmetrically and directed graphs, where edges join two vertices asymmetrically.

HAMILTONIAN THEOREM

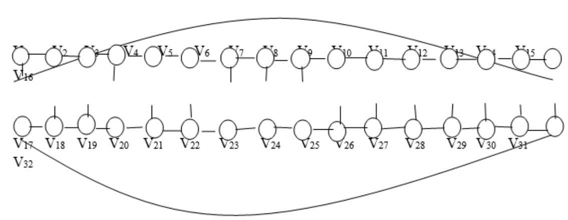
Theorem 1
The graph G (2n+2, 3n+3) for n≥6 which is regular of degree three, non-bipartite and planar is always Hamiltonian.

Proof: Now, we have to prove the value of n=6 then we can construct a graph of 14 vertices and 21 edges, which is also a regular graph of degree 3 and this graph contain at least one Hamiltonian circuit and hence it is Hamiltonian.



Theorem 2
The regular graph G(4n+4,6n+6) for n≥7 of degree three and planar of odd number of regions having four edges when n=7 and only two region covered by 2m+4 edges for m≥6 for simultaneous changes of n≥2 is always bi-colorable.

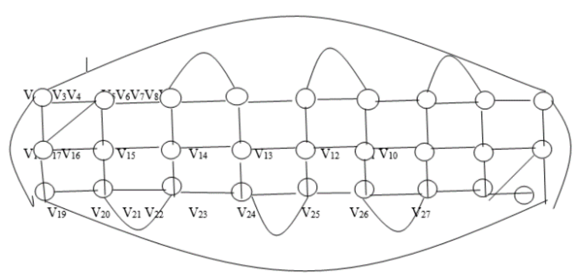
Proof: let us consider the value of n=7 for simultaneous changes of m=6 the graph constructed which contains 15 regions covering of four edges and two regions covering of 16 edges. The vertex v1 is colored by the color c1 and the vertex v2 is colored by the color c2. This graph be colored only by two colors c1&c2.



Theorem 3
The graph G(3m+6,12+6m) for m≥7, which is regular of degree four, non-bipartite and planar has two edge disjoint Hamiltonian cycle.

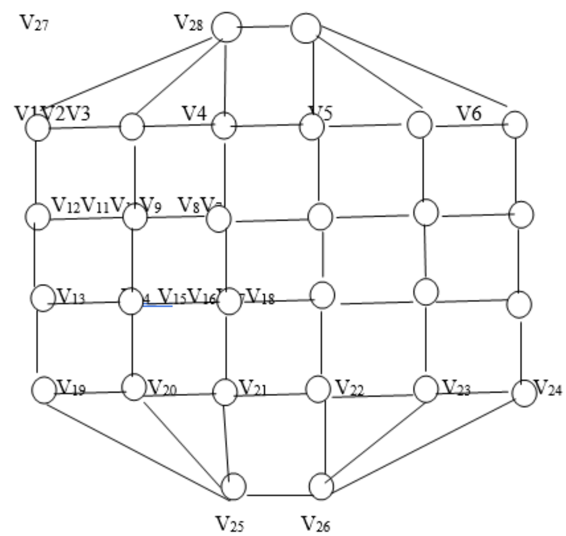
Proof: The graph G(3m+6,12+6m) is a planar graph as there is no intersection between their edges and it is a regular graph of degree four. The graph G(3m+6,12+6m) for m≥1 can be constructed as regular graph of degree four. The graph has least one Hamiltonian circuit and hence it is Hamiltonian.
Let us consider a Hamiltonian circuit

V1e1,V2e43,V10e9,V11e10,V12e29,V3e45,V4e31, V13e12,V14e33,V5e46,V6e35,V15e14,V16e37,V7e47,V8e39, V17e16e41,V9e52,V27e24,V26e23,V25e22,V24e21, V23e20,V22e19,V21e18,V20e17,V19e54V1 where V1,V2,V10,V11,V12,V3,V4,V13,V14,V5,V6,V15,V16,V7,V8,V17,V18,V9,V27,V26,V25,V24,V23,V22, V21, V20,V19 is vertices of the graph and e1,e43,e9,e10,e29,e45,e31,e12,e33,e46,e35,e14,e37,e47,e39,e16,e41,e52,e24,e23,e22,e21,e20,e19,e18,e17, e26are the edges of the graph.
Let us consider another Hamiltonian circuit V1e53,V9e8,V8e7,V7e6,V6e5,V5e4,V4e3,V3e2,V27 V11e2,V20e18e30,V12e11,V13e32,V12e20,V23e34, V14e13,V15e36,V24e22,V25where V1,V9,V8,V7,V6,V5,V4,V3,V2,V11,V20,V21,V12, V13,V22,V23,V14,V15,V24,V25,V16,V17,V26,V18,V27, V19,V16 are the vertices of the graph and e53,e8,e7,e6,e5,e4,e3,e2,e27,e28,e18,e30,e12,e32,e20,e34,e13,e36,e22,e28,e15,e40,e44,e42,e51,e26,e25 are the edges of the graph.



Theorem 4
The graph G(3m+7,6m+14) for m≥7, which is planner and regular of degree 4, and non-bipartite, has two edge-disjoint Hamiltonian circuit.

Proof: The graph G(3m+7,6m+14) is a planner graph for m≥7. Since degree of each vertex is 4, hence it is regular of degree 4 and it is non bipartite as it contains triangle.



Let us consider the one Hamiltonian circuit V1e49,V27e2,V3e3,V4e52,V28e53,V5e5,V6e45,V4e41,V18e46,V12e10,V11e9,V10e8,V9e7,V8e6,V4e4, V13e11,V14e12,V15e13,V16e14, V17e37,V23e38,V26e34,V22e18,V21e20,V25e17,V20e16,V19e47,V1 where vertices areV1,V2,V27,V3,V4,V28,V5,V24,V18,V12,V11,V10,V9,V8,V7,V13,V14,V15,V16,V17,V23,V26,V22, V21,V25, V20,V19 and the edges e49,e50,e2,e3,e52,e53,e5,e45,e41,e46,e10,e9,e8,e7,e6,e48,e11,e12,e13,e14,e37,e38,e34,e18,e30,e17,e16,e47.

Let us consider the another Hamiltonian circuit is V25e42,V19e23,V12e22,V7e21,V1e1,V2e24,V8e25, V14e26,V20e17,V21e29,V12e28,V9e27,V3e51,V27e54,V28e53V6e39,V12e46,V18e15,V17e36,V11e35, V5e4,V5e31,V10e32,V16e33,V22e19,V23e20,V24e44,V26e43V25 and the vertices are V25,V19,V12,V7,V1,V2,V8,V14,V20,V21,V12,V9, V3,V27,V28,V6,V12,V18,V17,V11,V5,V4,V10,V16,V23, V24,V26 and the edges of the graph are e42,e23,e22,e21,e1,e24,e25,e26,e17,e29,e28,e27,e51, e54,e53,e39,e46,e15,e36,e35,e4,e31,e32,e33,e19,e20, e44,e43.

VEHICLE ROUTING PROBLEM

Vehicle routing problem is based on the operations research. The traveling salesman problem which is a special case of vehicle routing problem. The objective of the VRP is minimize the total route cost.

Vehicle routing problem which is the application of Hamiltonian graph. vehicle routing problem demonstrate about a number of vehicles located at a central depot have to serve a set of geographically dispersed customers. The capacity will be given to each vehicle and demand will be given to each customer. The ambition is to minimize the total distance travel. Vehicle routing problem is more than one vehicle is require to serve all orders.

Type of decision
- Assigning(capacity)
- Routing(distance)

Find best vehicle route to serve a set of order from customers.
Best route may be
- Minimum cost
- Minimum distance
- Minimum travel time

Order may be
- Delivery from depot to customer
- Pickup at customer and return to depot.
- Take up at one place and deliver to another place.

Assumption for basic VRP
- Vehicles have the same capacity
- Vehicle based at a single depot station.
- Vehicle serve many different customers.
- Exactly one vehicle will deliver each customer demand.
- Need to calculate the reduction cost for collection of vehicle route from all starting and ending to be same depot, which contain all customers and do not violate vehicle capacity.

Mathematical description of vehicle routing problem
- Defined on undirected graph G= (V, E)
- V= {0,1….N} set of nodes, vertex 0 is the depot remaining nodes are customers.
- E is set of edges on the graph.
- Each customer I has demand di
- Cost Cijfor traveling route from i to j

Vehicle routing problem formulation
Minimum $\sum C_{ij}X_{ij}$

$\sum X_{ij}+\sum X_{N+1,j}=1$

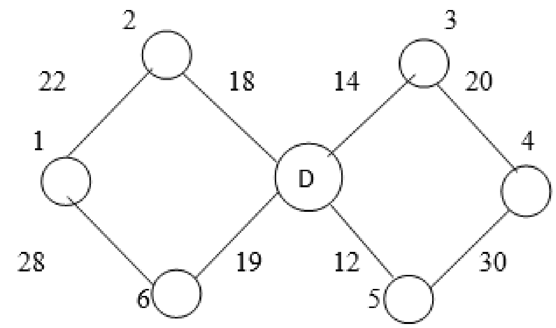
$\sum X_{ij}+\sum X_{i,\, N+2}=1$

$\sum X_{N+1,j}=V$

$\sum X_{i,N+2}=V$

$X_{ij}=0,1$

Single depot



There are 2 vehicle depends on distance
- D-3-4-5-D=14+20+30+12=76
- D-2-1-6-D=18+22+28+19=87

Total =163
Actually comprise 2 problem
Allocation problem– which of this customer will go to which vehicles
Routing problem - which is the order in which vehicle go to move such it travel minimum distance.

Depot/cities	0	1	2	3	4	5	6
0	-	20	18	14	16	12	19
1	-	-	22	18	30	26	28
2	-	-	-	32	20	22	21
3	-	-	-	-	20	22	21
4	-	-	-	-	-	30	22
5	-	-	-	-	-	-	26
6	-	-	-	-	-	-	-

Consider q is a customer required unit
q=[4 6 3 2 3 2]
where Q is vehicle capacity
Q= 10
D-2-1-6-D is not feasible because this capacity is more than 10.

D-3-4-5-D is feasible because this capacity is less than 10.
But overall problem is not feasible.
$(\sum q)/Q=20/(10) =2$
So, 2 vehicles are enough to supply customer to depot.
{1,2} – vehicle 1
{3,4,5,6}- vehicle 2
There are two 2 vehicle depends on distance
D-1-2-D=20+22+18=60
D-3-4-5-6-D=14+20+30+26+19=109
Total =169

CONCLUSION

Hamiltonian graph have been constructed and edge disjoint Hamiltonian cycle, vertex coloring, equal path partition has been discussed. It has been found that the application of edge disjoint Hamiltonian circuit has been prevailed in parallel computing, vertex coloring has been used in scheduling. It has been found that the intersection of Hamiltonian graph is not a Euler graph.it has been applied in transportation field. Hamiltonian graph has been used to solve vehicle routing problem.

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