Effect of Steady Shear Flow of Micro-Polar Fluid Through a Channel

Dr.Anis Fatima.H
Dept. of Mathematics Govt. College Kalaburgi

***Abstract* - Effect of steady shear flow of an incompressible micropolar fluid through a channel is investigated. The LAPLACE TRANSFORM TECHNIQE is employed to obtain the solution for velocity and micro-rotations subjected to the strong and weak limit of influence of surface. The field parameters, which do not appear in classical Newtonian fluid, are introduced and the effects of these on velocity and micro-rotations are presented graphically.**

INTRODUCTION

The classical Navier-Stocks theory does not adequately describe the flow properties of non-Newtonian fluids. A mathematical model for the description of micro-fluids which exhibit certain microscopic effects arising from local structure and micro motion of the fluid elements called micro fluids has been introduced by A.C.Eringen in 1966.

A subclass of microfluids called micro-polar fluid have the micro-rotational effect and micro-rotational inertia and these fluids can support couple stress and body couples only. This class of fluids possesses certain simplicity and elegance in their mathematical formulation which should attract to mathematicians.

The theory of micro-polar fluids yields two independent equations to describe the velocity and micro-rotation velocity fields. Thus, allowing the specification of both the velocity and micro-rotation velocity at the boundaries independently.

Mechanically, the micro-polar fluid theory differs from the Newtonian theory. In micro-polar fluid theory, the fluid flows with rotation but in Newtonian fluid, fluid flows linearly and we cannot find the rotation terms. Also, in recent experiments, we find that, fluids containing extremely small amount of polymeric additives indicate that the skin friction near a rigid body are considerably lower than the same fluids without additives. But the classical Navier-stokes theory is incapable of predicting these findings, since it contains no mechanism to explain these new physical phenomena.

Owing to its relative mathematically simplicity, the theory of micro-polar fluids has successfully applied to the analysis of wide variety of flow problems in fluid mechanics. Basic flow problem have investigated in the frame work of micro-polar fluid theory. Several investigators, prominent among them being A.C.Eringen, J.L.Bleustein, A.E.Green and T.Ariman etc.

A.C.Eringen[1] has given the theory of micro-polar fluids in 1967 and he has examined the study of f low of micro-polar fluids in a circular pipe under the influences of a constant pressure gradient and shown that the velocity profile is no longer parabolic and also in two dimensional flow through the channel, he shows that the flow speed is slower than the laminar flow for the same pressure gradient.

Recent experimental studies in blood indicate that under certain flow conditions blood flow may have strong deviation from Newtonian flow behavior. So that this theory has applied to blood flow, suspensions and lubrication. It is expected that the fluid microstructure may play an important role in flow of confined region and in non-symmetric flows.

Many authors have attempted to explain various anomalies associated with blood flow by proposing different models. Buglairello et al[2] showed in their experiment that there is a thin layer of plasma along the wall of the tube, which could give rise to a wall slip. Also, there are experimental results which show the dependence of effective viscosity of the blood on the tube radius and peripheral layer thickness.

Ariman et al[3] has reported that the theoretical results of polar fluid agreed with the experimental data of Buglairello and Sevilla.Chaturani et al[4] have studied, a two fluid model for blood flow through small diametertubes. Stokes[5] has studied couple stresses in fluids and Popel and Regirer [6] have studied acontinuum model of blood.

Two dimensional shear flow of linear micro polar fluids have been studied by Hudimoto,B and Tokvoka,T[7].T Ariman[8] has studied micro-polar and Dipolar fluids,P.Chaturani and P.N Kaloni[9] have studied two layered poiseuille flow model for blood flow through arteries of small diameter and arterioles.

Also this theory might be applied to electro-rheological fluids with many potential applications.

Here I proposed to analyses the fully developed steady shear flow of micro-polar fluid between two parallel plates. The polarity arising due to the presence of buoyant corpuscles in the fluid leads to give a theoretical insight to the study of b blood flow in large arteries.

The micro-rotational field depends on t the length parameter λ(size relation between the tube diameter and corpuscle) whereas the velocity field depends on both the parameters λ and K(ratio of vortex viscosity to shear viscosity).

Mathematical Formulation:

The basic fieldequation in vectorial forms is

$$\frac{\partial\rho}{\partial t} + \nabla.(\boldsymbol{\rho}v) = 0 \qquad (1)$$

$$\rho\frac{Dv}{Dt} = (\mu' + 2\mu)\nabla\nabla.v - (\mu+\mu_1)\nabla x\nabla xv + 2\,\mu_1\,\nabla x\sigma - \nabla_p + \boldsymbol{\rho}f \quad (2)$$

$$\rho J\frac{D\sigma}{Dt} = (\alpha+\beta+\gamma)\nabla\nabla.\sigma - \gamma\nabla x\nabla x\sigma + 2\,\mu_1\,\nabla xv - 4\mu_1\,\sigma + \boldsymbol{\rho}l \quad (3)$$

whereJ-is micro inertia

l-bodycoupleperunitmass

Boundary Conditions:

Velocity boundary conditions

The boundary conditions on velocity are the standard no-slip condition for viscous fluid. The no-slip condition implies that the velocity components vanish identically

i.e., U=V=W=0 at all the walls.

In other words,in case of viscous fluids, the velocity components of fluid at the boundary will be that of the boundary.

Formulation and Solution of Problem:

Consider an incompressible flow of micro polar fluid between two stationary parallel infinite plates at distance h apart. The flow throughout the region is maintained by a constant pressure gradient dp/dx =A.

The lower plate with origin is assumed to occupy theplane y=0 and upper plate y = h.

At sufficiently large distance from the entrance region the flow is supposed to be fully developed and at any point in the steady state velocity V and micro rotation σ have the components [u(y),0,0] and [0,0,Ω(y)] respectively. We assume the following,

i)Flow is incompressible

i.e; ∇.V=0

ii) The flow is axisymmetric such that

Vx = u(y), Vy = 0,Vz =0.

Sequently

σx = 0, σy = 0, σz= Ω(y).

c) No body forces and body couples are present.

i.e; f = l = 0

d) Steady flow-i.e; ∂/(∂t) = 0

using assumption (a) and (b),equation(1) is automatically satisfied.

Using the above assumption equation (2) and (3) becomes,

(μ+μ1) (d 2v)/dy2 + 2 μ(d Ω)/dx- dp/dx = 0 (4)

γ (d^2 Ω)/(dy^2) - 2 μ1 du/dy- 4 μ1Ω = 0 (5)

On introducing the following non-dimensional quantities

η=y/h U* = u/((Ah^2/2μ)) Ω *= Ω/((Ah/2μ))

K= (μ_1)/μ λ^(2)= h^2/((γ(μ+μ1)/4μμ1))

The equation (4) and(5) becomes

-1 + 1/2 (1+k) u_*^(,,) +k Ω_*^′=0 (6)

Ω_*^(′′′)=- 〖λ〗^2 〖Ω〗_*^′- 〖λ〗^(2)=0 (7)

where prime denotes differentiation with respect to η.

Byusing LAPLACE TRANSFORM TECHNIQUE, the above equations are solved under two limiting boundary conditions.No-spin boundary condition which corresponds to no-slip condition

Ω * (0) = Ω *(1) = 0

is based on the argument that the fluid solid interface with interaction is so strong that the microstructure does not rotate relative to plate and the next boundary condition is of weak limit of influence of surface on the rotation

Ω_*^′ (0)= Ω_*^′(1)= 0

And based on experimental results which emphasizes that the corpuscles rotate on the wall.

In view of the above two boundary conditions and no-slip condition for velocity,

u_*(0) = 〖 u〗_*(1) = 0

The solutions are

$\Omega^*s = \operatorname{cosech}\lambda \sinh\lambda\eta - \eta$ (8)

$\Omega^*w = \operatorname{cosech}\lambda/\lambda[\cosh\lambda\eta - \cosh(1-\eta)\lambda] - \eta$ (9)

$u_{(*s)} = 2k/(\lambda(1+k))\ [1-\eta+\eta \cosh\lambda - \cosh\eta] - \eta\ (1-\eta)$ (10)

$u_{(*w)} = 2k/(\lambda^{(2)}(1+k))\ [1 - \operatorname{cosech}\lambda \sinh\lambda\eta + \coth\lambda \sinh\lambda\eta - \cosh\lambda\eta] - \eta(1-\eta)$ (11)

Subscripts s and w denotes the values at strong and weak limit of interaction respectively.

Fig-1:Micro rotational field for strong limit

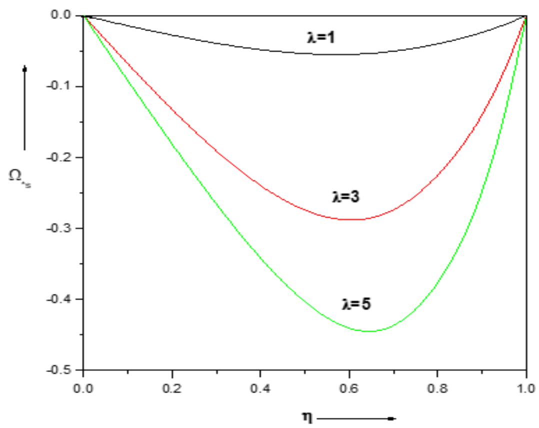


Fig-3:Field For Strong Limit K=0.2

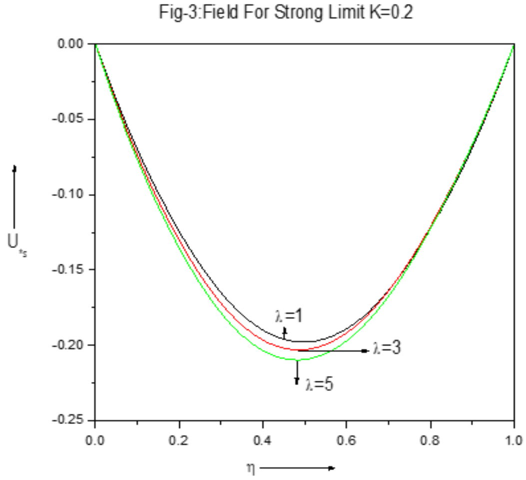


Fig-5:Velocity Field For weak Limit K=0.2

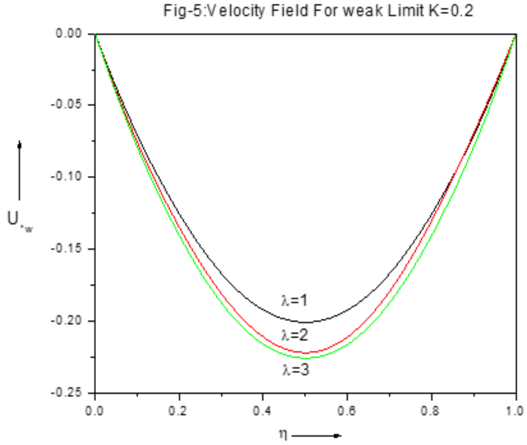


Fig-2: Micro rotational field for WEAK limit

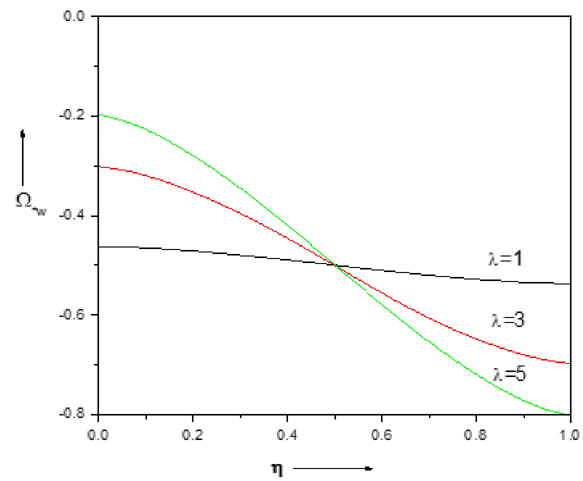


Fig-4:Velocity Field For Strong Limit K=1.0

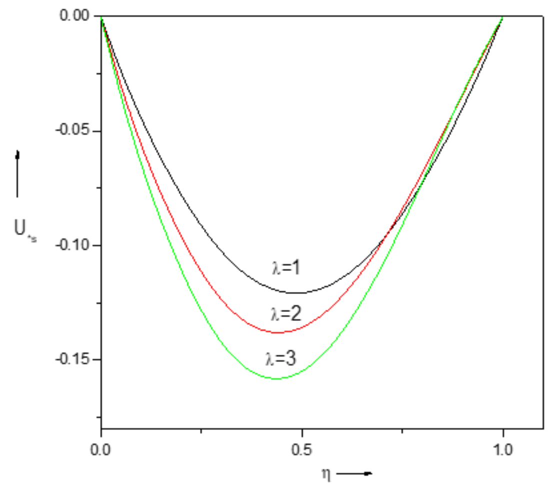
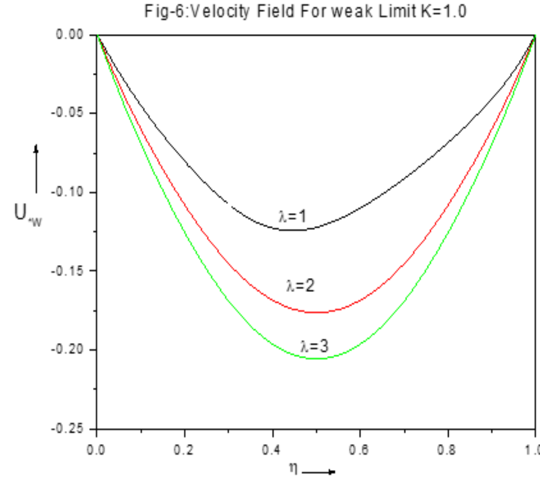


Fig-6:Velocity Field For weak Limit K=1.0



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