SUPER EDGE ANTI MAGIC GRACEFUL LABELING OF SOME SPECIAL GRAPHS

N. Mary Bernard
Assistant Professor, TDMNS College, T. Kallikulam.

Abstract- **Let $G(V, E)$ be a graph with p vertices and q edges. A Super Edge Anti Magic Graceful Labeling ($SEAMGL$) of a graph $G(p, q)$ is a one to one and onto function $f: E \to \{1, 2, 3, \dots, q\}$ with the property that for each $u \in V$, $\sum_{e \in N(u)} f(uv) = distinct$. Such a labeling is Super Edge Anti Magic Graceful ($SEAMG$) if $|f(u) + f(v) - f(uv)| = distinct$. A graph G is called Super Edge Anti Magic Graceful ($SEAMG$) if G admits Super Edge Anti Magic Graceful Labeling. In this paper we determine Friendship graph, Dumbbell graph, Middle graph and Prism graph which are Super Edge Anti Magic Graceful.** ($SEAMG$).

Index Terms- **Super edge anti-magic, Graceful labeling, Friendship graph, Dumbbell graph, Middle graph, Prism graph.**

AMS Subject Classification: 05C78

I. INTRODUCTION

A graph labelling [3] is a function which has domain as graph elements such as vertices and / or edges with co-domain as a set of numbers. Usually, the co-domain has been taken as integers. Many of graph labelings are defined and well-studied by so many authors.

In 1963, Sedlacek introduced the concept of magic labelling [8] in graphs. A graph $G(p, q)$ is magic if the edges of G can be labeled by the numbers $\{1, 2, 3, \dots, q\}$ so that the sum of labels of all the edges incident with any vertex is the same.

In 1967, Rosa introduced graceful labelling [11]. An injection f from the vertices of G to $\{0, 1, 2, \dots, q\}$ is called graceful labeling of G if when we assign each edge uv, the label $|f(u) - f(v)|$, the resulting edges are distinct.

In 1990, N. Hartsfield and G. Ringel introduced the concepts called **Anti magic labeling** and **Anti magic graphs [7]**. A graph G is Anti Magic if the q edges of G can be distinctly labeled in such a way that when taking the sum of the edges labels incident to each vertex they all will have distinct constants is called the anti-magic labelling.

Throughout, it is assumed that G is connected, finite, simple and undirected graph.

II. MAIN RESULT

A Super Edge Anti Magic Graceful Labeling ($SEAMGL$) of a graph $G(p, q)$ is a one to one and onto function $f: E \to \{1, 2, 3, \dots, q\}$ with the property that for each $u \in V$, $\sum_{e \in N(u)} f(uv) = distinct$. Such a labeling is Super Edge Anti Magic Graceful ($SEAMG$) if $|f(u) + f(v) - f(uv)| = distinct$.

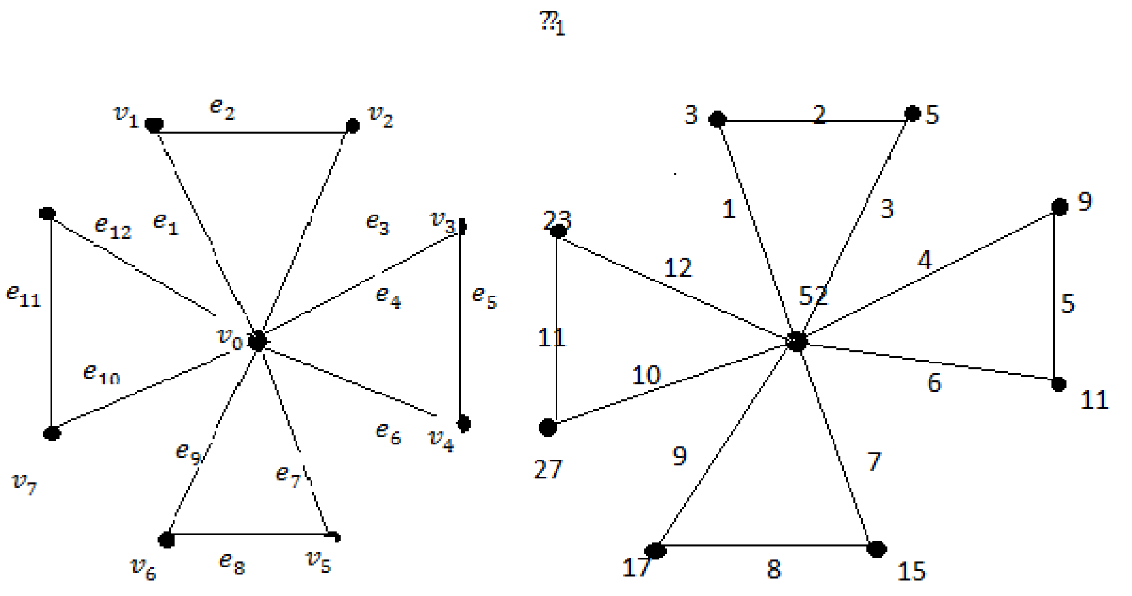
Definition:
A **friendship graph** Fn is a planar undirected **graph** with $2n + 1$ vertices and $3n$ edges. The **friendship graph** Fn can be constructed by joining n copies of the cycle **graph** C_3 with a common vertex.

Theorem :
A **friendship graph** Fn is $SEAMG$.
Proof:
Let $G = F_n$
Let p be the number of vertices, and q be the number of edges.
Let $v_1, v_2, v_3, \dots, v_n$ be the vertices of the Fan graph.
Let $E(G) = \{1,2,3, \dots, 3n\}$ and $V(G) = \{1,2,3, \dots, 2n + 1\}$ (ie) $V(G) = \sum_{e \in N(u)} f(uv)$.
Define a function $f : V(G) \cup E(G) \to \{1,2, \dots . qn + n\}$ as in the following way;
Fix, $f(e_1) = 1 . f(e_i) = i$ for $1 \le i \le n - 1$.

$$f(v_i) = f(e_i) + f(e_{i+1}), i = 1,2,3, \dots, n - 1$$
$$= i + i + 1 = 2i + 1, i = 1,2,4,5,7,8,10,11 \dots, i \ne 3t, t = 1,2,3, \dots$$

And $f(v_n) = qn + n$
It is easy to observe that the Fan graph F_n is $SEAMG$.

Illustration:



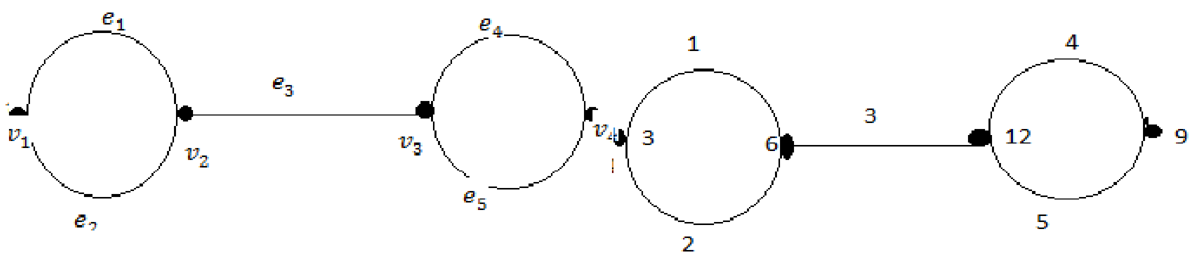
Definition:
A **dumbbell graph**, denoted by $D_{a,b.c}$ is a bicyclic **graph** consisting of two vertex- disjoint cycles C_a, C_b and a path P_{c+3} (c ⩾ −1) joining them having only its end-vertices in common with the two cycles

Theorem: A **dumbbell graph** admits $SEAMG$.

Proof:
Let $v_1, v_2, v_3, \dots, v_n$ be the vertices of the dumbbell graph,

Let $e_1, e_2, e_3, \dots, e_n$ be the edges of the dumbbell graph , such that
Label the edges are as follows:
f $(e_i) = i$ for $1 \le i \le n$, and
the vertices are labelled by $f(v_i) = \sum_{e \in N(u)} f(uv)$ $1 \le i \le n$.
Also $|f(u) + f(v) - f(uv)|$ is distinct.
It is clear that the dumbbell graph is $SEAMG$.

Illustration:

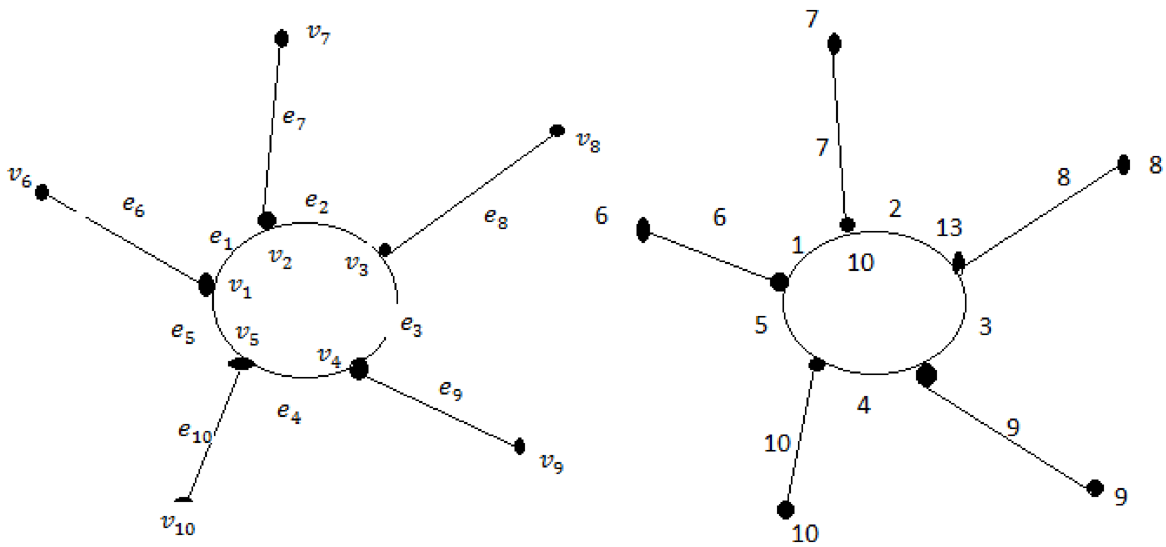


Definition:
The **middle graph** of a **graph** is the **graph** whose vertex set is where two vertices are adjacent if and only if they are either adjacent edges of G or one is a vertex and the other is an edge incident with it.

Theorem: A Middle graph is Super Edge Anti Magic Graceful.

Proof:
Let $G\ (p, q)$ be a Middle graph.
Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the Middle, graph
Let $e_1, e_2, e_3, \ldots, e_n$ be the edges of the Middle graph
The edges are label as $\{1,2,3, \ldots, q\}$, where q is the number of edges.
(i.e.) $f(e_1) = 1$, $f(e_2) = 2$, $f(e_3) = 3$, …, $f(e_n) = q$.
The vertices are label as,
$f(v_i) = i \quad for \quad 1 \le i \le \frac{q}{2}$ and
$f(v_{\frac{q}{2}} + i) = f\left(e_{\frac{q}{2}} + 1\right) + f\left(e_{\frac{q}{2}} + 2\right) + f(e_i)$,$1 \le i \le \frac{q}{2}$
hence all the vertex labels are distinct.
Also $|f(u) + f(v) - f(uv)|$ is distinct.
It is clear that G admits $SEAMGL$.
Thus, the middle graph is $SEAMG$.

Illustration:

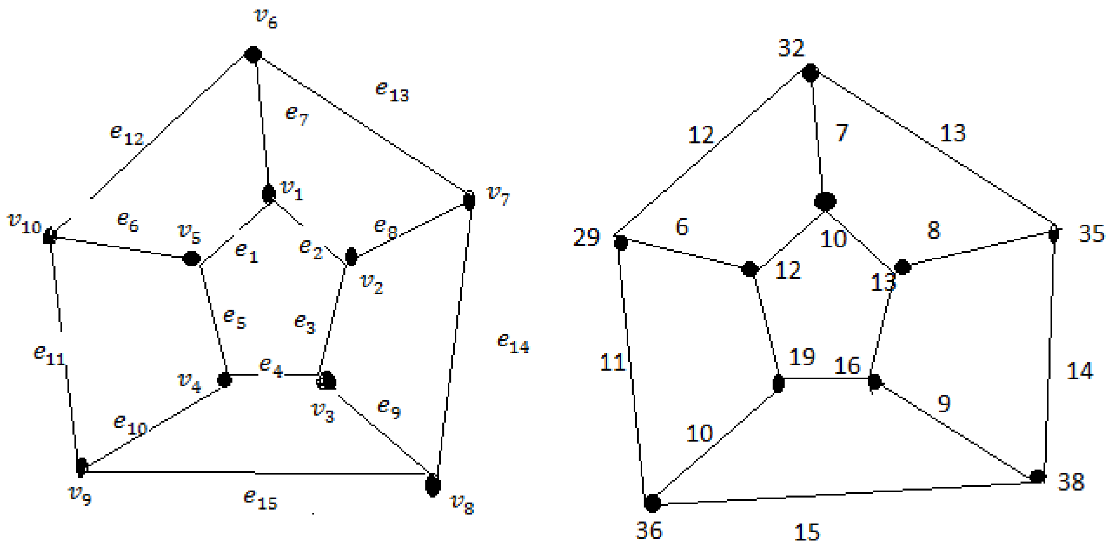


Definition:
The **prism graph** is the line **graph** of the complete bipartite **graph** . The **prism graph** is isomorphic with the cubical **graph**

Theorem 3.5: A Prism graph is Super Edge Anti Magic Graceful.

Proof:
Let G be a prism graph.
The edges of the graph are $\{e_1, e_2, e_3, \ldots, e_n\}$ and the vertices of a graph are $\{v_1, v_2, v_3, \ldots, v_n\}$.
The edges are label as, $f(e_i) = i$, $i = 1,2,3, \ldots, n$
and the vertex labels are $f(v_i) = \sum_{e \in N(u)} f(uv)$, $i = 1,2,3, \ldots, n$
Also $|f(u) + f(v) - f(uv)|$ is distinct.
It is clear that G admits $SEAMGL$.
Hence the prism graph is $SEAMG$.

Illustration:



REFERENCES

[1] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Elsevier, North Holland, New York, 1986.

[2] R. Frucht and F.Harary , On the corona two graphs,Aequationes Math,4(1970),322-325

[3] J.A. Gallian, A Dynamic Survey of Graph Labeling, Electron. J. Combin. (2017) #DS6.

[4] F. Harary, Graph Theory, Addison-Wesley, 1969.

[5] Y. Liang, X.Zhu Anti – magic labelling of cubic graphs. J.Graph Theory, 75(2014), 31-36.

[6] Y.Liang, T. Wong and X.Zhu,Anti-magic labelling of trees, Discrete Math.331(2014);9-14

[7] Y. Liang, X.Zhu Anti – magic labelling of Cartesian product of graphs, Theoret.Comput.Sci.477(2013), 1-5.

[8] J.A. MacDougall, M. Miller, Slamin and W.D. Wallis, Vertex-magic total labelings of graphs, Util. Math. 61 (2002) 3–21.9

[9] J.A. MacDougall, M. Miller and K.A. Sugeng, Super vertex-magic total labelings of graphs, Proceedings of the 15th Australian Workshop on Combinatorial Algorithms, (2004) 222–229.

[10] G. Marimuthu and M. Balakrishnan, E-super vertex magic labeling of graphs, Discrete Appl. Math. 160 (2012) 1766–1774.

[11] G. Marimuthu and M. Balakrishnan, Super edge magic graceful graphs, Inform. Sci., 287 (2014), 140-151.

[12] J. Sedla´cek, Problem 27, in Theory of Graphs and its Applications, Proc. Symposium ˇSmolenice, June (1963) 163–167.

[13] V. Swaminathan and P. Jeyanthi, Super vertex - magic labeling, Indian J. Pure Appl.Math. 34(6) (2003) 935–939.