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Modeling and Control of Inherently Unstable SIMO System

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Abstract*—SIMO system is more efficient control system than SISO system. SIMO system can also be regarded as several SISO systems. In SIMO system only one input or manipulated variable available to control two or more than two controlled or output variables. The most common system used to demonstrate the performance of the inherently unstable SIMO system is INVERTED PENDULUM system. An inverted pendulum is an inherently unstable system comprising of a linear cart and freely oscillating bob (mass) such that the mass is made to stand upright in the vertical plane by means of controlling the motion of cart in the horizontal plane thereby controlling the angle at which the mass would stand finally in upright position. Many control strategies are available to control the angle of inverted pendulum system like PID, Pole Placement controller, LQR controller, FUZZY controller etc.

***Index Terms*—SIMO system, Inverted pendulum, PID controller, ITAE method**

I. INTRODUCTION

An Inverted Pendulum is a linearly unstable system and has its mass above the pivoted point, which is mounted on a cart which can be moved horizontally. The inverted pendulum system is an example commonly found in control system textbooks and research literature [1]. Its popularity derives in part from the fact that it is unstable without control, that is, the pendulum will simply fall over if the cart isn't moved to balance it. Additionally, the dynamics of the system are nonlinear. The objective of the control system is to balance the inverted pendulum by applying a force to the cart that the pendulum is attached to. The pendulum is stable while hanging downwards, but the inverted pendulum is unstable and needs to be balanced when made upright.

In this case, the system has one input - the force applied to the cart, and two outputs - position of the cart and the angle of the pendulum, making it as a SIMO system. There are mainly three ways of balancing an inverted pendulum i.e. (i) by applying a torque at the pivoted point (ii) by moving the cart horizontally [2] (iii) by oscillating the support rapidly up and down. Just like a broom-stick, an Inverted Pendulum is an inherently unstable system. Force must be properly applied to keep the system intact. To achieve this, proper control theory is required. The Inverted Pendulum is a non-linear time variant open loop system. So the standard linear techniques cannot model the non-linear dynamics of the system.

II. MODELLING

Several techniques are used by the engineers to model the physical control systems like by using method of Free body diagram, Euler Lagrange equation etc. Euler Lagrange method is more suitable to model inverted pendulum system [4].

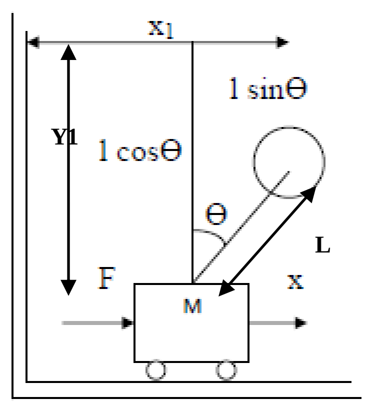

Figure 1

Figure 1 shows the model of cart control inverted pendulum system. The following section derives the transfer function with the Euler Lagrange method.

Parameter	Value
g- gravity	9.81 m/s^2
l- pole length	0.36 to 0.4 m - depending on the configuration
M- cart mass	2.4 kg
m- pole mass	0.23 kg
I- moment of interia of the pole	about 0.099 $kg{\cdot}m^2$ - depends on the configuration
b - cart friction coefficient	0.05 Ns/m
d - pendulum damping coefficient	although negligible,necessary in the model- 0.005 Nms/rad

Figure 2

As per shown in figure 2 includes all pendulum parameters. These are the parameters of the system provided by Feedback instrumentation [3].

The Lagrangian is defined as follows, where K is kinetic energy and P is potential energy [4],

$$L = K - P \tag{1}$$

Consider V1 as a resultant velocity,

$$v^2{}_1 = \dot{x}_1^2 + \dot{y}_1^2 \tag{2}$$

$$= \left(\dot{x} + l\cos\theta \times \dot{\theta}\right)^2 + \left(-l\sin\theta \times \dot{\theta}\right)^2$$

$$= \dot{x}^2 + 2\dot{x}l\cos\theta\dot{\theta} + l^2\dot{\theta}^2$$

Potential energy and kinetic energy of cart and pendulum are defined as follows,

$$K_{pend} = \frac{1}{2}m\left(\dot{x}^2 + 2\dot{x}l\cos\theta\dot{\theta} + l^2\dot{\theta}^2\right) + \frac{1}{2}i\dot{\theta}^2 \tag{3}$$

$$K_{cart} = \frac{1}{2}M\dot{x}^2 \tag{4}$$

$$P_{pen} = mgl\cos\theta \tag{5}$$

Potential energy of cart is zero initially. Lagrangian is defined as,

$$L = \left(K_{pen} + K_{cart}\right) - P_{pen}$$

$$L = \frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2 - mg\ell\cos\theta$$

$$L = \left[\frac{1}{2}\dot{x}^2\left(m+M\right) + \dot{x}m\ell\cos\theta + \frac{1}{2}m\ell^2 + \frac{1}{2}I\dot{\theta}^2\right] - mgl\cos\theta$$

The Euler-Lagrange's equation for the cart & system is given as[4],

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} + d\dot{\theta} = 0 \tag{6}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} + b\dot{x} = F \tag{7}$$

Partial derivatives are as follows,

$$\frac{\partial L}{\partial \dot{\theta}} = \dot{x}m\ell\cos\theta + m\ell^2\dot{\theta} + I\dot{\theta}$$

$$\frac{\partial L}{\partial \dot{x}} = \dot{x}\left(m+M\right) + m\ell\dot{\theta}\cos\theta$$

$$\frac{\partial L}{\partial \theta} = -\dot{x}m\ell\sin\theta + m\ell g$$

$$\frac{\partial L}{\partial x} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{d}{dt}\left(\dot{x}m\ell\cos\theta + m\ell^2\dot{\theta} + I\dot{\theta}\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \ddot{x}m\ell\cos\theta - \dot{x}m\ell\sin\theta\dot{\theta} + m\ell^2\ddot{\theta} + I\ddot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \ddot{x}\left(m+M\right) + m\ell\ddot{\theta}\cos\theta - m\ell\dot{\theta}\sin\theta$$

Now, If we put these derivatives in Euler-Lagrange's equation we gets following,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} + d\dot{\theta} = 0$$

$$\left(I + m\ell^2\right)\ddot{\theta} + m\ell\cos\theta\ddot{x} - mg\ell\sin\theta + d\dot{\theta} = 0 \tag{7}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} + b\dot{x} = F$$

$$\left(m+M\right)\ddot{x} + m\ell\ddot{\theta}\cos\theta - m\ell\dot{\theta}\sin\theta\dot{\theta}^2 + b\dot{x} = F \tag{8}$$

Equation (7) and (8) shows the dynamics of the system.

III. TRANSFER FUNCTION

Before derive the transfer function, first we have to linearize the model equations. So, we can substitute nonlinear functions (cosine and sine) to their linear equivalents [3]. For small angle variation around equilibrium position $\theta = \pi$ we can assume that $\cos\theta \cong -1$, $\sin\theta \cong -\theta$ and $\dot{\theta}^2 \cong 0$ .Thus motion equation (7 and 8) take the form,

$$(I + m\ell^2)\ddot{\theta} - mg\ell\theta - m\ell\ddot{x} + d\dot{\theta} = 0 \quad (9)$$

$$(m + M)\ddot{x} + b\dot{x} - m\ell\ddot{\theta} = F \quad (10)$$

Now by taking Laplace Transform of equations (9 and 10) we can obtain following Transfer Function,

$$\frac{\theta(s)}{F(s)} = \frac{\frac{m\ell}{q}s}{s^3 - \frac{b(I + m\ell^2)}{q}s^2 - \frac{(m+M)mg\ell}{q}s - \frac{bmg\ell}{q}}$$

$$\text{where}, q = [(m+M)(I+m\ell^2) - (m\ell)^2] \quad (11)$$

And,

$$\frac{X(s)}{F(s)} = \frac{(I + m\ell^2)s^2 - mg\ell}{((m+M)(I + M\ell^2) - m\ell^2)s^4 + [b(I + m\ell^2 I)]s^3 - [(m+M)mg\ell]s^2 - mg\ell bs} \quad (12)$$

Now, if frictional force is neglected and canceling common poles at origin, then equation 11 can be written as,

$$\frac{\theta(s)}{F(s)} = \frac{\frac{ml}{q}}{s^2 - \frac{(m+M)mgl}{q}}$$

$$= \frac{b_1}{s^2 - a^2} \quad (13)$$

Where,

$a = 2.5453$

$b_1 = 0.2511$

If we neglect frictional force and neglect pendulum mass (m) with compare to cart mass (M),we can write equation 12 as follows,

$$\frac{X(s)}{F(s)} \cong \frac{\frac{1}{(M+m)}}{s^2} = \frac{b_2}{s^2} \quad (14)$$

Where, $b_2 = 0.3802$

IV. CONTROLLER DESIGN

Now, we obtain transfer function of Inverted Pendulum system but, without controller we can not keep pendulum in upright position. Without controller pendulum and cart response with time is as shown in figure

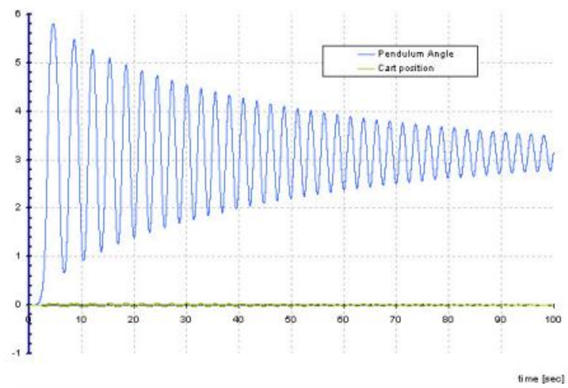

Figure 1

So, now to control the system we have to design a controller. Among all controller strategies available this time PID control strategy is most popular. In PID P stands for proportional I stands for Integral and D stands for Derivative actions. Transfer function for PID controller is

$$C(s) = \frac{Ds^2 + Ps + I}{s} \quad (15)$$

There are several methods engineers use to obtain PID parameters. We designed PID controller using Root Locus method. MATLAB provides SISO Tool to design PID compensator using root locus method[7]. The root locus of the uncompensated system is as shown in figure 4.

Now, to control this system we have to add poles and zeroes such that we can get desire response
Here, we tried to design system which has following characteristics,

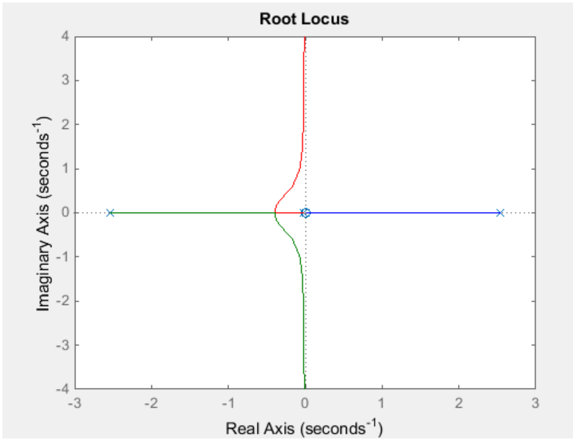

Figure 2

Overshoot=10%

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$T_s = \frac{4}{\xi\omega_n}$$

Ts=1 sec

$$\zeta = 0.591328$$

$$\omega_n = 6.76$$

$$\text{poles} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

So, dominant poles are

$$\text{poles} = -4 \pm j5.4553$$

If we need desired characteristic we have to add poles and zeroes accordingly to root locus. Our final result is as show in Figure 5.PID controller TF is,

$$C(s) = \frac{s^2+7s+10}{s}$$

For best performance and low peak overshoot as shown PID parameters and Controller gain are,

Kc=100 Kp=7 Ki=10 Kd=1

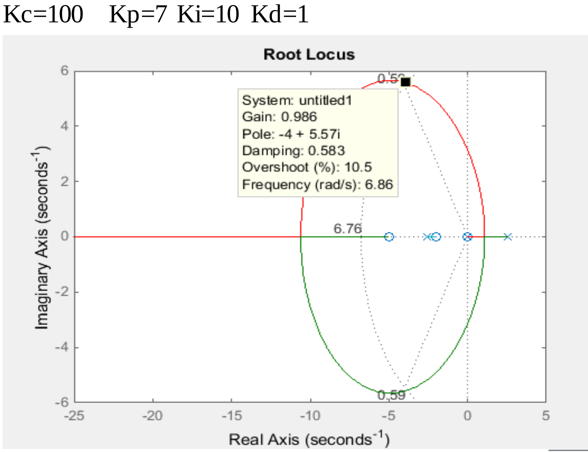

Figure 3

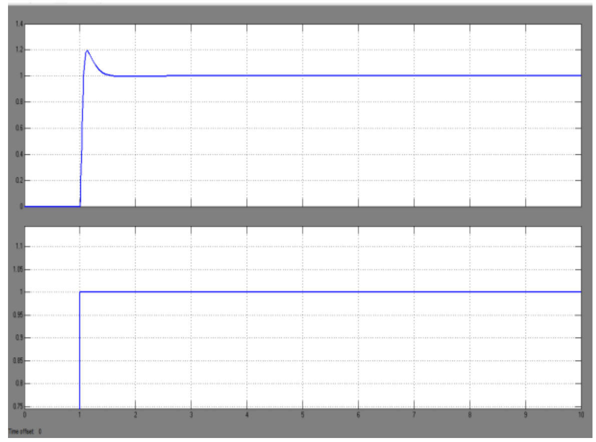

Figure 6

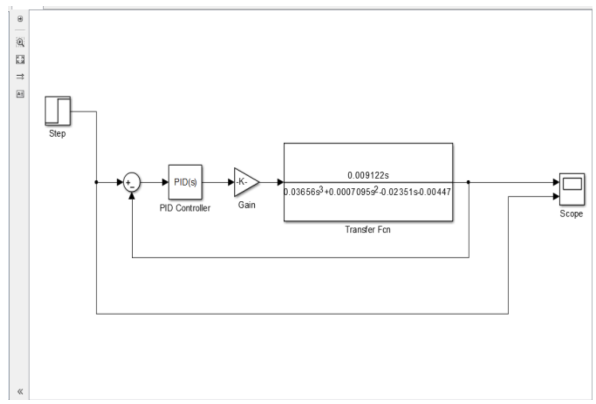

Figure 7

Figure6 and Figure7 shows the step response of the system in MATLAB Simulink program. This shows that pendulum angle is correctly control by this PID controller.

V. CONCLUSION

Thus by performing this experiment we could realize the physical system i.e. the instrument in mathematical model and perform simulation on it. We thus derived the transfer function of the system and design control strategy to control θ and to some extent cart position. But, by designing more precise controller we can control cart position to further level. Also by doing this project we defined the future course of action on the similar kind of SIMO systems. Finally this is one way we can control the Inverted pendulum system there are many other way by which we can do optimum control of SIMO system.

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