More Results On Power 3 Mean Cordial Labeling of Graphs

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***Abstract*- In this Paper, we investigate Power 3 Mean Cordial labeling of graphs for Triangular Snake, Quadrilateral Snake .**

Mathematics subject classification:05C78

***Index Terms*- Graph, Power 3 Mean Graph, Triangular Snake, Quadrilateral Snake ,Power 3 Mean Cordial graphs**

I. INTRODUCTION

We begin with simple, connected, undirected graph $G = V(G), E(G)$ without loops or parallel edges. For a detailed survey of labeling, we refer to J.A. Gallian[4]. For all other standard terminology and notations we follow[3]. The concept of Mean Cordial labeling was introduced in [1].

Definition 1.1

A graph G with p vertices and q edges is called a power -3 mean graph,if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2, \dots \dots, q + 1$ in such a way that in each edge $e = uv$ is labelled with $f(e = uv) = \left\lceil \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rceil$ or $\left\lfloor \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rfloor$. Then , the edge labels are distinct. In this case f is called Power 3 Mean labelling of G.

Definition 1.2

Let f be a function from $V(G)$ to $\{0,1,2,3,4\}$. For each edge $u\,v$ of G, assign the label $f(e = uv) = \left\lceil \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rceil$ or $\left\lfloor \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rfloor$ f is called a Power 3 Mean Cordial labeling of G, if $|V_f(i) - V_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$, $i, j \in \{0,1,2,3,4\}$ where $V_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with $x (x = 0,1,2,3,4)$ respectively. A graph with a Power 3 Mean Cordial labeling is called **a Power 3 Mean Cordial graph** .

II. MAIN RESULTS

Theorem 2.1

Triangular snake T_n is a power 3 mean cordial graph

Proof

Let P_n be a path of $u_1, u_2, \dots u_n$

Let $V(T_n) = V(P_n) \cup \{u_i, 1 \le i \le n - 1\}$ and $E(T_n) = E(P_n) \cup \{u_iv_i, u_{i+1}, v_i\ 1 \le i \le n -1\}$

Case (i)

$n \equiv 0 \pmod 5$ Let $n = 5t$

Define $f(u_i) = 0$ $1 \le i \le t$

$f(u_{t+i}) = 1$ $1 \le i \le t$
$f(u_{2t+i}) = 2$ $1 \le i \le t$
$f(u_{3t+i}) = 3$ $1 \le i \le t$
$f(u_{4t+i}) = 4$ $1 \le i \le t$
$f(v_i) = 0$ $1 \le i \le t$
$f(v_{t+i}) = 1$ $1 \le i \le t$
$f(v_{2t+i}) = 2$ $1 \le i \le t$
$f(v_{3t+i}) = 3$ $1 \le i \le t$
$f(v_{4t+i}) = 4$ $1 \le i \le t$
$f(w_i) = 0$ $1 \le i \le t$
$f(w_{t+i}) = 1$ $1 \le i \le t$
$f(w_{2t+i}) = 2$ $1 \le i \le t$
$f(w_{3t+i}) = 3$ $1 \le i \le t$
$f(w_{4t+i}) = 4$ $1 \le i \le t$

Then

$v_f(0) = v_f(1) = v_f(2) = 2t$ $v_f(3) = v_f(4) = 2t + 1$

and

$e_f(0) = 2t$ $e_f(1) = e_f(2) = 2t + 2$

$e_f(4) = e_f(4) = 4t$

Obviously f is a power 3 mean cordial

Case (ii)

$n \equiv 1 \pmod 5$

Assign the labels to the vertices u_i and v_i $(1 \le i \le n - 1)$ as in case (i)

Then assign the label 2 to 0 the vertex $u_n v_n$

Here

$v_f(0) = v_f(1) = 2t$

$v_f(2) = 2t + 2$

$v_f(3) = v_f(4) = 2t + 1$ and

$e_f(0) = 2t$ $e_f(1) = 2t + 2 e_f(3) = 2t + 3 e_f(3) = e_f(4) = 4t + 2$

Case (iii)

$n \equiv 2$ (mod 5

Let $n = 5t + 2$

Assign the label to the vertices u_i and v_i $(1 \le i \le n - 1)$ as in case (i) and then the label 2 to the vertices u_{n-1}, v_{n-1} and the label 1 to the vertices $u_n v_n$

Here

$v_f(0) = 2t$ $\quad v_f(1) = v_f(2) = 2t + 2$

$v_f(3) = v_f(4) = 2t + 1$

and

$e_f(0) = 2t$ $e_f(1) = 2t + 3$

$e_f(2) = 4t + 1$

$e_f(3) = e_f(4) = 4t + 2$

obviously f is a power 3 mean cordial.

Case (iv)

$n \equiv 3$ (mod 5)

Let $n = 5t + 3$

Assign the labels to the vertices u_i and v_i $(1 \le i \le n - 1)$ as in case (i) and then v_i $(1 \le i \le n - 1)$ as in case (i) then the label 2 to the vertices to u_{n-2}, v_{n-2} and v_{n-1} and the label 0 to the vertices $u_n v_n$.

Here

$v_f(0) = v_f(1) = v_f(2) = 2t + 2$

$v_f(3) = v_f(4) = 2t + 1$ and

$e_f(0) = 2t + 1$ $e_f(1) = e_f(2) = 4t + 1$

$e_f(3) = e_f(4) = 4t + 2$

Obliviously, f is a power 3 mean cordial labelling.

Case (v)

$n \equiv 4$ (mod 5)

Let $n = 4t + 5$

Assign to the vertices u_i and v_i $(1 \le i \le n - 1)$ as in case (i) and the label to the vetices u_{n-3} v_{n-3} and the label 2 to the vertices u_{n-2} v_{n-2} and the label to the vertices u_{n-1}, v_{n-1} and the label 0 to the vertices $u_n v_n$

Here

$v_f(0) = v_f(1) = v_f(2) = v_f(3) = 2t + 2$

$v_f(4) = 2t + 1$

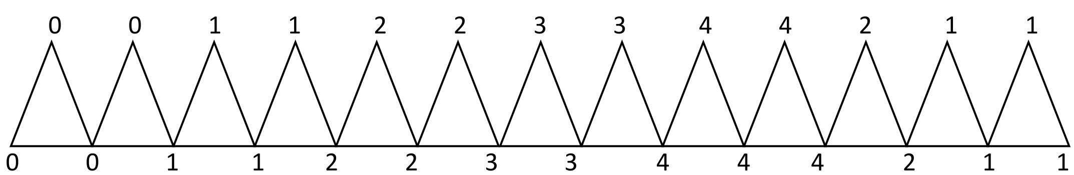
and

$e_f(0) = 2t + 1$ $e_f(1) = e_f(2) = e_f(3) = 4t + 1$

$e_f(4) = 4t + 2$

Example 2.2

Power 3 mean cordial labeling of triangular snake T_n as shown in figure



Theorem 2.3

Quadrilateral snake Q_n is a power 3 mean cordial labeling of graph

Proof:

Let P_n be the path $u_1, u_2, \ldots u_n$

Let $V(Q_n) = V(P_n) \cup \{v_i w_i, 1 \le i \le n\}$

and

$E(Q_n) = E(P_n) \cup \{u_i v_i, u_{i+1} w_i, v_i w_i, 1 \le i \le n - 1\}$

Case (i)

$n \equiv 0$ (mod5)

Let $n = 5t$

Define $f(u_i) = 0 \quad 1 \le i \le t$

$f(u_{t+i}) = 1 \quad 1 \le i \le t$

$f(u_{2t+i}) = 2 \quad 1 \le i \le t$

$f(u_{3t+i}) = 3 \quad 1 \le i \le t$

$f(u_{4t+i}) = 4 \quad 1 \le i \le t$

$f(v_i) = 0 \quad 1 \le i \le t$

$f(v_{t+i}) = 1 \quad 1 \le i \le t$

$f(v_{2t+i}) = 2 \quad 1 \le i \le t$

$f(v_{3t+i}) = 3 \quad 1 \le i \le t$

$f(v_{4t+i}) = 4 \quad 1 \le i \le t$

$f(w_i) = 0 \quad 1 \le i \le t$

$f(w_{t+i}) = 1 \quad 1 \le i \le t$

$f(w_{2t+i}) = 2 \quad 1 \le i \le t$

$f(w_{3t+i}) = 3 \quad 1 \le i \le t$

$f(w_{4t+i}) = 4 \qquad 1 \leq i \leq t$

Then

$v_f(0) = v_f(1) = v_f(2) = 2t + 2$

and $v_f(3) = v_f(4) = 2t + 3$ and

$e_f(0) = 2t + 2$

$e_f(1) = e_f(2) = 4t$

$e_f(3) = e_f(4) = 4t + 2$

Obliviously f is a power 3 mean cordial labeling

Case (ii)

$n \equiv 1 \pmod 5$

$n = 5t + 1$

Assign the labels to the vertices u_i, v_i and w_i $(1 \leq i \leq n - 1)$ as in case (i). Then assign the label 2 to the vertex u_n, v_n and w_n.

Here

$v_f(0) = v_f(1) = 2t + 2$

$v_f(2) = 4t + 1$ $v_f(3) = v_f(4) = 2t + 3$

and

$e_f(0) = 2t + 2 \qquad e_f(1) = 4t$

$e_f(2) = 4t + 2 \qquad e_f(3) = e_f(4) = 4t + 4$

Case (iii)

$n \equiv 2 \pmod 5$

Let $n = 5t + 2$

Assign the label to the vertices u_i, v_i and w_i $(1 \leq i \leq n - 1)$ as in case (i) and then the label 2 to the vertices u_{n-1}, v_{n-1}, w_{n-1} and the label 1 to the vertices u_n, v_n, w_n.

Here

$v_f(0) = 2t + 2 \qquad v_f(1) = v_f(2) = 4t + 1$

$v_f(3) = 2t + 3$ and

$e_f(0) = 2t + 2 \qquad e_f(1) = 4t + 2$

$e_f(2) = e_f(3) = e_f(4) = 4t + 2$

obviously f is a power 3 mean cordial labeling.

Case (iv)

$n \equiv 3 \pmod 5$

Let $n = 5t + 3$

Assign to the vertices u_i v_i and w_i $(1 \leq i \leq n - 1)$ as in case (i) and then the labels 2 to the vertex u_{n-2}, v_{n-2}, w_{n-2} and the label 0 to the vertices to the vertices u_{n-1}, v_{n-1}, w_{n-1} and the label to the vertices u_n, v_n, w_n.

Here

$v_f(0) = v_f(1) = v_f(2) = 4t + 1$

$v_f(3) = v_f(4) = 2t + 3$ and

$e_f(0) = 4t \qquad e_f(1) = e_f(2) = e_f(3) = e_f(4) = 4t + 4$

Case (v)

$n \equiv 4 \pmod 5$

Let $n = 5t + 4$

Assign the labels to the vertices u_i v_i and w_i $(1 \leq i \leq n - 1)$ as in case (i) and then the label to the vertices u_{n-3}, v_{n-3} and w_{n-3} and the label 2 to the vertices u_{n-2}, v_{n-2}, w_{n-2} and the label 1 to the vertices u_{n-1}, v_{n-1}, w_{n-1} and label 0 to the vertices $u_n v_n w_n$.

Here

$v_f(0) = v_f(1) = v_f(2) = v_f(3) = 4t + 1$

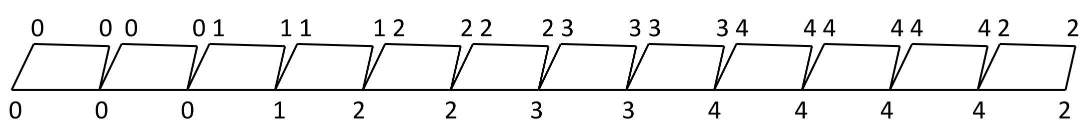
$v_f(4) = 2t + 1$

and $e_f(0) = 4t$ $e_f(1) = e_f(2) = e_f(3) = e_f(4) = 4t + 4$

clearly f is a power 3 mean cordial labeling.

Example 2.4

Power 3 mean cordial labeling of quadrilateral snake Q_n is as shown in figure.



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