Alfven waves in gravitationally confined magnetized quantum plasma

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***Abstract*—Our research centers on exploring Alfven waves within a fermionic spin-1/2 multispecies quantum plasma, confined by magnetic and gravitational forces. Through the integration of the fundamental equations of Hall magnetohydrodynamics and the inclusion of quantum corrections, we derive a series of modified Zakharov-like equations uniquely crafted to characterize Alfven waves in this particular system. Our objective is to comprehensively investigate and comprehend the features of these Alfven waves. As part of our investigation, we establish the dispersion relation, offering significant insights into the wave properties and behavior exhibited within the plasma under examination.**

***Index Terms*— Alfven wave, Quantum plasma, QHD model, Zakharov equation**

I. INTRODUCTION

In recent years, Alfven waves have garnered significant attention and have become the focus of extensive research. These waves are characterized by low-frequency oscillations of ions and the magnetic field within a plasma, with frequencies much lower than the ion gyrofrequency. The inertia of ion mass density contributes to the wave dynamics, while the restoring force is provided by magnetic field line tension. Notably, Alfven waves propagate parallel to the direction of the magnetic field, while ion motion and magnetic field perturbations occur perpendicular to the direction of propagation.
The substantial interest in Alfven waves arises from their critical roles in energy transport and heating processes in laboratory and astrophysical plasmas. These waves are known to contribute to plasma heating and have been thoroughly investigated both theoretically and experimentally. In fusion plasma devices, "Alfven wave heating" has been explored as an additional method to enhance plasma heating. Additionally, Alfven waves have been proposed as a model for understanding the heating mechanisms observed in the coronae of the Sun and other stars. Research on Alfven waves has thus provided valuable insights into energy transfer and heating phenomena in various plasma systems.
Alfven waves and magnetoacoustic waves are fundamental in facilitating the transfer of magnetic energy within solar and stellar winds. They also contribute to the pulsations observed in Earth's magnetosphere and serve as scattering mechanisms for accelerating cosmic rays in astrophysical shock waves. The applications of these waves extend beyond these examples, finding relevance in various fields such as laboratory experiments, space physics, and astrophysics. Extensive literature in these areas explores the diverse range of applications and phenomena associated with Alfven waves and magnetoacoustic waves.
In recent years, Alfven waves, being one of the fundamental low-frequency modes of magnetized plasmas, have been extensively studied due to their crucial role in energy transportation and heating in magneto plasmas, including laboratory, space, and astrophysical plasmas. The ideal magnetohydrodynamic (MHD) model, first utilized for deriving Alfven waves, is a fluid model employed to describe magnetized plasmas, treating the entire plasma as a single fluid entity. Alternatively, the two-fluid model considers the plasma to consist of ion and electron fluids, with electrons assumed to be magnetized while ions are not. The inclusion of the Hall term introduces dispersion when the plasma comprises ion and electron fluids.
The field of quantum plasma has emerged as a vibrant area of research due to its potential applications in various practical fields. Quantum plasmas find applications in ultrasmall electronic devices, plasmas generated through laser-matter interactions, ultra-cold plasmas, and extremely dense astrophysical objects such as neutron stars and white dwarfs. To explore quantum effects in plasmas, the Quantum

Hydrodynamic (QHD) model is utilized, serving as the quantum counterpart to the classical fluid model. In this model, the momentum equation of charged particles is modified to incorporate Fermi pressure and the Bohm potential term. An ideal magnetohydrodynamic (MHD) model, incorporating statistical effects and quantum diffraction effects, has been developed, particularly relevant to dense astrophysical objects like the interiors of white dwarfs. Additionally, the QHD model of spin-1/2 charged particles, including defined physical quantities for all species containing particles with spin-up and spin-down, has been recently studied with all possible electrostatic and electromagnetic modes. It has been observed that dispersion resulting from diffraction effects gives rise to a new type of dispersive Alfvenic wave.

This research paper focuses on the identification of Alfven waves in a degenerate multispecies quantum plasma, consisting of fermionic particles with a spin of ½ and confined by magnetic and gravitational fields. To investigate this system, we utilize the fundamental equations derived from quantum Hall-magnetohydrodynamics (QMHD). Our objective is to comprehend the behavior and properties of Alfven waves within this specific quantum plasma environment. Through the modified Zakharov approach, we derive a set of nonlinear equations that govern the system under examination. Additionally, we obtain a modified linear dispersion relation tailored specifically for Alfven waves with finite amplitudes. This dispersion relation accounts for the effects of spin magnetization, enabling us to gain deeper insights into the wave behavior in the presence of spin-induced magnetization.

II. THEORY

Commencing from the momentum equations governing ions and electrons within a magnetized plasma, we can proceed to formulate the quantum magnetohydrodynamic (QMHD) model,

$$m_H \frac{du_H}{dt} = e(E + u_H \times B) \tag{1a}$$

$$m_{He} \frac{du_{He}}{dt} = e(E + u_{He} \times B) \tag{1b}$$

$$m_C \frac{du_C}{dt} = e(E + u_C \times B) \tag{1c}$$

$$m_O \frac{du_O}{dt} = e(E + u_O \times B) \tag{1d}$$

Owing to the large inertia of ions, their quantum effects can be ignored. Due to the slowly varying nature of the Alfven waves, the electron inertia is ignored in the electron equation of motion,

$$-eE - e(u_e \times B) - \frac{\nabla P_{Fe}}{n_e} + F_Q = 0 \tag{2}$$

The Fermi pressure for degenerate electrons can be expressed as $P_{Fe} = \frac{(3\pi^2)^{2/3} h^2}{5m_e} n_e^{5/3}$. The last term in eq. (2) is the quantum force on the electron which is given by

$$F_Q - \frac{h^2}{2m_e} \nabla \left[\frac{\nabla^2 \sqrt{n_e}}{\sqrt{n_e}}\right] + \left(\frac{\mu_B^2 B}{\varepsilon_{Fe}}\right) \nabla B \tag{3}$$

The ion continuity equation can be written as

$$\frac{\partial n_H}{\partial t} + \nabla.(n_H u_H) = 0 \tag{4a}$$

$$\frac{\partial n_{He}}{\partial t} + \nabla.(n_{He} u_{He}) = 0 \tag{4b}$$

$$\frac{\partial n_C}{\partial t} + \nabla.(n_C u_C) = 0 \tag{4c}$$

$$\frac{\partial n_O}{\partial t} + \nabla.(n_O u_O) = 0 \tag{4d}$$

The Ampere's law in the presence of spin magnetization is

$$\nabla \times B = \mu_0 (J_p + J_m) \tag{5}$$

Similarly, the Faraday's law is

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{6}$$

For the derivation of the basic equations of the QMHD model, we substitute from (5) into (2) and get

$$E = -\left(u_H + \frac{1}{en} \nabla \times M - \frac{1}{en\mu_0} \nabla \times B\right) \times B - \frac{\nabla P_{Fe}}{en} + \frac{F_Q}{e} \tag{7a}$$

$$E = -\left(u_{He} + \frac{1}{en} \nabla \times M - \frac{1}{en\mu_0} \nabla \times B\right) \times B - \frac{\nabla P_{Fe}}{en} + \frac{F_Q}{e} \tag{7b}$$

$$E = -\left(u_C + \frac{1}{en}\nabla\times M - \frac{1}{en\mu_0}\nabla\times B\right)\times B - \frac{\nabla P_{Fe}}{en} + \frac{F_Q}{e} \quad (7c)$$

$$E = -\left(u_O + \frac{1}{en}\nabla\times M - \frac{1}{en\mu_0}\nabla\times B\right)\times B - \frac{\nabla P_{Fe}}{en} + \frac{F_Q}{e} \quad (7d)$$

where, we have used the quasi-neutrality condition($n{\sim}n_i{\sim}n_o$). Eliminating E from (1) and (6) by using (7), we can write the normalized effective one fluid momentum equation as

$$\frac{du_H}{dt} = \frac{1}{n}(\nabla\times B)B - \frac{\beta}{2n^{2/3}}\nabla n^{2/3} + \frac{H_e^2}{2}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) + \frac{\varepsilon_0^2\beta}{2}B\nabla B - \frac{1}{n}(\nabla\times M)\times B \quad (8a)$$

$$\frac{du_{He}}{dt} = \frac{1}{n}(\nabla\times B)B - \frac{\beta}{2n^{2/3}}\nabla n^{2/3} + \frac{H_e^2}{2}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) + \frac{\varepsilon_0^2\beta}{2}B\nabla B - \frac{1}{n}(\nabla\times M)\times B \quad (8b)$$

$$\frac{du_C}{dt} = \frac{1}{n}(\nabla\times B)B - \frac{\beta}{2n^{2/3}}\nabla n^{2/3} + \frac{H_e^2}{2}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) + \frac{\varepsilon_0^2\beta}{2}B\nabla B - \frac{1}{n}(\nabla\times M)\times B \quad (8c)$$

$$\frac{du_O}{dt} = \frac{1}{n}(\nabla\times B)B - \frac{\beta}{2n^{2/3}}\nabla n^{2/3} + \frac{H_e^2}{2}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) + \frac{\varepsilon_0^2\beta}{2}B\nabla B - \frac{1}{n}(\nabla\times M)\times B \quad (8d)$$

Eliminating again E between (1) and (6), the normalized magnetic field induction equation (with Hall term) takes the form

$$\frac{\partial B}{\partial t} = \nabla\times(u_H\times B) - \nabla\times\frac{1}{n}\begin{bmatrix}(\nabla\times B)\times B\\ -(\nabla\times M)\times B\end{bmatrix} \quad (9a)$$

$$\frac{\partial B}{\partial t} = \nabla\times(u_{He}\times B) - \nabla\times\frac{1}{n}\begin{bmatrix}(\nabla\times B)\times B\\ -(\nabla\times M)\times B\end{bmatrix} \quad (9b)$$

$$\frac{\partial B}{\partial t} = \nabla\times(u_C\times B) - \nabla\times\frac{1}{n}\begin{bmatrix}(\nabla\times B)\times B\\ -(\nabla\times M)\times B\end{bmatrix} \quad (9c)$$

$$\frac{\partial B}{\partial t} = \nabla\times(u_O\times B) - \nabla\times\frac{1}{n}\begin{bmatrix}(\nabla\times B)\times B\\ -(\nabla\times M)\times B\end{bmatrix} \quad (9d)$$

Convenience to deal with circularly polarized Alfven waves. We can combine the x and y components of Eq. (9) by using $M_\pm = M_x + iM_y, B_\pm = B_x + iB_y$and $u_\pm = u_x + iu_y$ to get

$$\frac{\partial^2 B_\pm}{\partial t^2} + \frac{\partial}{\partial z}\left[u_z\frac{\partial B_\pm}{\partial t} + \frac{d}{dt}(u_zB_\pm)\right] - \frac{\partial}{\partial z}\left[\frac{1}{n}\frac{\partial B_\pm}{\partial z} - \frac{\varepsilon_0^2\beta}{n}\frac{\partial(nB_\pm)}{\partial z}\right] + H(1-\varepsilon_0^2\beta)\frac{\partial}{\partial z}\left[\frac{d}{dt}\left(\frac{1}{n}\frac{\partial B_\pm}{\partial z}\right)\right] = 0 \quad (10a)$$

$$\frac{\partial^2 B_\pm}{\partial t^2} + \frac{\partial}{\partial z}\left[u_z\frac{\partial B_\pm}{\partial t} + \frac{d}{dt}(u_zB_\pm)\right] - \frac{\partial}{\partial z}\left[\frac{1}{n}\frac{\partial B_\pm}{\partial z} - \frac{\varepsilon_0^2\beta}{n}\frac{\partial(nB_\pm)}{\partial z}\right] + He(1-\varepsilon_0^2\beta)\frac{\partial}{\partial z}\left[\frac{d}{dt}\left(\frac{1}{n}\frac{\partial B_\pm}{\partial z}\right)\right] = 0 \quad (10b)$$

$$\frac{\partial^2 B_\pm}{\partial t^2} + \frac{\partial}{\partial z}\left[u_z\frac{\partial B_\pm}{\partial t} + \frac{d}{dt}(u_zB_\pm)\right] - \frac{\partial}{\partial z}\left[\frac{1}{n}\frac{\partial B_\pm}{\partial z} - \frac{\varepsilon_0^2\beta}{n}\frac{\partial(nB_\pm)}{\partial z}\right] + C(1-\varepsilon_0^2\beta)\frac{\partial}{\partial z}\left[\frac{d}{dt}\left(\frac{1}{n}\frac{\partial B_\pm}{\partial z}\right)\right] = 0 \quad (10c)$$

$$\frac{\partial^2 B_\pm}{\partial t^2} + \frac{\partial}{\partial z}\left[u_z\frac{\partial B_\pm}{\partial t} + \frac{d}{dt}(u_zB_\pm)\right] - \frac{\partial}{\partial z}\left[\frac{1}{n}\frac{\partial B_\pm}{\partial z} - \frac{\varepsilon_0^2\beta}{n}\frac{\partial(nB_\pm)}{\partial z}\right] + O(1-\varepsilon_0^2\beta)\frac{\partial}{\partial z}\left[\frac{d}{dt}\left(\frac{1}{n}\frac{\partial B_\pm}{\partial z}\right)\right] = 0 \quad (10d)$$

The parallel component of Eq. (8) by using the above definitions can be written as

$$\frac{du_z}{dt} = -\frac{1}{2n}\frac{\partial {B_\pm}^2}{\partial z} + \frac{\varepsilon_0^2\beta}{2}\frac{\partial {B_\pm}^2}{\partial z} + \frac{\varepsilon_0^2\beta}{n}{B_\pm}^2\frac{\partial n}{\partial z} - \frac{\beta}{3n}\frac{\partial n}{\partial z} + \frac{H_e^2}{2}\frac{\partial}{\partial z}\left(\frac{\frac{\partial^2}{\partial z^2}\sqrt{n}}{\sqrt{n}}\right) \quad (11)$$

From the equation of continuity

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(nu_z) = 0 \quad (12)$$

Taking the time derivative of Eq. (12) and using Eq. (11), we have

$$\frac{\partial^2 n}{\partial t^2} - \frac{\beta}{3}\frac{\partial^2 n}{\partial z^2} - \frac{1}{2}(1-\varepsilon_0^2\beta n)\frac{\partial^2 {B_\pm}^2}{\partial z^2} - \frac{H_e^2}{2}\frac{\partial}{\partial z}\left[n\frac{\partial}{\partial z}\left(\frac{\frac{\partial^2}{\partial z^2}\sqrt{n}}{\sqrt{n}}\right)\right] \quad (13)$$

Equations (10), (11), and (13) are nonlinear equations in a Fermionic spin-1/2 quantum plasma. In the case when n and uz are constants, Eq. (10) becomes

$$\left[\frac{\partial^2}{\partial t^2} - V_A^2(1-\varepsilon_0^2\beta n)\frac{\partial^2}{\partial z^2} + i\frac{V_A^2}{\Omega_i}\begin{pmatrix}1\\ -\varepsilon_0^2\beta\end{pmatrix}\frac{\partial^3}{\partial z^3\,\partial t}\right]B_\pm = 0 \quad (14)$$

This gives a dispersion relation for wave solutions with arbitrary amplitudes which is given by

$$\omega^2 + \frac{\omega\omega_A^2}{\Omega_i} - \omega_A^2 = 0 \quad (15)$$

where, $\omega_A = k_A V_A \sqrt{(1-\varepsilon_0^2\beta)}$ is a spin modified Alfven wave frequency with corresponding wave number k_A. For a low regime of frequency, i.e., $\frac{\omega_A}{\Omega_i} <$ < 1, E q. (15) will become

$$\omega_{\pm} = \omega_A \left(1 \mp \frac{\omega_A}{2\Omega_i}\right) \quad (16)$$

III. MATH

In order to discuss the dispersion relation of parametrically in spin ½ multi species quantum plasma magnetically and gravitationally confined, we use the observed values of certain astrophysical scenarios, like that of dense plasmas (atmosphere of neutron stars, interior of massive white dwarfs). For these regions, the usual plasma parameters may be $n_0 \approx 10^{30}$–10^{35} m^{-3} and $B_0 \approx 10^5$–10^{10} T. [24–29] The existence of high magnetic fields (10^6G) in white dwarfs was also predicted by Blackett and Ginzburg [43,44] and has been verified with the help of Zeeman spectroscopy. A number of white dwarfs have also been observed having magnetic fields in the range of a few hundred Mega gauss. Since in degenerate quantum plasmas, the Fermi energy, quantum tunnelling, quantum statistical parameter, and magnetization energy are the functions of density as well as of magnetic field, thus, any change in n_0 and B_0 will consequently alter the wave dynamics. Using SI units, the quantum statistical parameter is $\beta \sim 10^{44}$ ($n_0^{5/3}$ / B_0^2) and the normalized Zeeman energy due to electron spin is $\mathcal{E}_0 \sim 10^{15}$ ($B_0/n^{2/3}$) 18 In the context of dense astrophysical objects, the quantum statistical parameter β takes on a finite value, and the normalized Zeeman energy resulting from the electron spin effect, denoted as Ɛ0, is typically on the order of unity or smaller. Utilizing quantum magnetohydrodynamics (QMHD), we derive a dispersion relation specifically for a spin-1/2 multispecies quantum plasma that is subjected to either magnetic or gravitational confinement. This dispersion relation provides valuable insights into the behaviour and properties of the plasma under such conditions.

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