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Transformation

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I. INTRODUCTION

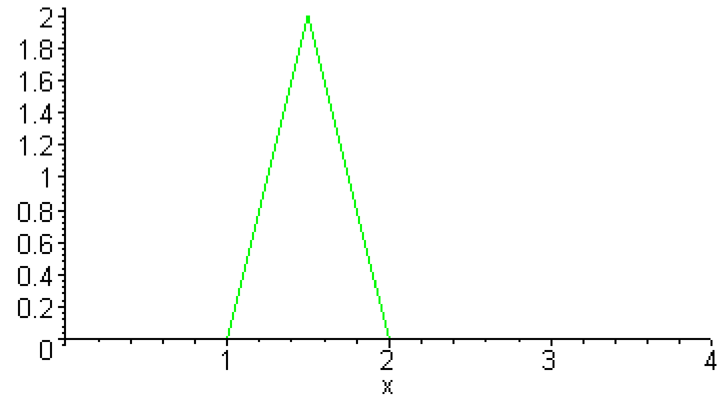
One of the most common and important tasks in computer graphics is to transform the coordinates (position, orientation and size) of either objects within the graphical scene or the camera that is viewing the scene. It is also frequently necessary to transform coordinates from one coordinate system to another, (e.eg. world coordinates to viewpoint coordinates to screen coordinates). All of these transformations can be effectively and sufficiently handled using some simple matrix representations, which we will see can be particularly useful for combining multiple transformations into a single composite transform matrix. There are 4 main types of transformations that one can perform in 2 dimensions:

- Translations
- Scaling
- Rotation
- Shearing

These basic transformations can also be combined to obtain more complex transformations. In order to make the representation of these complex transformations easier to understand and more efficient, we introduce the idea of homogeneous coordinates.

II. REPRESENTATION OF POINTS/OBJECTS

A point **p** in 2D is represented as a pair of numbers: **p**= (x, y) where x is the x-coordinate of the point **p** and y is the y-coordinate of **p** . 2D objects are often represented as a set of points (vertices), $\{p_1,p_2,...,p_n\}$, and an associated set of edges $\{e_1,e_2,...,e_m\}$. An edge is defined as a pair of points e = $\{p_i,p_j\}$. What are the points and edges of the triangle below?



We can also write points in vector/matrix notation as

$$p = \begin{bmatrix} x \\ y \end{bmatrix}$$

III. TRANSLATIONS

Since a translation is an affine transformation but not a transformation, homogeneous are normally used to represent the translation operator by a matrix and thus to make it linear. Thus we write the 3-dimensional vector **w** = (w_x, w_y, w_z) using 4 homogeneous coordinates as **w** = $(w_x, w_y, w_z, 1)$.

To translate an object by a vector **v**, each homogeneous vector **p** (written in homogeneous coordinates) would need to be multiplied by this **translation matrix**:

$$T_{\mathbf{v}} = \begin{bmatrix} 1 & 0 & 0 & v_x \\ 0 & 1 & 0 & v_y \\ 0 & 0 & 1 & v_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As shown below, the multiplication will give the expected result:

$$T_{\mathbf{v}}\mathbf{p} = \begin{bmatrix} 1 & 0 & 0 & v_x \\ 0 & 1 & 0 & v_y \\ 0 & 0 & 1 & v_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} p_x + v_x \\ p_y + v_y \\ p_z + v_z \\ 1 \end{bmatrix} = \mathbf{p} + \mathbf{v}$$

The inverse of a translation matrix can be obtained by reversing the direction of the vector:

$$T_{\mathbf{v}}^{-1} = T_{-\mathbf{v}}.$$

Similarly, the product of translation matrices is given by adding the vectors:

$$T_{\mathbf{u}}T_{\mathbf{v}} = T_{\mathbf{u}+\mathbf{v}}.$$

Because addition of vectors is commutative, multiplication of translation matrices is therefore also commutative (unlike multiplication of arbitrary matrices).

IV. SCALING

Scaling is a linear transformation that enlarges (increases) or shrinks (diminishes) objects by a scale factor that is the same in all directions. The result of uniform scaling is similar (in the geometric sense) to the original. A scale factor of 1 is normally allowed, so that congruent shapes are also classed as similar. Uniform scaling happens, for example, when enlarging or reducing a photograph, or when creating a scale model of a building, car, airplane, etc.

A scaling can be represented by a scaling matrix. To scale an object by a vector $v = (v_x, v_y, v_z)$, each point $p = (p_x, p_y, p_z)$ would need to be multiplied with this scaling matrix:

$$S_v = \begin{bmatrix} v_x & 0 & 0 \\ 0 & v_y & 0 \\ 0 & 0 & v_z \end{bmatrix}.$$

As shown below, the multiplication will give the expected result:

$$S_v p = \begin{bmatrix} v_x & 0 & 0 \\ 0 & v_y & 0 \\ 0 & 0 & v_z \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} v_x p_x \\ v_y p_y \\ v_z p_z \end{bmatrix}.$$

Such a scaling changes the diameter of an object by a factor between the scale factors, the area by a factor between the smallest and the largest product of two scale factors, and the volume by the product of all three.

The scaling is uniform if and only if the scaling factors are equal ($v_x = v_y = v_z$). If all except one of the scale factors are equal to 1, we have directional scaling.

In the case where $v_x = v_y = v_z = k$, scaling increases the area of any surface by a factor of k^2 and the volume of any solid object by a factor of k^3.

V. ROTATION

A rotation is a rigid body movement which, unlike a translation, keeps a point fixed. This definition applies to rotations within both two and three dimensions (in a plane and in space, respectively.)

All rigid body movements are rotations, translations, or combinations of the two.

A rotation is simply a progressive radial orientation to a common point. That common point lies within the axis of that motion. The axis is 90 degrees perpendicular to the plane of the motion. If the axis of the rotation lies external of the body in question then the body is said to orbit. There is no fundamental difference between a “rotation” and an “orbit” and or "spin". The key distinction is simply where the axis of the rotation lies, either within or outside of a body in question. This distinction can be demonstrated for both “rigid” and “non rigid” bodies.

VI. SHEARING

A shear is a transformation that distorts the shape of an object along either or both of the axies. Like scale and translate, a shear can be done along just one or along both of the coordinate axes. A shear along one axis (say, the x-axis) is performed in terms of the point's coordinate in the other axis (the y-axis). Thus a shear of 1 in the x-axis will cause the x-coodinate of the point ot distort by 1*(y-coordinate).

To shear in the ***x*** direction the equation is:

$x1 = x + ay$
$y1 = y$

Where $b = 0$

Where x1 and y1 are the new values, x and y are the original values, and a is the scaling factor in the x direction. The matrix is as follows.

$$\begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Shearing in the ***y*** direction is similar except the roles are reversed.

$x1 = x$
$y1 = y + bx$

Where $a = 0$.

Where $x1$ and $y1$ are the new values, x and y are the original values, and b is the scaling factor in the y direction. The matrix is as follows.

$$\begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

VII. SUMMARY

Transformaions are a fundamental part of computer graphics. Transformations are used to position objects, to shape objects, to change viewing positions, and even to change how something is viewed (e.g. the type of perspective that is used).

There are 4 main types of transformations that one can perform in 2 dimensions:

- translations
- scaling
- rotation
- shearing