AN OVERVIEW ON SETS

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Abstract- **A set is a well defined collection of distinct objects. The objects that make up a set (also known as the elements or members of a set) can be anything: numbers, people, letters of the alphabet, other sets, and so on. A set is a gathering together into a whole of definite, distinct objects of our perception or of our thought—which are called elements of the sets. Sets are conventionally denoted with capital letters. In this research paper we will discuss about the two forms of sets that is set builder form and roaster form. We will also discuss about different types of sets, their properties, identities and basic operations. We will also discuss about the vienn diagrams and De morgans' laws.**

Index Terms- **elements, properties, identities, operators ,De morgans' laws.**

I. INTRODUCTION

A set is a well defined collection of distinct objects. The objects that make up a set (also known as the elements or members of a set) can be anything: numbers, people, letters of the alphabet, other sets, and so on. A set is a gathering together into a whole of definite, distinct objects of our perception or of our thought—which are called elements of the sets. Sets are conventionally denoted with capital letters. A ***set*** can be defined as a collection of ***things*** that are brought together because they obey a certain ***rule***.

Curly brackets $\{\ldots\}$ are used to stand for the phrase 'the set of ...'. These braces can be used in various ways. A set is an unordered list of *elements*. (An *element* may also be referred to as a *member*). An element may be any mathematical entity.

We can denote a set directly by listing all of its elements between curly brackets, as in the following two examples:

- $\{7, 3, 15, 31\}$ is a set holding the four numbers 3, 7, 15, and 31.
- $\{a, c, b\}$ is the set containing 'a','b', and 'c'.

When it is desired to denote a set that contains elements from a regular sequence an ellipses notation may be employed, as shown in the next two examples.

- $\{1, 2, 3, \ldots, 100\}$ is the set of integers between 1 and 100 inclusive.
- $\{0, 1, 2, \ldots\}$ is the set of natural numbers.

II. FORMS OF SETS

In this we will discuss about the forms called set builder form and roasted form.

2.1 SET BUILDER FORM

2.1 ROSTER FORM

III. TYPES OF SETS

3.1Empty Set or Null Set

A set which does not contain any element is called an empty set, or the null set or the void set and it is denoted by ∅ and is read as phi. In roster form, ∅ is denoted by {}. An empty set is a finite set, since the number of elements in an empty set is finite, i.e., 0.

3.2 Singleton Set

A set which contains only one element is called a singleton set.

For example:• A = {x : x is neither prime nor composite}
It is a singleton set containing one element, i.e., 1.
• B = {x : x is a whole number, x < 1}
This set contains only one element 0 and is a singleton set.
• Let A = {x : x ∈ N and x^2 = 4}
Here A is a singleton set because there is only one element 2 whose square is 4.
• Let B = {x : x is a even prime number}
Here B is a singleton set because there is only one prime number which is even, i.e., 2.

3.3 Finite Set

A set which contains a definite number of elements is called a finite set. Empty set is also called a finite set.

For example: • The set of all colors in the rainbow.
• N = {x : x ∈ N, x < 7}
• P = {2, 3, 5, 7, 11, 13, 17, ... 97}

3.4 Infinite Set

The set whose elements cannot be listed, i.e., set containing never-ending elements is called an infinite set.

For example:• Set of all points in a plane
• A = {x : x ∈ N, x > 1}
• Set of all prime numbers
• B = {x : x ∈ W, x = 2n}

3.5 Cardinal Number of a Set

The number of distinct elements in a given set A is called the cardinal number of A. It is denoted by n(A).

For example: • A {x : x ∈ N, x < 5}
A = {1, 2, 3, 4}
Therefore, n(A) = 4
• B = set of letters in the word ALGEBRA
B = {A, L, G, E, B, R}
Therefore, n(B) = 6

3.6 Equivalent Sets

Two sets A and B are said to be equivalent if their cardinal number is same, i.e., n(A) = n(B). The symbol for denoting an equivalent set is '↔'.
For example :A = {1, 2, 3}Here n(A) = 3
B = {p, q, r} Here n(B) = 3
Therefore, A ↔ B

IV. OPERATIONS

4.1Unions

Two sets can be "added" together. The *union* of *A* and *B*, denoted by $A \cup B$, is the set of all things that are members of either *A* or *B*.

Examples:

- {1, 2} ∪ {1, 2} = {1, 2}.
- {1, 2} ∪ {2, 3} = {1, 2, 3}.
- {1, 2, 3} ∪ {3, 4, 5} = {1, 2, 3, 4, 5}

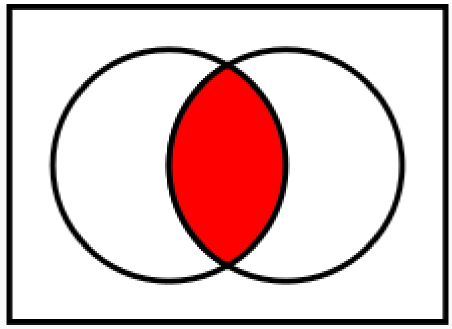
Some basic properties of unions:

$A \cup B = B \cup A$.

- $A \cup (B \cup C) = (A \cup B) \cup C$. The **union** of *A* and *B*, denoted$A \cup B$
-
- $A \subseteq (A \cup B)$.
- $A \cup A = A$.
- $A \cup \emptyset = A$.
- $A \subseteq B$ if and only if $A \cup B = B$.

4.2Intersections

A new set can also be constructed by determining which members two sets have "in common". The *intersection* of *A* and *B*, denoted by $A \cap B$, is the set of all things that are members of both *A* and *B*. If $A \cap B = \emptyset$, then *A* and *B* are said to be *disjoint*.



The **intersection** of *A* and *B*, denoted $A \cap B$.

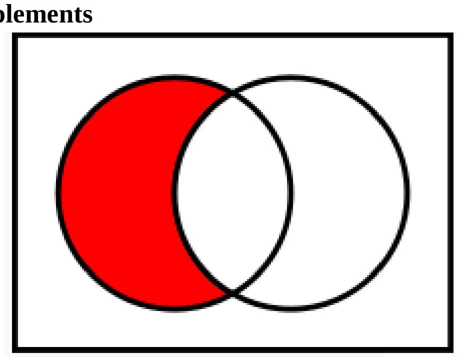
Examples:

- {1, 2} ∩ {1, 2} = {1, 2}.
- {1, 2} ∩ {2, 3} = {2}.

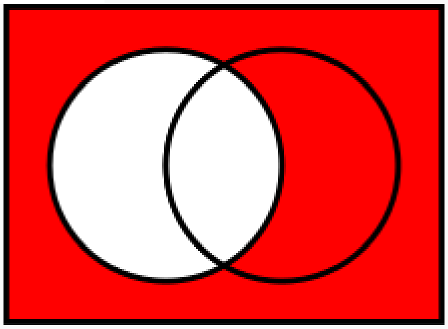
Some basic properties of intersections:

- $A \cap B = B \cap A$.
- $A \cap (B \cap C) = (A \cap B) \cap C$.
- $A \cap B \subseteq A$.
- $A \cap A = A$.
- $A \cap \emptyset = \emptyset$.
- $A \subseteq B$ if and only if $A \cap B = A$.

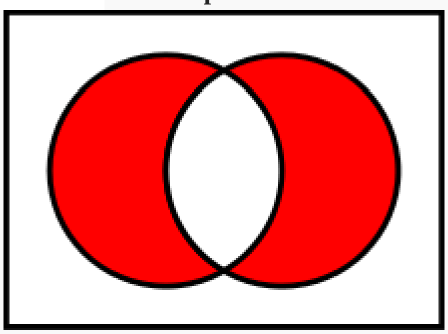
4.3Complements



The **relative complement** of B in A



The **complement** of A in U



Two sets can also be "subtracted". The **symmetric difference** of A and$B$

Main arti relative complement of B in A (also called the *set-theoretic difference* of A and B), denoted by$A \setminus B$ (or $A - B$), is the set of all elements that are members of A but not members of B. Note that it is valid to "subtract" members of a set that are not in the set, such as removing the element *green* from the set {1, 2, 3}; doing so has no effect.

In certain settings all sets under discussion are considered to be subsets of a given universal set U. In such cases, $U \setminus A$ is called the*absolute complement* or simply *complement* of A, and is denoted by A'.

Examples:

- $\{1, 2\} \setminus \{1, 2\} = \emptyset$.
- $\{1, 2, 3, 4\} \setminus \{1, 3\} = \{2, 4\}$.
- If U is the set of integers, E is the set of even integers, and O is the set of odd integers, then $U \setminus E = E' = O$.

Some basic properties of complements:

- $A \setminus B \neq B \setminus A$ for $A \neq B$.
- $A \cup A' = U$.
- $A \cap A' = \emptyset$.
- $(A')' = A$.
- $A \setminus A = \emptyset$.
- $U' = \emptyset$ and $\emptyset' = U$.
- $A \setminus B = A \cap B'$.

An extension of the complement is the symmetric difference, defined for sets A, B as

$$A\,\Delta\,B = (A \setminus B) \cup (B \setminus A).$$

For example, the symmetric difference of {7,8,9,10} and {9,10,11,12} is the set {7,8,11,12}.

4.4Cartesian product[edit]

A new set can be constructed by associating every element of one set with every element of another set. The *Cartesian product* of two sets A and B, denoted by $A \times B$ is the set of all ordered pairs (a, b) such that a is a member of A and b is a member of B.

Examples:

- {1, 2} × {red, white} = {(1, red), (1, white), (2, red), (2, white)}.
- {1, 2} × {red, white, green} = {(1, red), (1, white), (1, green), (2, red), (2, white), (2, green) }.
- {1, 2} × {1, 2} = {(1, 1), (1, 2), (2, 1), (2, 2)}.

Some basic properties of cartesian products:

- $A \times \emptyset = \emptyset$.
- $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
- $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

Let A and B be finite sets. Then

- $|A \times B| = |B \times A| = |A| \times |B|$.

4.4 Venn diagram

A **Venn diagram** or **set diagram** is a diagram that shows all possible logical relations between a finite collection of sets. Venn diagrams were conceived around 1880 by John Venn. They are used to teach elementary set theory, as well as illustrate simple set relationships in probability, logic, statistics, linguistics and computer science. A Venn diagram is constructed with a collection of simple closed curves drawn in a plane. According to Lewis, the "principle of these diagrams is that set be represented by regions in such relation to one another that all the possible logical relations of these classes can be indicated in the same diagram. That is, the diagram initially leaves room for any possible relation of the classes, and the actual or given relation, can then be specified by indicating that some particular region is null or is not-null". Venn diagrams normally comprise overlapping circles. The interior of the circle symbolically represents the elements of the set, while the exterior represents elements that are not members of the set. For instance, in a two-set Venn diagram, one circle may represent the group of all wooden objects, while another circle may represent the set of all tables. The overlapping area or *intersection* would then represent the set of all wooden tables. Shapes other than circles can be employed as shown below by Venn's own higher set diagrams. Venn diagrams do not generally contain information on the relative or absolute sizes (cardinality) of sets; i.e. they are schematic diagrams.

V. DE MORGAN'S LAWS

In propositional logic and boolean algebra, De Morgan's laws are a pair of transformation rules that are both valid rules of inference. The rules allow the expression of conjunctions and disjunctions purely in terms of each other via negation. The rules can be expressed in English as:

The negation of a conjunction is the disjunction of the negations. The negation of a disjunction is the conjunction of the negations or informally as:"***not (A and B)***" is the same as "***(not A) or (not B)***" and also, "***not (A or B)***" is the same as "***(not A) and (not B)***".