IGSR Continuity and IGSR Compactness in Intuitionistic Topological Spaces

Mohana K[1], Stephy Stephen[2]
[1]Assistant Professor, Department of Mathematics, Nirmala College for Women
[2] PG Student, Department of Mathematics, Nirmala College for Women

Abstract- **In this paper, intuitionistic generalized semi regular continuity and intuitionistic generalized semi regular compactness are studied. Also its several properties are outlined.**

Index Terms- **Intuitionistic set, IGSR continuous, IGSR compact.**

1. INTRODUCTION

Levine [5] was the first to introduce the concept of g-closed set in general topology. Coker [2] implemented the generalization of sets in general topology to intuitionistic topology which he derived from the concepts of intuitionistic fuzzy topology developed by Atassanov [1]. Coker and Selma [3,7,8] made a study of the concepts of intuitionistic sets, its connectedness, continuity, compactness. Also Younis[11] and Asmaa studied the generalized closed set and its continuity in intuitionistic topology. Gnanambal[4,9] and Selvanayaki made a similar study in intuitionistic topological spaces dealing with the generalized pre regular closed sets and discussed about its continuity and compactness. Thakur[10] worked on the generalized continuity in intuitinistic fuzzy topological spaces. Therefore, these research works have implanted the idea of studying about the continuity and compactness of intuitionistic generalized semi regular closed sets which is discussed in this paper.

1.1 PRELIMINARIES

We recall some basic definitions and results which will be useful for this study. In this study, a space X means an intuitionistic topological space(X,τ) and Y means an intuitionistic topological space (Y,δ).

Definition 1.1: [4] Let X be a non empty set. An intuitionistic set (IS) A is an object having the form $A = < X, A_1, A_2>$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \varphi$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

Definition 1.2: [4] Let X be a non empty set and let A, B are intuitionistic sets in the form $A = < X, A_1, A_2>$, $B = < X, B_1, B_2>$ respectively. Then

(a) $A \subseteq B$ iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$

(b) $A = B$ iff $A \subseteq B$ and $B \supseteq A$

(c) $\bar{A} = < X, A_2, A_1>$

(d) $[\ ]A = < X, A_1, (A_1)^c>$

(e) $A - B = A \cap \overline{B}$.

(f)$\underset{\sim}{\varphi} = < X, \phi, X >$, $\underset{\sim}{X} = < X, X, \phi>$

(g) $A \cup B = < X, A_1 \cup B_1, A_2 \cap B_2 >$.

(h) $A \cap B = < X, A_1 \cap B_1, A_2 \cup B_2 >$.

Furthermore, let $\{A_i: i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = < X, A_i^{(1)}, A_i^{(2)} >$. Then

(i) $\cap A_i = < X, \cap A_i^{(1)}, \cup A_i^{(2)} >$.

(j) $\cup A_i = < X, \cup A_i^{(1)}, \cap A_i^{(2)} >$.

Definition 1.3: [4] An intuitionistic topology (IT in short) on a nonempty set X is a family τ of IS's in X containing $\underset{\sim}{\varphi}$, $\underset{\sim}{X}$ and closed under finite infima and arbitrary suprema. The pair (X,τ) is called an intuitionistic topological space (ITS in short). Any intuitionistic set in τ is known as intuitionistic open set (IOS) in X and the complement of IOS is called intuitionistic closed set (ICS) in X.

Proposition 1.4: [9] Let (X,τ) be an ITS in X and $A = < X, A_1, A_2 >$ be an IS in X, then the several topologies [(a),(b)] and general topologies [(c),(d)] are generated by (X,τ) are

(a)$\tau_{0,1} = \{ [\ ]A: A\in \tau \}$

(b) $\tau_{0,2} = \{ < >A: A\in \tau \}$

(c)$\tau_1 = \{A_1: < X, A_1, A_2 > \in \ \}$

(d) $\tau_2 = \{(A_2)^c: < X, A_1, A_2 > \in \tau \}$

Definition 1.5: [4] Let (X,τ) be an ITS and $A = < X, A_1, A_2 >$ be an IS in X. Then the interior and closure of A are defined as

$Icl(A) = \cap \{K : K$ is an ICS in X and $A \subseteq K\}$
$Iint(A) = \cup \{G : G$ is an IOS in X and $G \subseteq A\}$.
It can be shown that Icl(A) is an ICS and Iint(A) is an IOS in X and A is an ICS in X iff Icl(A) = A and is an IOS in X iff Iint(A) = A.

Definition 1.6: [9] Let (X,τ) be an ITS, then an intuitionistic set A of X is called intuitionistic regular open (intuitionistic regular closed) if A= Iint(Icl(A)) (A=Icl(Iint(A))).

Definition 1.7 [6]: Let (X,τ) be a non empty intuitionistic topological space and let $A = < X, A_1, A_2 >$ be an intuitionistic set. Then A is said to be

(i) Intuitionistic α −generalized closed(Iαg-closed) if Iαcl(A)$\subseteq$U whenever A$\subseteq$U and U is intuitionistic open in X.

(ii) intuitionistic generalized semiclosed (Igs closed) if Iscl(A)$\subseteq$U whenever A$\subseteq$U and U is intuitionistic open in X.

Definition 1.8 [6]: Let (X,τ) be an intuitionistic topological space and let $A = < X, A_1, A_2 >$ be an intuitionistic set. Then A is said to be intuitionistic generalized semi regular closed (Igsr-closed) if $Iscl(A) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic regular open in X.

Definition 1.9: [9] Let X, Y be two non empty sets and f: X→Y be a function. If $B = < Y, B_1 ,B_2 >$ is an IS in Y, then the preimage of B is denoted by $f^{-1}(B)$, where $f^{-1}(B) = < X, f^{-1}(B_1), f^{-1}(B_2) >$. If $A = < X, A_1, A_2 >$ is an IS in X, then the image of A under f is denoted by f (A) is the IS in Y defined by $f (A) = < Y, f (A_1), f (A_2) >$.

Definition 1.10: [9] Let (X,) and (Y,δ) be two intuitionistic topological spaces and let a function f: X→Y be defined, then f is said to be continuous iff the preimage of each ICS in Y is intuitionistic closed in X.

Corollary 1.11: [9] Let A , A_i ($i \in J$) be IS's in X and B, B_j ($j \in K$) be IS's in Y and f: X→Y be a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f (A_1) \subseteq f (A_2)$, $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

(b) $A \subseteq f^{-1}(f (A))$ and if f is injective, then $A = f^{-1}(f (A))$

(c) $f (f^{-1}(B)) \subseteq B$ and if f is surjective then $f(f^{-1}(B)) = B$

(d) $f(\cup A_i) = \cup f(A_i)$; $f(\cap A_i) \subseteq \cap f(A_i)$ and if f is injective then $f(\cap A_i) = \cap f(A_i)$

(e) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$; $f^{-1}(\cap B_i) \subseteq \cap f^{-1}(B_i)$

(f)If f is surjective then $\overline{f(A)} \subseteq f(\bar{A})$. Further if f is injective then $\overline{f(A)} = f(\bar{A})$.

(g) $f^{-1}(\bar{B}) = \overline{f^{-1}(B)}$

Definition 1.12: [9] If there exists an intuitionistic regular open set A in X such that $\varphi_{\sim} \neq A \neq X_{\sim}$, then X is called super disconnected. X is called super connected, if X is not super disconnected.

2. INTUITIONISTIC GENERALIZED SEMI REGULAR CONTINUITY:

Definition 2.1: Let f: $(X,\tau) \to (Y,\delta)$ be a mapping, then f is said to be Igsr-continuous if the preimage of every I-closed set of Y is Igsr-closed in X, i.e., $f^{-1}(V)$ is Igsr-closed in (X,τ) for every I-closed set V of (Y,δ).

Theorem 2.2: A mapping f: $(X,\tau) \to (Y,\delta)$ is Igsr-continous iff the preimage of every intuitionistic open set of Y is Igsr-open in X.

Proof: Let $B = < Y, B_1, B_2 >$ be an intuitionistic open set of Y. Then $f^{-1}(B^c) = f^{-1}(< Y, B_2, B_1 >) = (< X, f^{-1}(B_2), f^{-1}(B_1) >)$. Also $(f^{-1}(B))^c = (f^{-1}(< Y, B_1, B_2 >)^c = (< X, f^{-1}(B_1), f^{-1}(B_2) >)^c = < X, f^{-1}(B_2), f^{-1}(B_1) >$. Since $f^{-1}(B^c) = (f^{-1}(B))^c$, for every intuitionistic set B of Y, $f^{-1}(B^c)$ is Igsr-closed in Y. So $f^{-1}(B)$ is Igsr-open in X. Hence f is Igsr-continuous.

Proposition 2.3: Let (X,τ) and (Y,δ) be two intuitionistic topological spaces. If f: $(X,\tau_{0,1}) \to (Y,\delta_{0,1})$ and f : $(X,\tau_{0,2}) \to (Y,\delta_{0,2})$ are Igsr-continuous , then f: $(X,\tau) \to (Y,\delta)$ is Igsr-continuous.

Proof: Let $B = < Y, B_1, B_2 >$ be an intuitionistic open set of Y. By hypothesis, f: $(X,\tau_{0,1}) \to (Y,\delta_{0,1})$ and f : $(X,\tau_{0,2}) \to (Y,\delta_{0,2})$ are Igsr-continuous. So there exists an Igsr-open sets $f^{-1}(< Y, B_1, (B_1)^c >) = < X, f^{-1}(B_1), f^{-1}(B_1)^c >$ in $(X,\tau_{0,1})$ and $f^{-1}(< Y, (B_2)^c ,B_2 >) = < X, f^{-1}(B_2)^c, f^{-1}(B_2) >$ in $(X,\tau_{0,2})$. Since $B_2 \subset (B_1)^c$ and $B_1 \subset (B_2)^c$, $< X, f^{-1}(B_1), f^{-1}(B_2) > \subseteq < X, f^{-1}(B_1), f^{-1}(B_1)^c >$ or $< X, f^{-1}(B_2)^c, f^{-1}(B_2) >$. Hence $< X, f^{-1}(B_1), f^{-1}(B_2) >$ is Igsr-open in X and so f: $(X,\tau) \to (Y,\delta)$ is Igsr-continuous.

Proposition 2.4: Let (X,τ) and (Y,δ) be two intuitionistic topological spaces. If f: $(X,\tau_1) \to (Y,\delta_1)$ and f: $(X,\tau_2) \to (Y,\delta_2)$ are Igsr-continuous then f: $(X,\tau) \to (Y,\delta)$ is Igsr-continuous.

Proof: Let $B = < Y, B_1, B_2 >$ be an intuitionistic open set of Y. By hypothesis, f: $(X,\tau_1) \to (Y,\delta_1)$ and f : $(X,\tau_2) \to (Y,\delta_2)$ are Igsr-continuous. So there exists an

Igsr-open sets $f^{-1}(< Y, B_1, (B_1)^c >) = < X, f^{-1}(B_1), f^{-1}(B_1)^c > = f^{-1}(B_1)$ in (X,τ_1) and $f^{-1}(< Y, (B_2)^c, B_2 >) = < X, f^{-1}(B_2)^c, f^{-1}(B_2) > = (f^{-1}(B_2))^c$ in (X,τ_2). Since $B_2 \subset (B_1)^c$ and $B_1 \subset (B_2)^c$, $f^{-1}(B_1) \subseteq f^{-1}(B_2)^c$. Hence $< X, f^{-1}(B_1), f^{-1}(B_2) >$ is Igsr-open in X and so f: $(X,)\rightarrow(Y,\delta)$ is Igsr continuous.

Theorem 2.5: If f: $(X,\tau) \rightarrow (Y,\delta)$ is intuitionistic continuous, then f is Igsr-continuous but not conversely.

Proof: Let f: $(X,\tau) \rightarrow (Y,\delta)$ be intuitionistic continuous and let A be any intuitionistic closed set in Y. Then the inverse image $f^{-1}(A)$ is intuitionistic closed in X. Since every I-closed set is Igsr-closed, $f^{-1}(A)$ is Igsr-closed in X. Hence f is Igsr-continuous.

Converse of the above proposition is not true and is proved in the following example:

Example 2.6: Let X={a,b} with $\tau = \{\phi_{\sim}, X_{\sim}, < X, \{a\}, \phi >, < X, \{a\}, \{b\} >, < X, \phi, \{b\} >\}$ and $Y = \{\alpha, \beta\}$ with $\delta = \{\phi_{\sim}, Y_{\sim}, < Y, \phi, \{\alpha\} >, < Y, \{\beta\}, \phi >\}$. Define f: $(X,\tau)\rightarrow(Y,\delta)$ by $f(a) = \alpha$ and $f(b) = \beta$, then f: $(X,\tau)\rightarrow(Y,\delta)$ is Igsr-continuous but not intuitionistic continuous.

Theorem 2.7: Let f: $(X,\tau) \rightarrow (Y,\delta)$ be Irg-continuous, then f is Igsr-continuous.

Proof: Consider f: $(X,\tau) \rightarrow (Y,\delta)$ to be Irg-continuous and let A be I-closed in (Y,δ). Since f is Irg-continuous, $f^{-1}(A)$ is Irg-closed in (X,τ). We know that every Irg-closed is Igsr-closed. So $f^{-1}(A)$ is Igsr-closed, which implies that f is Igsr-continuous.

Converse is not true and is shown by the following example:

Example 2.8: Let X={a,b} with $\tau = \{\phi_{\sim}, X_{\sim}, < X, \{a\}, \phi >, < X, \{a\}, \{b\} >, < X, \phi, \{b\} >\}$ and $Y = \{1, 2\}$ with $\delta = \{\phi_{\sim}, Y_{\sim}, < Y, \{1\}, \phi >, < Y, \phi, \{1\} >$. Define a function f: $(X,\tau)\rightarrow(Y,\delta)$ with $f(a) = 2$, $f(b) = 1$. Then f is Igsr-continuous but not Irg-continuous.

Theorem 2.9: Let f: $(X,\tau) \rightarrow (Y,\delta)$ be Iαg-continuous , then f is Igsr-continuous.

Proof: Let A be I-closed in (Y,δ). Since f is Iαg-continuous, $f^{-1}(A)$ is Iαg-closed in (X,τ). It is clear that every Iαg-closed set is Igsr-closed. Hence $f^{-1}(A)$ is Igsr-closed. Therefore, f is Igsr-continuous.

Converse is proved to be not true from the example:

Example 2.10: Assume X = {a,b} with $\tau = \{\phi_{\sim}, X_{\sim}, < X, \{a\}, \phi >, < X, \{a\}, \{b\} >, < X, \phi, \{b\} >\}$ and Y={1,2} with $\delta = \{\phi_{\sim}, Y_{\sim}, < Y, \{2\}, \phi >, < Y, \{1\}, \{2\} >, < Y, \phi, \{2\} >$. Let us define f: $(X,\tau)\rightarrow(Y,\delta)$ by $f(a) = 1$, $f(b) = 2$, then f is Igsr-continuous but not Iαg-continuous.

Theorem 2.11: If f: $(X,\tau) \rightarrow (Y,\delta)$ is Ig-continuous, then f is Igsr-continuous.

Proof: Consider A be an I-closed in (Y,δ). Given that f is Ig-continuous, therefore $f^{-1}(A)$ is Ig-closed in (X,τ). It is known that every Ig-closed set is Igsr-closed. Hence $f^{-1}(A)$ is Igsr-closed. Therefore, f is Igsr-continuous.

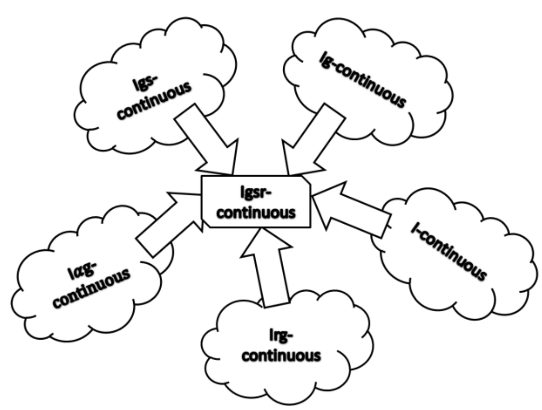
Example 2.12: Let X={a,b} with $\tau = \{\phi_{\sim}, X_{\sim}, < X, \{a\}, \phi >, < X, \{a\}, \{b\} >, < X, \phi, \{b\} >\}$ and $Y=\{\alpha,\beta\}$ with $\delta = \{\phi_{\sim}, Y_{\sim}, < Y, \phi, \{\alpha\} >, < Y, \{\alpha\}, \phi\}$. By defining f: $(X,\tau)\rightarrow(Y,\delta)$ by $f(a) = \beta$ and $f(b) = \alpha$, we get that f is Igsr-continuous but not Ig-continuous.

Theorem 2.13: Let f: $(X,\tau) \rightarrow (Y,\delta)$ be Igs-continuous, then f is Igsr-continuous.

Proof: Let f: $(X,\tau) \rightarrow (Y,\delta)$ be Igs-continuous and assume A to be an I-closed set in (Y,δ). Since f is Igs-continuous, $f^{-1}(A)$ is Igs-closed in (X,τ). Every Igs-closed is Igsr-closed set, therefore, $f^{-1}(A)$ is Igsr-closed. Hence f is Igsr-continuous.

Example 2.14: Let X={a,b} with $\tau = \{\phi_{\sim}, X_{\sim}, <X, \{a\}, \phi >, < X, \{a\}, \{b\} >, < X, \phi, \{b\} >\}$ and Y={1,2} with $\delta = \{\phi_{\sim}, Y_{\sim}, < Y, \phi, \{2\} >, < Y, \{2\}, \{1\} >, < Y, \{2\}, \phi >\}$. By defining f: $(X,\tau)\rightarrow(Y,\delta)$ by $f(a) = 1$ and $f(b) = 2$, we get that f is Igsr-continuous, but not Igs-continuous.

The diagrammatic representation of the above theorems is :



Proposition 2.15: A mapping f: $(X,\tau) \rightarrow (Y,\delta)$ is Igsr-continuous if (X,τ) is intuitionistic super connected.

Proof: Given that f is a mapping from $(X,\tau) \rightarrow (Y,\delta)$ and (X,τ) is intuitionistic super connected. Since (X,τ) is intuitionistic super connected, the only intuitionistic regular open sets are $X_{\sim}$ and $\phi_{\sim}$, hence all the subsets of X are Igsr-closed. Therefore the preimage of every I-closed set of Y is Igsr-closed in X.

Theorem 2.16: Let f: $(X,\tau) \to (Y,\delta)$ be Igsr-continuous, then $f(Igsrcl(A)) \subset Icl(f(A))$ for every intuitionistic subset of X.

Proof: Consider A to be an intuitionistic subset of X. We have Icl(f(A)) is intuitionistic closed in (Y,δ). Since f is Igsr-continuous, $f^{-1}(Icl(f(A)))$ is Igsr-closed in X. We know that $A \subset f^{-1}(f(A)) \subset f^{-1}(Icl(f(A)))$ $\Rightarrow Igsrcl(A) \subseteq Igsrcl(f^{-1}(Icl(f(A))))$. Therefore $f(Igsrcl(A)) \subseteq f(Igsrcl(f^{-1}(Icl(f(A))))) \subset Icl(f(A))$.

Definition 2.17: Let (X,τ) be an intuitionistic topological space, then $\tau^* = \{A \subset X / Igsrcl(\underset{\sim}{X}-A)=\underset{\sim}{X}-A\}$.

Theorem 2.18: Every Igsr-closed set is intuitionistic closed iff $\tau^* = \tau$ holds.

Proof: Let $A \in \tau^*$. Then $Igsrcl(\underset{\sim}{X}-A) = \underset{\sim}{X}-A$. Since $\tau^* = \tau$, every Igsr-closed set is intuitionistic closed.

Conversely, let every Igsr-closed set is intuitionistic closed. Let A be Igsr-closed set in τ^* $(A \subset X)$, then A is I-closed. $\Rightarrow Igsrcl(\underset{\sim}{X}-A) = \underset{\sim}{X}-A$. Hence $\tau^* = \tau$.

Remark 2.19: If $\tau^* = \tau$, then intuitionistic continuity and Igsr-continuity are the same.

Proof: Let f: $(X,\tau) \to (Y,\delta)$ be Igsr-continuous. Then $f^{-1}(A)$ is Igsr closed for every intuitionistic closed set A of Y. Given that $\tau^* = \tau$, from theorem 2.18, we have $f^{-1}(A)$ is intuitionistic closed $\Rightarrow$ f is intuitionistic continuous.

Theorem 2.20: If IGSRO(X) forms a topology in intuitionistic space, then τ^* is also a topology.

Proof: Given that IGSRO(X) forms a topology in intuitionistic space, i.e., for every $A \subset X$, the sets are intuitionistic generalized semi regular open, which implies that τ^* contains subsets that are IGSRO. Since IGSRO forms a topology, τ^* also forms a topology.

Theorem 2.21: Let f: $(X,\tau^*) \to (Y,\delta)$ be a mapping, then the following statements are equivalent:

a) For every intuitionistic subset A of X, $f(Igsrcl(A)) \subset Icl(f(A))$.
b) If τ^* is a topology, then f: $(X,\tau^*) \to (Y,\delta)$ is intuitionistic continuous.

Proof: a) $\Rightarrow$ b)

Let A be an intuitionistic closed set in (Y,δ).

From a) $f(Igsrcl(A)) \subset Icl(f(A))$

$\Rightarrow f(Igsrcl(f^{-1}(A))) \subset Icl(f(f^{-1}(A))) \subset Icl(A) = A$

Therefore, $f(Igsrcl(f^{-1}(A))) \subset A$.

$Igsrcl(f^{-1}(A)) \subset f^{-1}(A)$.

Also $f^{-1}(A) \subset Igsrcl(f^{-1}(A))$.

Thus $(f^{-1}(A))^c \in \tau^*$.

$\Rightarrow f^{-1}(A)$ is intuitionistic closed in (X,τ^*). So f is intuitionistic continuous.

b) $\Rightarrow$ a)

For every subset A of X, Icl(f(A)) is I-closed in (Y,δ).

Then given that if τ^* is a topology, $\Rightarrow$ f is I-continuous.

Therefore $f^{-1}(Icl(f(A)))$ is I-closed in τ^*.

So $Igsrcl(f^{-1}(Icl(f(A)))) = f^{-1}(Icl(f(A)))$

which implies that $f(Igsrcl(f^{-1}(Icl(f(A))))) \subset Icl(f(A))$.

Since f is I-continuous, $A \subset Icl(A) \subset Icl(f^{-1}(f(A))) \subset f^{-1}(Icl(f(A)))$.

Hence $f(Igsrcl(A)) \subset f(Igsrcl(f^{-1}(Icl(f(A))))) \subset Icl(f(A))$.

3. INTUITIONISTIC GENERALIZED SEMI REGULAR COMPACTNESS:

Definition 3.1: Let (X,) be an intuitionistic topological space. If a family $\{< X, K_i^{(1)}, K_i^{(2)} > ; i \in \wedge\}$ of Igsr-open sets in X satisfies the condition $\cup\{< X, K_i^{(1)}, K_i^{(2)} > ; i \in \wedge\} = \underset{\sim}{X}$, then it is called an Igsr-open cover of X.

A finite subfamily of an Igsr-open cover $\{< X, K_i^{(1)}, K_i^{(2)} > ; i \in \wedge\}$ of X, which is also an Igsr-open cover of X is called a finite subcover of $\{< X, K_i^{(1)}, K_i^{(2)} > ; i \in \wedge\}$.

Definition 3.2: An ITS (X,τ) is called Igsr-compact iff each Igsr-open cover has a finite subcover.

Definition 3.3: Let (X,τ) be an intuitionistic topological space and let A be an IS in X. The family $\{< X, K_i^{(1)}, K_i^{(2)} > ; i \in \wedge\}$ of Igsr-open sets in X is called a Igsr-open cover of A if $A \subseteq \cup\{< X, K_i^{(1)}, K_i^{(2)} > ; i \in \wedge\}$.

Definition 3.4: Let $A = < X, A^{(1)}, A^{(2)} >$ be an intuitionistic set in an intuitionistic topological space, then A is called Igsr-compact iff every Igsr-open cover of A has a finite sub cover.

In other words, A is Igsr-compact iff for each family $\mathcal{K}=\{K_i : i \in \wedge\}$ where $K_i = \{< X, K_i^{(1)}, K_i^{(2)} > ; i \in \wedge\}$ of Igsr-open sets in X, $A^{(1)} \subseteq \cup_{i \in \wedge} K_i^{(1)}$ and $A^{(2)} \supseteq \cup_{i \in \wedge}$

$K_i^{(2)}$, there exists a finite sub family $\{K_i : i=1,2,\ldots\ldots n\}$ of $\mathcal{K}$ such that $A^{(1)} \subseteq \cup_{i\in\wedge} K_i^{(1)}$ and $A^{(2)} \supseteq \cup_{i\in\wedge} K_i^{(2)}$.

Proposition 3.5: Let (X,τ) be an intuitionistic topological space, then (X,τ) is Igsr-compact iff the ITS $(X,\tau_{0,1})$ is Igsr-compact.

Proof:

Necessity: Let (X,τ) be Igsr-compact and let $\{ [\,]K_j : j\in\wedge\}$ of X in (X,τ) be an Igsr-open cover in $(X,\tau_{0,1})$. Since $\cup([\,]K_j) = \underset{\sim}{X}$, we have $\cup K_j^{(1)} = X$ and hence $K_j^{(2)} \subseteq (K_j^{(1)})^c \Rightarrow \cap K_j^{(2)} \subseteq (\cup K_j^{(1)})^c = \varphi \Rightarrow \cup K_j = \underset{\sim}{X}$. Since (X,τ) is Igsr-compact, there exists $K_1, K_2, K_3, \ldots\ldots K_n$ such that $\cup_{i=1}^{n} K_i = \underset{\sim}{X}$ which implies $\cup_{i=1}^{n} K_i^{(1)} = \underset{\sim}{X}$ and $\cup_{i=1}^{n} K_i^{(2)} = \phi$. So $(X,\tau_{0,1})$ is Igsr-compact.

Suffiency: Let $(X,\tau_{0,1})$ is Igsr-compact. Assume $\{ K_j : j\in\wedge\}$ of X to be an Igsr-open cover in (X,τ). Since $\cup(K_j) = \underset{\sim}{X}$, we obtain $\cup K_j^{(1)} = X$ and hence $\cap(K_j^{(1)})^c = \phi \Rightarrow \cap K_j^{(2)} \subseteq (\cup K_j^{(1)})^c = \varphi \Rightarrow \cup K_j = \underset{\sim}{X}$. Given $(X,\tau_{0,1})$ is Igsr compact, so there exists $K_1,K_2,K_3\ldots\ldots K_n$ such that $\cup_{i=1}^{n}[\,]K_i = \underset{\sim}{X} \Rightarrow \cup_{i=1}^{n} K_i^{(1)} = X$ and $(\cap_{i=1}^{n} K_i^{(1)})^c = \varphi$. Therefore $K_i^{(1)} \subseteq (K_i^{(2)})^c \Rightarrow X = \cup_{i=1}^{n} K_i^{(1)} \subseteq \cap_{i=1}^{n}(K_i^{(2)})^c \Rightarrow \cap_{i=1}^{n} K_i^{(2)} = \varphi$. Thus $\cup_{i=1}^{n} K_i = \underset{\sim}{X}$. Hence (X,τ) is Igsr-compact.

Proposition 3.6: Let $f : (X,\tau)\rightarrow(Y,\delta)$ be a surjective Igsr-continuous mapping. If (X,τ) is Igsr-compact, then (Y,δ) is I-compact.

Proof: Let $\{< X, K_i^{(1)}, K_2^{(2)} > : i\in\wedge\}$ be an open cover of Y. Since f is Igsr-continuous, $\{f^{-1}(K_i) ; i\in\wedge\}$ is an Igsr-open cover of X. Given (X,τ) is Igsr-compact, hence it has a finite subcover $\{f^{-1}(K_1), f^{-1}(K_2),\ldots. f^{-1}(K_n)\}$ such that $\cup_{i=1}^{n} f^{-1}(K_i^{(i)}) = \underset{\sim}{X}$ and $\cap_{i=1}^{n} f^{-1} K_i^{(2)} = \underset{\sim}{\varphi}$. This implies that $f^{-1}(\cup_{i=1}^{n} K_i^{(1)}) = X$ and $f^{-1}(\cap_{i=1}^{n}(K_i^{(2)}) = \varphi$. $\Rightarrow \cup_{i=1}^{n} K_i^{(1)} = f(X)$ and $\cap_{i=1}^{n} K_i^{(2)} = f(\varphi)$. Since f is a surjective $\{K_1,K_2,K_3\ldots\ldots K_n\}$ is an open cover of Y and hence (Y,δ) is intuitionistic compact.

Corollary 3.7: Let $f: (X,\tau) \rightarrow(Y,\delta)$ be Igsr-continuous. If A is Igsr-compact in (X,τ), then f(A) is intuitionistic compact in (Y,δ).

Proof: Let $\{< X, K_i^{(1)}, K_2^{(2)} > : i\in\wedge\}$ be an open cover of Y. Since f is Igsr-continuous, $\{f^{-1}(K_i) ; i\in\wedge\}$ is an Igsr-open cover of X. Given (X,τ) is Igsr-compact, hence it has a finite subcover $\{f^{-1}(K_1), f^{-1}(K_2),\ldots. f^{-1}(K_n)\}$ such that $\cup_{i=1}^{n} f^{-1}(K_i^{(i)}) = \underset{\sim}{X}$ and $\cap_{i=1}^{n} f^{-1} K_i^{(2)} = \underset{\sim}{\varphi}$. This implies that $f^{-1}(\cup_{i=1}^{n} K_i^{(1)}) = X$ and $f^{-1}(\cap_{i=1}^{n}(K_i^{(2)}) = \varphi$. $\Rightarrow \cup_{i=1}^{n} K_i^{(1)} = f(X)$ and $\cap_{i=1}^{n} K_i^{(2)} = f(\varphi)$. By definition we have $K_i^{(1)} \subseteq \cup_{i\in\wedge} G_i^{(1)}$ and $K_i^{(2)} \supseteq \cup_{i\in\wedge} G_i^{(2)}$ where G_i are family of Igsr-open sets in X, there exists a finite sub family $K_i^{(1)} \subseteq \cup_{i\in\wedge} G_i^{(1)}$ and $K_i^{(2)} \supseteq \cup_{i\in\wedge} G_i^{(2)}$. Hence (Y,δ) is intuitionistic compact.

Definition 3.8: Let (X,τ) and (Y,δ) be two intuitionistic topological spaces and $f: (X,\tau) \rightarrow(Y,\delta)$, f is said to be Igsr-irresolute if the preimage of every Igsr-closed set of Y is Igsr-closed in X.

Proposition 3.9: Let $f: (X,\tau) \rightarrow(Y,\delta)$ be an Igsr-irresolute mapping and if A is Igsr-compact relative to X, then f(A) is Igsr-compact relative to Y.

Proof: Let $\{ K_i : i\in\wedge\}$ be an Igsr-open set of Y such that $f(A) \subseteq\cup\{K_i : i\in\wedge\}$, then $A \subseteq \cup\{f^{-1} (K_i) : i\in\wedge\}$ where $f^{-1}(K_i)$ is an Igsr-open set in X. Given A is Igsr-compact relative to X, there exists a finite sub collection $\{K_1,K_2,K_3\ldots\ldots K_n\}$ such that $A \subset \cup\{f^{-1}(K_i) : i=1,2,\ldots n\}$ i.e., $f(A) \subset \cup\{K_i : i=1,2,\ldots n\}$. Hence f(A) is Igsr-compact relative to Y.

Proposition 3.10: Let $f: (X,\tau) \rightarrow(Y,\delta)$ be an Igsr-irresolute mapping. If X is Igsr-compact then Y is also an Igsr-compact space.

Proof: Let $f: (X,\tau) \rightarrow(Y,\delta)$ be an Igsr-irresolute mapping from (X,τ) onto (Y,δ) where (X,τ) is Igsr-compact. Let $\{K_i : i\in\wedge\}$ be an Igsr-open cover of Y, then $f^{-1}(K_i)$ is an Igsr-open cover of X. Since X is Igsr-compact, $\{f^{-1}(A_{i1}), f^{-1}(A_{i2}),\ldots. f^{-1}(A_{in})\}$ is a finite sub family such that $\cup_{j=1}^{n} K_{ij} = \underset{\sim}{X}$. Since f is onto, $f(\underset{\sim}{X}) = \underset{\sim}{X}$ and $f(\cup_{j=1}^{n} f^{-1} K_{ij}) = \cup_{j=1}^{n} f(f^{-1}(K_{ij})) = \cup_{j=1}^{n} K_{ij}$. Therefore $\cup_{j=1}^{n} K_{ij} = \underset{\sim}{X}$ and $\{ K_{i1},K_{i2},K_{i3}\ldots\ldots K_{1n} \}$ is an intuitionistic finite subcover of $\{K_i : i\in\wedge\}$. Hence (Y,δ) is Igsr-compact.

REFERENCES

[1] Atanassov KT, Intuitionistic fuzzy sets, Fuzzy sets and systems, 1986; 20(1), 87-96.

[2] Coker D, A note on intuitionistic sets and intuitionistic points, Turkish J Math., 1996; 20(3), 343-351.

[3] Coker D, An Introduction to intuitionistic toptological spaces, Busefal, 2000, 81:51-56.

[4] Gnanambal Ilango, S Selvanayaki, Generalized Pre regular closed sets in intuitionistic

topological spaces, Internat. J.Math.Archive, 2014, 5(4):1-4.

[5] Levine N, Generalized closed sets in topology, Rend.Circ.Mat Palermo,19 (1970), 89-96.

[6] Mohana K, Stephy Stephen, On generalized semi regular closed sets in intuitionistic topological spaces, "unpublished".

[7] Selma Ozcag, Coker D, On connectedness in intuitionistic fuzzy special topological spaces, Internat.J. Math & Math Sci., 1998,21(1):33-40

[8] S Ozcag,D Coker, A note on connectedness in intuitionistic fuzzy special topological spaces, Internat.J.Math & Math Sci., 2000, 23(1):45-54.

[9] Selvanayaki, Gnanambal Ilango, IGPR Continuity and compactness in intuitionstic topological space, British Jour.Math Csci. 2015,11(2):1-4.

[10] Thakur S.S and Jyoti Pandey Bajpai, On intuitionistic fuzzy gpr-closed sets, Fuzzy Inf.Eng.4(2012), 425-444.

[11] Younis J Yaseen and Asmaa G Raouf, On generalization closed set and generalized continuity on intuitionistic topological spaces, J.of Al-anbar university for pure sci., Vol 3, No.1(2009).