The effects of cross-diffusion and stratification in unsteady currents on vertical plate with power law variation

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***Abstract*—In this paper, the influence of viscous dissipation effects, Dufour and Soret effects, stratification effects along with power law variation of both temperature and concentration effects on unsteady natural convective flow of an electrically conducting fluid in a Brinkman porous medium has been analyzed along with vertical plate. By using the non-dimensional transformations in to the governing differential equations, the system expressed into a set of non-linear coupled differential equations along with the boundary conditions. The unconditionally stable implicit finite difference scheme of Crank-Nicolson type has been used to solve the reduced unsteady nonlinear boundary value problem. The Numerical results for velocity, temperature and concentration profiles are analyzed in detail and depicted graphically for various physical parameter values after comparison with available results in literature with good agreement.**

***Index Terms*—Brinkman porous medium, Crank-Nicolson method, Double stratification, Dufour and Soret effects, Unsteady, Variable temperature and concentration.**

I. INTRODUCTION

In a porous medium, the natural convective transport has wide significance in heat and mass transfer problems due to their increasing applications in different areas such as scientific, biological, various engineering and industrial technology areas. In the monographs by Ingham and Pop [1], a detailed discussion of these applications are available. Much of the work related to porous media transport phenomena has been presented in the handbook of porous media by Vafai [2]. The solutal and thermal stratification of fluid arises due to concentration differences, temperature variations and in the form of the existence of different type fluids. Similarly, it is favorable in particular engineering areas such as thermal energy storage systems, heat rejection into the environment and heat transfer from thermal sources.

With this inspiration, most of the researchers have begun their studies in the area of doubly stratified convective flows. By Introducing the thermal stratification in the energy equation provides more realistic scenario than the conventional models as it produces a coupling effect on both the temperature and velocity fields. Nakayama and Koyama [3] discussed the thermal stratification effect, whereas Srinivasacharya and Ramreddy [4] and Murthy *et al.* [5] analysed the double stratification effects on natural convection flow. An influence of electrophoresis and chemical reaction on unsteady convection doubly stratified flow past a vertical plate in the presence of a heat source/sink is presented by Ganesan and Suganthi [6]. Recently Srinivasacharya and Upendar [7] analyzed and presented, mixed convection in MHD doubly stratified micropolar fluid.

The mass fluxes caused by temperature gradients and energy flux caused by a concentration gradient are popularly known as thermal-diffusion (Soret) and diffusion-thermo (Dufour) effects respectively. A broad investigation of these effects is considered theoretically and experimentally in both gas and fluids. These are found to be very important in different areas like hydrology, petrology and Geosciences (eg., see Eckert and Drake [8] and references cited therein). Dursunkaya and Worek [9] studied about cross-diffusion effects in unsteady and steady free convection, whereas Srinivasacharya and RamReddy [10] discussed the steady convective flows in Newtonain and non-Newtoanian fluids with cross diffusion effects. Nield and Kuznetsov [11] investigated and discussed the cross-diffusion in nanofluids, with the aim of making a detailed comparison with regular cross diffusion effects. The cross-diffusion impacts are peculiar to nanofluids, and at the same time investigating the interaction between these effects when the base fluid of the nanofluid is itself a binary fluid like salty water. Recently, Loganathan *et al.* [12] observed that the local heat transfer rate enhances with an enhancement in Dufour and Soret numbers for both air and water by analyzing the problem of the cross-diffusion effects on the unsteady natural convection and Srinivasacharya and Surender [13] studied and presented the effect of double stratification along with Dufour and Soret type diffusivities on mixed convection boundary layer flow of a nanofluid past a vertical plate in a porous medium.

Studying the effects of cross-diffusion along with double stratification in porous medium on unsteady natural convection fluid past vertical plate with variable surface temperature and concentration is one of the interesting domain. Recently, Javaherdeh e*t al.* [14] worked on Natural convection heat and mass transfer in MHD fluid flow past a moving vertical plate with variable surface temperature and variable surface concentration in a porous medium. Whereas Ekambavanan, and Ganesan [15] studied about Finite difference solution of an unsteady natural convection boundary layer flow over an inclined plate with only variable surface temperature. And Ibrahim and Palani [16] investigated the effects of magnetohydrodynamic flow past a vertical plate with only variable surface temperature. Shanker and Kishan [17] presented the effect of mass transfer on the MHD flow past in an impulsively started vertical plate along with variable temperature but constant heat flux.

The aim of the present study is to investigate the effects of cross-diffusion in the Brinkman porous medium on unsteady natural convection doubly stratified fluid past vertical plate with variable surface temperature and concentration using an implicit finite difference method of Crank-Nicolson type. The influence of velocity, temperature and concentration are analyzed and exhibited graphically for variations in governing parameters in detail.

II. MATHEMATICAL FORMULATION

Consider a problem of two-dimensional, unsteady, laminar free convection flow past a vertical plate of a viscous incompressible doubly stratified fluid saturated porous medium in an electrically conducting fluid with variable surface temperature and concentration under the influence of magnetic field is formulated mathematically in this section by taking into account the effect of viscous dissipation. The coordinate system is chosen to represent the *x*-axis along the vertical plate and the *y*-axis as upward normal to the plate. The fluid and the plate are assumed to be at the constant temperature and constant concentration initially at $t'=0$, The ambient medium is assumed to be vertically linearly stratified with respect to both temperature and concentration in the form $T_\infty(x) = T_{\infty,0}+Ax$ and $C_\infty(x) = C_{\infty,0}+Bx$ respectively. Then, under these assumptions, the governing boundary layer equations of mass, momentum, energy and species concentration for free convection flows with Boussinesq's and Brinkman porous medium approximation are as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{1}{\varepsilon}\frac{\partial u}{\partial t'}+\frac{u}{\varepsilon^2}\frac{\partial u}{\partial x}+\frac{v}{\varepsilon^2}\frac{\partial u}{\partial y}=\frac{\upsilon}{\varepsilon}\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{k}u-\frac{\sigma B_0^{\,2}}{\rho}\frac{u}{\varepsilon}$$
$$+g[\beta_T(T'-T_\infty(x))+\beta_C(C'-C_\infty(x))] \tag{2}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2}$$
$$+\frac{\mu}{\rho Cp}\left(\frac{\partial u}{\partial y}\right)^2+\frac{DK_T}{CsCp}\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}+\frac{DK_T}{T_m}\frac{\partial^2 T'}{\partial y^2} \tag{4}$$

Where u and v are Darcy velocity components along the x and y directions respectively, ρ is the density, g is the acceleration due to gravity, C_p is the specific heat, μ is the coefficient of viscosity, σ is the electrical conductivity, k is the permeability, ε is the porosity, T' is the temperature, C' is the concentration, β_T and β_C are the coefficients of thermal and solutal expansions, α is the thermal diffusivity, D is the mass diffusivity and T_m is the mean fluid temperature.

The boundary conditions are

$$t'\le 0: u(x,y,t')=0,\ v(x,y,t')=0,$$
$$T'(x,y,t')=T_\infty(x),\ C'(x,y,t')=C_\infty(x)\,.$$

$$t' > 0 : u(x,0,t') = 0,\ \ v(x,0,t') = 0,$$
$$T'(x,0,t') = T_{\infty,0} + (T_w - T_{\infty,0})(ax)^m,$$
$$C'(x,0,t') = C_{\infty,0} + (C_w - C_{\infty,0})(ax)^n.$$
$$t' > 0 : u(0,y,t') = 0,\ \ v(0,y,t') = 0,$$
$$T'(0,y,t') = T_{\infty,0},\quad C'(0,y,t') = C_{\infty,0}.$$
$$t' > 0 : u(x,\infty,t') \to 0,\ T'(x,\infty,t') \to T_\infty(x),$$
$$C'(x,\infty,t') \to C_\infty(x). \tag{5}$$

Using the following non-dimensional variables

$$X = \frac{x}{L},\ Y = \frac{y}{L}Gr^{1/4},$$
$$U = \frac{uL}{\upsilon}Gr^{-1/2},\ V = \frac{vL}{\upsilon}Gr^{-1/4},$$
$$t = \frac{t'\upsilon}{L^2}Gr^{1/2},\ T = \frac{T' - T_\infty(x)}{T_w - T_{\infty,0}},$$
$$C = \frac{C' - C_\infty(x)}{C_w - C_{\infty,0}},\ a = \frac{1}{L}. \tag{6}$$

in to the (1) to (4), we obtain the following system of non-dimensional partial differential equations

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{7}$$

$$\frac{1}{\varepsilon}\frac{\partial U}{\partial t} + \frac{U}{\varepsilon^2}\frac{\partial U}{\partial X} + \frac{V}{\varepsilon^2}\frac{\partial U}{\partial Y} + \frac{1}{DaGr^{1/2}}U + \frac{1}{\varepsilon}\frac{M}{Gr^{1/2}}U = \frac{1}{\varepsilon}\frac{\partial^2 U}{\partial Y^2} + T + NC \tag{8}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} + \varepsilon_1 U = \frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2} + Ec\left(\frac{\partial U}{\partial Y}\right)^2 + Df\frac{\partial^2 C}{\partial Y^2} \tag{9}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} + \varepsilon_2 U = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} + Sr\frac{\partial^2 T}{\partial Y^2} \tag{10}$$

along with the corresponding initial and boundary conditions in a non-dimensional form are

$$t \le 0 : U(X,Y,t) = 0,\ V(X,Y,t) = 0,$$
$$T(X,Y,t) = 0,\ C(X,Y,t) = 0.$$
$$t > 0 : U(X,0,t) = 0, V(X,0,t) = 0,$$
$$T(X,0,t) = X^m - \varepsilon_1 X, C(X,0,t) = X^n - \varepsilon_2 X$$
$$t > 0 : U(0,Y,t) = 0, V(0,Y,t) = 0,$$
$$T(0,Y,t) = 0, C(0,Y,t) = 0$$
$$t > 0 : U(X,\infty,t) \to 0, T(X,\infty,t) \to 0, C(X,\infty,t) \to 0 \tag{11}$$

where $Gr = g\beta_T L^3 (T_w - T_{\infty,0})/\upsilon^2$ is the Grashof number, $N = \beta_C(C_w - C_{\infty,0})/(\beta_T(T_w - T_{\infty,0}))$ is the buoyancy ratio, $Da = k\upsilon/(\mu L^2)$ is the Darcy number, $M = \sigma B_0^2 L^2/(\rho\upsilon)$ is the magnetic parameter, $EC = n_0^2/(C_p(T_w - T_{\infty,0}))$ is the Eckert number, $n_0 = (\mu/(\rho L))Gr^{1/2}$, $\alpha = k/(\rho C_P)$ is the thermal diffusivity, $Pr = \upsilon/\alpha$ and $Sc = \upsilon/D$ are the Prandtl and Schmidt numbers, $Sr = DK_T(T_w - T_{\infty,0})/(T_m\upsilon(C_w - C_{\infty,0}))$ and $Df = DK_T(C_w - C_{\infty,0})/(C_S C_p\upsilon(T_w - T_{\infty,0}))$ are the Soret and Dufour numbers, $\varepsilon_1 = AL/(T_w - T_{\infty,0})$ and $\varepsilon_2 = BL/(C_w - C_{\infty,0})$ are the thermal and solutal stratification parameters.

The non-dimensional forms of physical parameters of interest local skin friction, Nusselt number and Sherwood number are obtained as

$$\tau_X = Gr^{3/4}\left(\frac{\partial U}{\partial Y}\right)_{Y=0},\ Nu_X = -\frac{X\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}},$$
$$Sh_X = -\frac{X\left(\frac{\partial C}{\partial Y}\right)_{Y=0}}{C_{Y=0}} \tag{12}$$

The non-dimensional forms of average skin friction, Nusselt number and Sherwood number are obtained as

$$\bar{\tau} = Gr^{3/4}\int_0^1\left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX,\ \overline{Nu} = -\int_0^1 \frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}} dX,$$
$$\overline{Sh} = -\int_0^1 \frac{\left(\frac{\partial C}{\partial Y}\right)_{Y=0}}{C_{Y=0}} dX \tag{13}$$

III. RESULTS AND DISCUSSION

The reduced unsteady, nonlinear and coupled (7) to (10) with conditions (11) are solved numerically by using the implicit finite difference method known as Crank-Nicolson type scheme (for more details on this method, one can refer the work of Loganathan et al. [18] along with citations therein). In order to verify the accuracy of the present results, the velocity profiles of the present problem are compared with the existing solution of Gebhart and Pera [19] in the absence of doubly stratified porous medium with $Pr=0.71$ $Sc=0.94$, $N=1.0$, $Ec=0.0$, $Sr=0.0$, $Df=0.0$, $\varepsilon=0.0$, $m=0.0$, $n=0.0$ and $M=0.0$.

The effects of various parameters on the physical quantities are analyzed by taking fixed values of $Pr=0.71$, $Gr=5.0$, $Ec=1.0$, $Sc=0.22$, $N=0.5$ and $Da=0.1$. All These computations are carried out at steady state time t.

The effects of various parameters on non-dimensional velocity, temperature and concentration are analyzed and plotted with fixed values $Pr=0.71$, $Gr=5.0$, $Ec=1.0$, $Sc=0.22$, $N=1.0$ at steady state time t.

Figs. 1. (a) to 1. (b) represent profiles of non-dimensional temperature and concentration for various estimations of cross-diffusion parameters for $\varepsilon=0.6$, $M=0.5$, $\varepsilon_1=0.1$, $\varepsilon_2=0.5$, $m=0.5$, $n=0.5$.

Figs. 1. (a) to 1. (b) show temperature and concentration profiles with an upgrade in Dufour number and simultaneous decrease in Soret number for steady state.

The variations in non-dimensional velocity, temperature for various values of thermal and solutal stratification parameters are investigated and plotted with $\varepsilon=0.6$, $M=0.5$, $m=0.5$, $n=0.5$, $Sr=2.0$, $Df=0.03$ in Figs. 2. (a) to 2. (b).

From Fig. 3. (a) to Fig. 3. (b) demonstrated the impacts of exponents in the power law variation of the both temperature and concentration m and n values on non-dimensional velocity and temperature, profiles with $\varepsilon=0.6$, $M=0.5$, $\varepsilon_1=0.1$, $\varepsilon_2=0.5$, $Sr=2.0$, $Df=0.03$.

The development of non-dimensional velocity for various values of the porosity and magnetic parameter are illustrated and plotted with $\varepsilon_1=0.1$, $\varepsilon_2=0.5$, $m=0.5$, $n=0.5$, $Sr=2.0$, $Df=0.03$ in Fig. 4. (a).

IV. CONCLUSION

This paper analyses the problem of unsteady MHD free convective flow past a vertical plate with variable surface temperature and concentration with doubly stratified porous medium in the presence of porosity, Soret and Dufour effects. The resulting non-dimensional governing coupled partial differential equations are solved numerically by using the unconditionally stable implicit finite difference method of Crank-Nicolson type. The important findings of this study are listed as follows:

a) The decrease in the concentration profile and an increase in the temperature and velocity profiles with an increase in the value of the Dufour number (or simultaneous decrease in the Soret number).

b) For increasing value of the thermal stratification parameter, temperature depict the reverse trend while both velocity and concentration depicts the same trend. In case of a rise in value of the solutal stratification parameter, concentration, depict the reverse trend while velocity, temperature, depicts the same trend. More over it is also observed that, the value of time t at steady state decreases with an increase in the value of thermal stratification parameter and increases with an increase in the value of solutal stratification parameter.

c) The velocity and temperature decreases with an increase in the exponent m. But except temperature, velocity and concentration decrease with the rise in the exponent n. Further, to reach the steady state values, the required time is more when the exponents m, n are more.

d) For increasing value of the porosity, velocity depict the same trend while both temperature and concentration depicts the reverse trend. In case of a rise in the value of magnetic parameter, temperature, concentration depict the same trend while the velocity depicts the reverse trend. Furthermore, to reach the steady state values, the required time is more when the porosity and magnetic parameters are more.

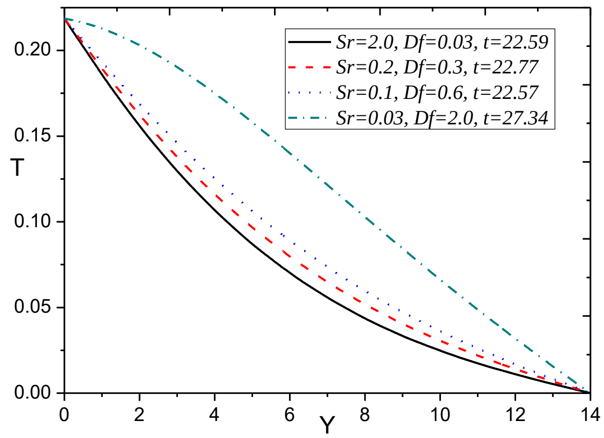

Fig. 1. (a) Temperature profiles at *X=1*.

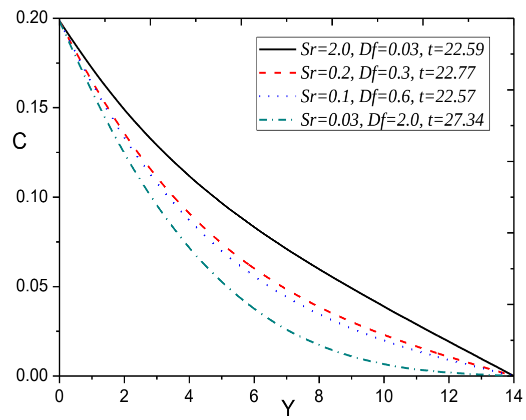

Fig. 1. (b) Concentration profiles at *X=1*.

Fig.1. Profiles with the effect of cross-diffusion for $\varepsilon=0.6$, $M=0.5$, $\varepsilon_1=0.1$, $\varepsilon_2=0.5$, $m=0.5$, $n=0.5$.

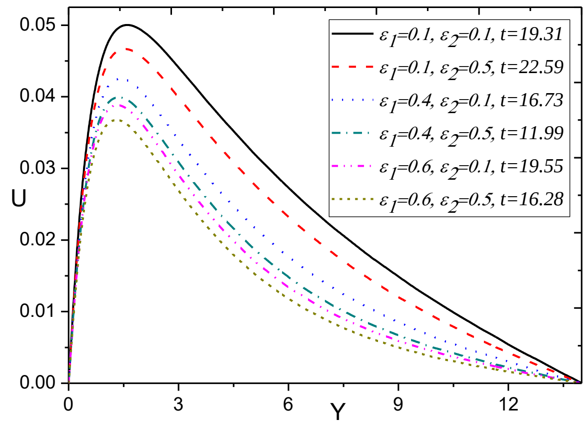

Fig. 2. (a) Velocity profiles at *X=1*.

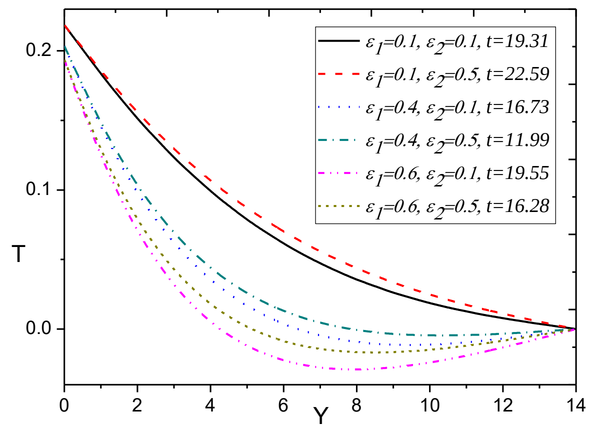

Fig. 2. (b) Temperature profiles at *X=1*.

Fig. 2. Profiles with the effect of double stratification for $\varepsilon=0.6$, $M=0.5$, $m=0.5$, $n=0.5$, $Sr=2.0$, $Df=0.03$.

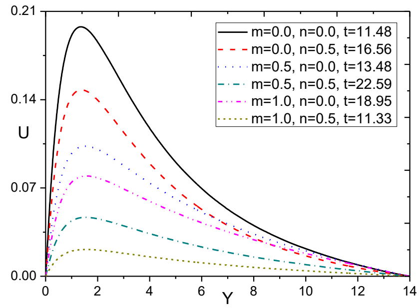

Fig. 3. (a) Velocity profiles at *X=1*.

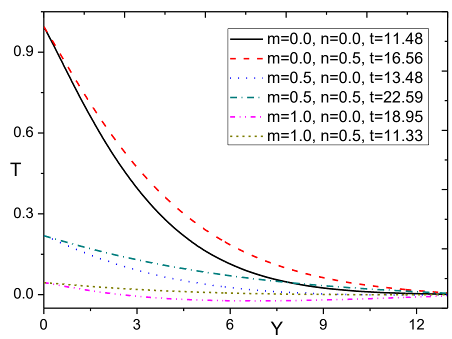

Fig. 3. (b) Temperature profiles at *X=1*.

Fig.3. Profiles with the effect of exponents *m*, *n* for $\varepsilon=0.6, M=0.5, \varepsilon_1=0.1, \varepsilon_2=0.5$, $Sr=2.0$, $Df=0.03$.

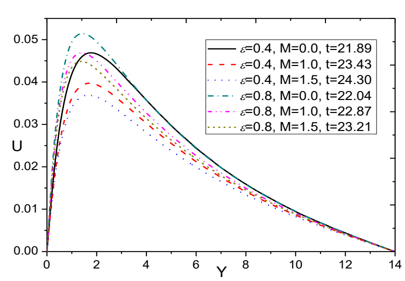

Fig. 4. (a) Velocity profiles at *X=1*.

Fig. 4. Profiles with the effect of the porosity and *M* for $\varepsilon_1=0.1$, $\varepsilon_2=0.5$, $m=0.5$, $n=0.5$, $Sr=2.0$, $Df=0.03$.

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