A Note on Adjoint of Trapezoidal Fuzzy Number Matrices

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Abstract- **The fuzzy sets whose membership values are fuzzy sets on the interval (0,1). This concept was proposed by Zadeh, as an extension of fuzzy sets. The fuzzy matrices play an important role in scientific developments. In this paper, some elementary operations on proposed trapezoidal fuzzy numbers (TrFNs) are defined. We also defined some operations on trapezoidal fuzzy matrices(TrFMs). A notion on adjoint of trapezoidal fuzzy matrices (ATrFMs) is proposed. Some more special properties of adjoint of TrFM have also been verified. Numerical example is also verified.**

Index Terms- **Fuzzy Arithmetic, Fuzzy number, Trapezoidal fuzzy number (TrFN), Trapezoidal fuzzy matrix(TrFM), Adjoint of Trapezoidal fuzzy matrix(ATrFM).**

I. INTRODUCTION

Fuzzy sets have been introduced by Lofti.A.Zadeh[18] Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Hisdal [3] discussed the IF THEN ELSE statement and interval-valued fuzzy sets if higher type. Interval arithmetic was first suggested by Dwyer [2] in 1951, by means of Zadeh's extension principle [19], the usual Arithmetic operations on real numbers can be extended to the ones defined on Fuzzy numbers. Dubosis and Prade [1] has defined any of the fuzzy numbers. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single – valued numbers. Jhon [5] studied an appraisal of theory and applications on type-2 fuzzy sets.

Trapezoidal fuzzy number's (TrFNs) are frequently used in application. It is well known that the matrix formulation of a mathematical formula gives extra facility to study the problem. Due to the presence of uncertainty in many mathematical formulations in different branches of science and technology. A presented new ranking function and arithmetic operations on type-2 generalized type-2 trapezoidal fuzzy numbers by Stephen Dinagar and Anbalagan [15].

We introduce trapezoidal fuzzy matrices (TrFMs). To the best of our knowledge, no work is available on TrFMs, through a lot of work on fuzzy matrices is available in literature. A brief review on fuzzy matrices is given below.

Fuzzy matrix has been proposed to represent fuzzy relation in a system based on fuzzy set theory [11]. Fuzzy matrices were introduced for the first time by Thomason [17] who discussed the convergence of power of fuzzy matrix. Several authors had presented a number of results on the convergence of power sequences of fuzzy matrices. Several authors have presented a number of results on the convergence of power sequence of fuzzy matrices [4,9]. Fuzzy matrices play an important role in scientific development. Two new operations and some applications of fuzzy matrices are given in [12,13,14].

Kim [6] presented some important results on determinant of square fuzzy matrices. He defined the determinant of a square fuzzy matrix and contributed with very research works [7,8]. Mamoni Dhar [10] presented a note on Determinant and Adjoint of fuzzy Square Matrix. Stephen Dinagar et.al [16] presented a study on Adjoint of type-2 triangular fuzzy matrices. The adjoint of square fuzzy fuzzy matrix was defined by Thomson [17] and Kim [7].

The paper organized as follows, Firstly in section 2, we recall the definition of Trapezoidal fuzzy number and some operations on trapezoidal fuzzy numbers (TrFNs). In section 3, we have reviewed the definition of trapezoidal fuzzy matrix (TrFM) and some operations on Trapezoidal fuzzy matrices

(TrFMs). In section 4, we defined the notion of Adjoint of trapezoidal fuzzy matrix (ATrFM). In section 5, we have presented some properties of Adjoint of trapezoidal fuzzy matrix (ATrFM). In section 6, relevant numerical examples presented. Finally in section 7, conclusion is included.

II. PRELIMINARIES

In this section, We recapitulate some underlying definitions and basic results of fuzzy numbers.

Definition 2.1 fuzzy set

A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

A={(x,□ A(x)) ; x□X}

Here □_A : X □ [0,1] is a mapping called the degree of membership function of the fuzzy set A and □_A(x) is called the membership value of x□X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2.2 Normal fuzzy set

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one x□X such that □_A (x) =1.

Definition 2.3 Convex fuzzy set

A fuzzy set A={(x,□_A (x))}□X is called Convex fuzzy set if all A_∝ are Convex set (i.e.,) for every element x_1□ A_∝ and x_2□ A_∝ for every □□[0,1], □x_1+(1-□) x_2□ A_□ for all □□[0,1] otherwise the fuzzy set is called non-convex fuzzy set

Definition 2.4 Fuzzy number

A fuzzy set A˜ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

A˜ is normal

A˜ is convex

The support of A˜ is closed and bounded then A˜ is called fuzzy number.

Definition 2.5 Trapezoidal fuzzy number

A fuzzy number = is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}^{TzL}}(x)=\begin{cases} 0 & ; \ x\le a_1 \\ \dfrac{x-a_1}{a_2-a_1} & ; \ a_1\le x\le a_2 \\ 1 & ; \ a_2\le x\le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & ; \ a_3\le x\le a_4 \\ 0 & ; \ x>a_4 \end{cases}$$

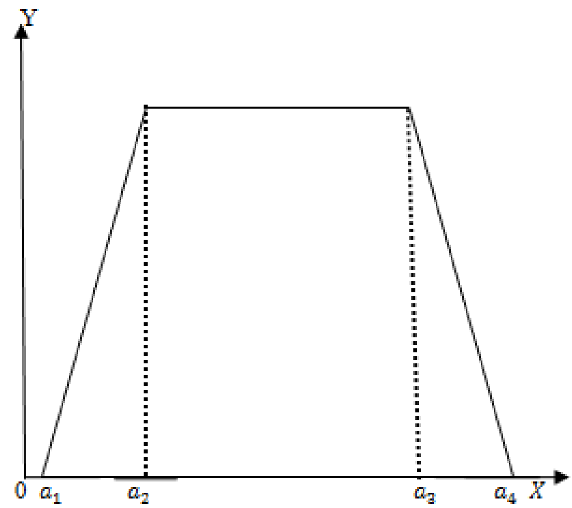

Fig 1: Trapezoidal Fuzzy Number

Definition 2.6 Ranking function

We defined a ranking function $\Re$: F(R)→R which maps each fuzzy numbers to real line F(R) represent the set of all trapezoidal fuzzy number. If R be any linear ranking function

$$\Re\,(\tilde{A}^{TzL}) = \left(\frac{a_1,a_2,a_3,a_4}{4}\right)$$

Also we defined orders on F(R) by

$\Re\,(\tilde{A}^{TzL}) \ge \Re\,(\tilde{B}^{TzL})$ if and only if $\tilde{A}^{TzL} \ge_R \tilde{B}^{TzL}$

$\Re\,(\tilde{A}^{TzL}) \le \Re\,(\tilde{B}^{TzL})$ if and only if $\tilde{A}^{TzL} \le_R \tilde{B}^{TzL}$

$\Re\,(\tilde{A}^{TzL}) = \Re\,(\tilde{B}^{TzL})$ if and only if $\tilde{A}^{TzL}$? $=_R \tilde{B}^{TzL}$

Definition 2.7 Arithmetic operations on trapezoidal fuzzy numbers (TrFNs)

Let $\tilde{A}^{TzL} = (a_1, a_2, a_3, a_4)$ and $\tilde{B}^{TzL} = (b_1, b_2, b_3, b_4)$ be trapezoidal fuzzy numbers (TrFNs) then we defined, Addition

$\widetilde{A}^{TzL} + \widetilde{B}^{TzL} = (a_1+b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

Subtraction

$\widetilde{A}^{TzL} - \widetilde{B}^{TzL} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

Multiplication

$\widetilde{A}^{TzL} \times \widetilde{B}^{TzL} = (a_1 \Re(B),\ a_2 \Re(B),\ a_3 \Re(B),\ a_4 \Re(B))$

where $\Re(\widetilde{B}^{TzL}) = \left(\frac{b_1+b_2+b_3+b_4}{4}\right)$

or $\Re(\widetilde{b}^{TzL}) = \left(\frac{b_1+b_2+b_3+b_4}{4}\right)$

Division

$\widetilde{A}^{TzL} / \widetilde{B}^{TzL} =$

$$\left(\frac{a_1}{\Re(\widetilde{B}^{TzL})}, \frac{a_2}{\Re(\widetilde{B}^{TzL})}, \frac{a_3}{\Re(\widetilde{B}^{TzL})}, \frac{a_4}{\Re(\widetilde{B}^{Tzl})}\right)$$

Where $\Re(\widetilde{B}^{TzL}) = \left(\frac{b_1+b_2+b_3+b_4}{4}\right)$ or

$\Re(\widetilde{b}^{TzL}) = \left(\frac{b_1+b_2+b_3+b_4}{4}\right)$

Scalar multiplication

$$K\widetilde{A}^{TzL} = \begin{cases} (ka_1, ka_2, ka_3, ka_4) & if\ K \geq 0 \\ (ka_4, ka_3, ka_2, ka_1) & if k < 0 \end{cases}$$

Definition 2.8 Zero trapezoidal fuzzy number

If $\widetilde{A}^{TzL}$ = (0,0,0,0) then $\widetilde{A}^{TzL}$ is said to be zero trapezoidal fuzzy number. It is defined by $\widetilde{0}^{TzL}$.

Definition 2.9 Zero equivalent trapezoidal fuzzy number

A trapezoidal fuzzy number $\widetilde{A}^{TzL}$ is said to be a zero equivalent trapezoidal fuzzy number if $\Re(\widetilde{A}^{TzL}) = 0$. It is defined by $\widetilde{0}^{TzL}$.

Definition 2.10 Unit trapezoidal fuzzy number

If $\tilde{A}^{TzL}$ = (1,1,1,1) then $\widetilde{A}^{TzL}$ is said to be a unit trapezoidal fuzzy number. It is denoted by $\widetilde{1}^{TzL}$.

Definition 2.11 Unit equivalent trapezoidal fuzzy number

A trapezoidal fuzzy number $\widetilde{A}^{TzL}$ is said to be unit equivalent triangular fuzzy number. If $\Re(\widetilde{A}^{TzL}) = 1$. It is denoted by $\widetilde{1}^{TzL}$.

Definition 2.12 Inverse of trapezoidal fuzzy number

If $\widetilde{a}^{TzL}$ is trapezoidal fuzzy number and $\widetilde{a}^{TzL} \neq \widetilde{0}^{TzL}$ then we define.

$$\widetilde{a}^{Tzl^{-1}} = \frac{\widetilde{1}^{Tzl}}{\widetilde{a}^{TzL}}$$

III. TRAPEZOIDAL FUZZY MATRICES (TrFMs)

In this section, we introduced the trapezoidal fuzzy matrix and the operations of the matrices some examples provided using the operations.

Definition 3.1 Trapezoidal fuzzy matrix (TrFM)

A trapezoidal fuzzy matrix of order m×n is defined as $A = (\widetilde{a}_{ij}^{TzL})_{m\times n}$, where $a_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ is the ij^{th} element of A.

Definition 3.2 Operations on Trapezoidal Fuzzy Matrices (TrFMs)

As for classical matrices. We define the following operations on trapezoidal fuzzy matrices. Let A = ($\widetilde{a}_{ij}^{TzL}$) and B =($\widetilde{b}_{ij}^{TzL}$) be two trapezoidal fuzzy matrices (TrFMs) of same order. Then, we have the following

i. Addition

$$A+B = \left(\widetilde{a}_{ij}^{TzL} + \widetilde{b}_{ij}^{TzL}\right)$$

ii. Subtraction

$$A-B = \left(\widetilde{a}_{ij}^{TzL} - \widetilde{b}_{ij}^{TzL}\right)$$

iii. For $A = \left(\widetilde{a}_{ij}^{TzL}\right)_{m\times n}$ and $B = \left(\widetilde{b}_{ij}^{TzL}\right)_{n\times k}$ then $AB = \left(\widetilde{c}_{ij}^{TzL}\right)_{m\times k}$ where $\widetilde{c}_{ij}^{TzL} = \sum_{p=1}^{n} \widetilde{a}_{ip}^{TzL} \cdot \widetilde{b}_{pj}^{TzL}$, i=1,2,…,m and j=1,2,…,k.

iv. A^T or $A^1 = \left(\widetilde{a}_{ji}^{TzL}\right)$

v. $KA = \left(K\, \tilde{a}_{ij}^{TzL} \right)$ Where K is scalar.

Definition 3.3 Equal Trapezoidal fuzzy matrices

Two trapezoidal fuzzy matrices A = ($\tilde{a}_{ij}^{TzL}$) and B = ($\tilde{b}_{ij}^{TzL}$) of the same order are said to be equal if the rak of their elements in the corresponding positions are equal. Also it is denoted by $A = B$.

IV. ADJOINT OF TRAPEZOIDAL FUZZY MATRIX (ATrM)

Definition 4.1 Determinant of trapezoidal fuzzy matrix

The determinant of a square of trapezoidal fuzzy matrix $A =$ ($\tilde{a}_{ij}^{TzL}$) is denoted by (A) or det(A) and is defined as follows:

$$|A| = \sum_{r\in S_n} sign\ r\ \prod_{i=1}^{n} \tilde{a}_{ir(i)}^{TzL}$$
$$= \sum_{r\in S_n} sign\ r\ \tilde{a}_{1r(1)}^{TzL}\ \tilde{a}_{2r(2)}^{TzL} \dots \tilde{a}_{nr(n)}^{TzL}$$

Where $\tilde{a}_{ir(i)}^{TzL}$ are TrFNs and S_n denotes the symmetric group of all permutations of the indices {1,2,3,…,n} and sign r =1 or -1 according as the permutation $r = \begin{pmatrix} 1 & 2 & \dots\ \dots & n \\ r(1) & r(2) & \dots\ \dots & r(n) \end{pmatrix}$ is even or odd respectively.

Definition 4.2 Minor

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square Trapezoidal fuzzy matrix of order n. The minor of an element $\tilde{a}_{ij}^{TzL}$ in A is a determinant of order (n−1) × (n−1) which is obtained by deleting the i^{th} row and the j^{th} column from A and is denoted by $\tilde{M}_{ij}^{TzL}$.

Definition 4.3 Cofactor

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square Trapezoidal fuzzy matrix of order n. The cofactor of an element $\tilde{a}_{ij}^{TzL}$ in A is denoted by $\tilde{A}_{ij}^{TzL}$ and is defined as $\tilde{A}_{ij}^{TzL} = (-1)^{i+j}\ \tilde{M}_{ij}^{TzL}$.

Definition 4.4 Aliter definition for determinant

Alternatively, the determinant of a square Trapezoidal square fuzzy matrix $A = \left(\tilde{a}_{ij}^{TzL}\right)$ of order n may be expanded in the form

$$|A| = \sum_{j=1}^{n} \tilde{a}_{ij}^{TzL}\ \tilde{A}_{ij}^{TzL},\ i \in \{1,2,\dots,n\}$$

Where $\tilde{A}_{ij}^{TzL}$ is the cofactor of $\tilde{a}_{ij}^{TzL}$.

Thus the determinant is the sum of the products of the elements of any row (or column) and the cofactor of the corresponding elements of the same row (or column).

Definition 4.5 Adjoint

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square Trapezoidal fuzzy matrix of order n. Find the cofactor $\tilde{A}_{ij}^{TzL}$ of every elements $\tilde{a}_{ij}^{TzL}$ in A and replace every $\tilde{a}_{ij}^{TzL}$ by its cofactor $\tilde{A}_{ij}^{TzL}$ in A and let it be B. i.e, B = $(\tilde{A}_{ij}^{TzL})$. Then the transpose of B is called the adjoint or adjugate of A and is denoted by adjA.

ie, $B' = \left(\tilde{A}_{ji}^{TzL}\right) = adjA$.

V. SOME PROPERTIES OF ADJOINT OF TRAPEZOIDAL FUZZY MATRIX

In this section, we introduced the property of ATrFM.

5.1 Properties of ATrFM (Adjoint of Trapezoidal Fuzzy matrix)

Property 5.1.1:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ is a square TrFM then $A(adjA)$ is a diagonal-equivalent TrFM.

Proof:

Let $A = \begin{pmatrix} \tilde{a}_{11}^{TzL} & \tilde{a}_{12}^{TzL} \\ \tilde{a}_{21}^{TzL} & \tilde{a}_{22}^{TzL} \end{pmatrix}$

Then $adjA = \begin{pmatrix} \tilde{A}_{11}^{TzL} & \tilde{A}_{21}^{TzL} \\ \tilde{A}_{12}^{TzL} & \tilde{A}_{22}^{TzL} \end{pmatrix}$

Now $A(adjA) = \begin{pmatrix} \tilde{a}_{11}^{TzL} & \tilde{a}_{12}^{TzL} \\ \tilde{a}_{21}^{TzL} & \tilde{a}_{22}^{TzL} \end{pmatrix} \begin{pmatrix} \tilde{A}_{11}^{TzL} & \tilde{A}_{21}^{TzL} \\ \tilde{A}_{12}^{TzL} & \tilde{A}_{22}^{TzL} \end{pmatrix}$.

$$= \begin{pmatrix} \tilde{a}_{11}^{TzL}\tilde{A}_{11}^{TzL} + \tilde{a}_{12}^{TzL}\tilde{A}_{12}^{TzL} & \tilde{a}_{11}^{TzL}\tilde{A}_{21}^{TzL} + \tilde{a}_{12}^{TzL}\tilde{A}_{22}^{TzL} \\ \tilde{a}_{21}^{TzL}\tilde{A}_{11}^{TzL} + \tilde{a}_{22}^{TzL}\tilde{A}_{12}^{TzL} & \tilde{a}_{21}^{TzL}\tilde{A}_{21}^{TzL} + \tilde{a}_{22}^{TzL}\tilde{A}_{22}^{TzL} \end{pmatrix}$$

$$= \begin{pmatrix} \tilde{a}_{11}^{TzL}\tilde{A}_{11}^{TzL} + \tilde{a}_{12}^{TzL}\tilde{A}_{12}^{TzL} & \tilde{a}_{11}^{TzL}(-\tilde{a}_{12}^{TzL}) + \tilde{a}_{12}^{TzL}\tilde{a}_{11}^{TzL} \\ \tilde{a}_{21}^{TzL}\tilde{a}_{22}^{TzL} + \tilde{a}_{22}^{TzL}(-\tilde{a}_{21}^{TzL}) & \tilde{a}_{21}^{TzL}\tilde{A}_{21}^{TzL} + \tilde{a}_{22}^{TzL}\tilde{A}_{22}^{TzL} \end{pmatrix}$$

$$= \begin{pmatrix} |A| & \tilde{0}^{TzL} \\ \tilde{0}^{TzL} & |A| \end{pmatrix}$$

Which is a diagonal-equivalent TrFM

Property 5.1.2:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ is a square TrFM then $(adjA)A$ is a diagonal-equivalent TrFM.

Proof:

Let $A = \begin{pmatrix} \tilde{a}_{11}^{TzL} & \tilde{a}_{12}^{TzL} \\ \tilde{a}_{21}^{TzL} & \tilde{a}_{22}^{TzL} \end{pmatrix}$

Then $adjA = \begin{pmatrix} \tilde{A}_{11}^{TzL} & \tilde{A}_{21}^{TzL} \\ \tilde{A}_{12}^{TzL} & \tilde{a}_{22}^{TzL} \end{pmatrix}$

Now $(adjA)A = \begin{pmatrix} \tilde{A}_{11}^{TzL} & \tilde{A}_{21}^{TzL} \\ \tilde{A}_{12}^{TzL} & \tilde{a}_{22}^{TzL} \end{pmatrix}\begin{pmatrix} \tilde{a}_{11}^{TzL} & \tilde{a}_{12}^{TzL} \\ \tilde{a}_{21}^{TzL} & \tilde{a}_{22}^{TzL} \end{pmatrix}$.

$= \begin{pmatrix} \tilde{A}_{11}^{TzL}\tilde{a}_{11}^{TzL} + \tilde{A}_{21}^{TzL}\tilde{a}_{21}^{TzL} & \tilde{A}_{11}^{TzL}\tilde{a}_{12}^{TzL} + \tilde{A}_{21}^{TzL}\tilde{a}_{22}^{TzL} \\ \tilde{A}_{12}^{TzL}\tilde{a}_{11}^{TzL} + \tilde{A}_{22}^{TzL}\tilde{a}_{21}^{TzL} & \tilde{A}_{12}^{TzL}\tilde{a}_{12}^{TzL} + \tilde{A}_{22}^{TzL}\tilde{a}_{22}^{TzL} \end{pmatrix}$

$= \begin{pmatrix} \tilde{A}_{11}^{TzL}\tilde{a}_{11}^{TzL} + \tilde{A}_{21}^{TzL}\tilde{a}_{21}^{TzL} & \tilde{a}_{22}^{TzL}\tilde{a}_{12}^{TzL} + (-\tilde{a}_{12}^{TzL})\tilde{a}_{22}^{TzL} \\ (-\tilde{a}_{21}^{TzL})\tilde{a}_{11}^{TzL} + \tilde{a}_{11}^{TzL}\tilde{a}_{21}^{TzL} & \tilde{A}_{12}^{TzL}\tilde{a}_{12}^{TzL} + \tilde{A}_{22}^{TzL}\tilde{a}_{22}^{TzL} \end{pmatrix}$

$= \begin{pmatrix} |A| & \tilde{0}^{TzL} \\ \tilde{0}^{TzL} & |A| \end{pmatrix}$

Which is a diagonal-equivalent TrFM

Property 5.1.3**:**

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ is a square TrFM of order2, then $A(adjA) = (adjA)A\ c = \check{R}(|A|)I_2$.

Proof:

By property:1, we have

$A(adjA) = \begin{pmatrix} |A| & \tilde{0}^{TzL} \\ \tilde{0}^{TzL} & |A| \end{pmatrix}$

$c = \begin{pmatrix} \check{R}|A| & \tilde{0}^{TzL} \\ \tilde{0}^{TzL} & \check{R}|A| \end{pmatrix}$

$= \check{R}(|A|)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \check{R}(|A|)I_2$

i.e., $A(adjA) = \check{R} \longrightarrow (*)$

$(adjA)A = \begin{pmatrix} |A| & \tilde{0}^{TzL} \\ \tilde{0}^{TzL} & |A| \end{pmatrix}$

$c = \begin{pmatrix} \check{R}|A| & \tilde{0}^{TzL} \\ \tilde{0}^{TzL} & \check{R}|A| \end{pmatrix}$

$= \check{R}(|A|)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \check{R}(|A|)I_2$

i.e.,$A(adjA) = \check{R}(|A|)I_2 \longrightarrow (**)$

From $(*)$ and $(**)$, we have

$$A(adjA) = (adjA)A\ c = \check{R}(|A|)I_2$$

Property 5.1.4:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ is a square TrFM of order n. If A contains a row with zero-equivalent trapezoidal fuzzy numbers then $(adjA)A$ is a null-equivalent TrFM.

Proof:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square TrFM of order n and let $B = \left(\tilde{b}_{ij}^{TzL}\right) = adjA$. Then by the definition of adjoint matrix, th ij^{th} element $\tilde{b}_{ij}^{TzL}$of B is $\tilde{A}_{ji}^{TzL}$, where $\tilde{A}_{ji}^{TzL}$ is the cofactor of $\tilde{a}_{ji}^{TzL}$ in A which is obtained by deleting the j^{th} row and the i^{th} column from A. without loss of generality we assume that the r^{th} row of A be $\tilde{0}^{TzL}$. Therefore all the element $\tilde{a}_{rj}^{TzL} = \tilde{0}^{TzL}$ for all j. We know that if all the elements of a row or (column) of A are $\tilde{0}^{TzL}$ then $|A|$ is also $\tilde{0}^{TzL}$. Hence all the element of $adjA$ are $\tilde{A}_{ji}^{TzL} = \tilde{0}^{TzL}$ except $j \neq r$.

Let . $C = (adjA)A$. The the ij^{th} elemet $\tilde{c}_{ij}{}^{TzL}$ of C is

$$\tilde{c}_{ij}^{TzL} = \sum_{k=1}^{n} \tilde{A}_{ik}^{TzL}\ \tilde{a}_{kj}^{TzL}$$
$$= \sum_{k\neq r} \tilde{A}_{ik}^{TzL}\ \tilde{a}_{kj}^{TzL} + \tilde{A}_{ir}^{TzL}\ \tilde{a}_{rj}^{TzL}$$

Now all $\tilde{A}_{ik}^{TzL} = \tilde{0}^{TzL}, k \neq r$ and $\tilde{a}_{rj}^{TzL} = \tilde{0}^{TzL}$.

Hence $\tilde{c}_{ij}^{TzL} = \tilde{0}^{TzL}$ for all $i, j = 1,2, \dots, n$.

Then $C = (adjA)A$ is a null-equivalent TrFM.

Property 5.1.5:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ is a square TrFM of order n. If A contains a column with zero-equivalent trapezoidal fuzzy numbers then $A(adjA)$ is a null-equivalent TrFM.

Proof:

Let $A = \left(\tilde{a}_{ij}^{TzL}\right)$ be a square TrFM of order n and let $B = \left(\tilde{b}_{ij}^{TzL}\right) = adjA$. Then by the definition of adjoint matrix, th ij^{th} element $\tilde{b}_{ij}^{TzL}$of B is $\tilde{A}_{ji}^{TzL}$, where $\tilde{A}_{ji}^{TzL}$ is the cofactor of $\tilde{a}_{ji}^{TzL}$ in A which is obtained by deleting the j^{th} row and the i^{th} column from A. without loss of generality we assume that the r^{th} column of A be $\tilde{0}^{TzL}$. Therefore all the element $\tilde{a}_{ir}^{TzL} = \tilde{0}^{TzL}$ for all i. We know that if all the elements of a row or (column) of A are $\tilde{0}^{TzL}$ then $|A|$ is also $\tilde{0}^{TzL}$. Hence all the element of $adjA$ are $\tilde{A}_{ij}^{TzL} = \tilde{0}^{TzL}$ except $i \neq r$.

Let . $C = A(adjA)$. The the ij^{th} elemet $\tilde{c}_{ij}^{TzL}$ of C is

$$\tilde{c}_{ij}^{TzL} = \sum_{k=1}^{n} \tilde{A}_{ik}^{TzL}\ \tilde{a}_{kj}^{TzL}$$
$$= \sum_{k\neq r} \tilde{A}_{ik}^{TzL}\ \tilde{a}_{kj}^{TzL} + \tilde{A}_{ir}^{TzL}\ \tilde{a}_{rj}^{TzL}$$

Now all $\tilde{A}_{kj}^{TzL} = \tilde{0}^{TzL}, k \neq r$ and $\tilde{a}_{ir}^{TzL} = \tilde{0}^{TzL}$.

Hence $\tilde{c}_{ij}^{TzL} = \tilde{0}^{TzL}$ for all $i, j = 1,2, \dots, n$.

Then $C = A(adjA)$ is a null-equivalent TrFM.

Property 5.1.6:

Let $A = (\tilde{a}_{ij}^{TzL})$ be a square TrFM of order n. If A is symmetric TrFM then $adjA$ is also Symmetric TrFM.

Proof:

Let $A = (\tilde{a}_{ij}^{TzL})$ be a symmetric TrFM and let $B = (\tilde{b}_{ij}^{TzL}) = adjA$. Then by definition of adjoit matrix the ij^{th} element $\tilde{b}_{ij}^{TzL}$ of B is $\tilde{A}_{ji}^{TzL}$, where $\tilde{A}_{ji}^{TzL}$ is the cofactor of $\tilde{a}_{ji}^{TzL}$ in A.

Since A is symmetric TrFM,

$\tilde{a}_{ij}^{TzL} = \tilde{a}_{ji}^{TzL}$ for all $i, j = 1,2, \dots, n$.

Hence $\tilde{A}_{ij}^{TzL} = \tilde{A}_{ji}^{TzL}$ for all $i, j = 1,2, \dots, n$.

i.e., $\tilde{b}_{ji}^{TzL} = \tilde{b}_{ij}^{TzL}$ for all $i, j = 1,2, \dots, n$.

i.e., $B = (\tilde{b}_{ij}^{TzL})$ is a symmetric TrFM.

Property 5.1.7:

Let $A = (\tilde{a}_{ij}^{TzL})$ be a square TrFM of order n. If A is null-equivalet TrFM then $adjA$ is also null-equivalent TrFM.

Proof:

Let $A = (\tilde{a}_{ij}^{TzL})$ be a null-equivalent TrFM and let $B = (\tilde{b}_{ij}^{TzL}) = adjA$.

Then by $\tilde{b}_{ij}^{TzL} = \tilde{A}_{ji}^{TzL}$, where $\tilde{A}_{ji}^{TzL}$ is the cofactor of $\tilde{a}_{ji}^{TzL}$ in A.

Since A is null-equivalent TrFM, all

$\tilde{a}_{ij}^{TzL} = \tilde{0}^{TzL}$ for all $i, j = 1,2, \dots, n$.

Hence $\tilde{A}_{ij}^{TzL} = \tilde{0}^{TzL}$ for all $i, j = 1,2, \dots, n$.

i.e., $\tilde{b}_{ji}^{TzL} = \tilde{0}^{TzL}$ for all $i, j = 1,2, \dots, n$.

i.e., $B = (\tilde{b}_{ij}^{TzL}) = adjA$ is also null-equivalent TrFM.

Property 5.1.8:

For a square TrFM of order n, $adj(A') = (adjA')$.

Proof:

Let $A = (\tilde{a}_{ij}^{TzL})$ be a square TrFM of order n. Then by definitions $adjA = (\tilde{A}_{ji}^{TzL})$.

Hence $(adjA)' = (\tilde{A}_{ij}^{TzL}) \longrightarrow (***)$

Also,

$A' = (\tilde{a}_{ji}^{TzL})$. Now, $adj(A') = (\tilde{A}_{ij}^{TzL}) \longrightarrow (****)$

From $(***)$ and $(****)$, we have $adj(A') = (adjA)'$.

Property 5.1.9:

If A is an unit-equivalet TrFM of order n, then $adjA$ is also an unit-equivalent TrFM of order n .

Proof:

Let $A = (\tilde{a}_{ij}^{TzL})$ be an unit-equivalent TrFM of order n. Then all the entries in the principal diagonal $\tilde{a}_{ii}^{TzL} = \tilde{1}^{TzL}$ and the remaining entries outside the principal diagonal are $\tilde{0}^{TzL}$.

Hence the cofactor $\tilde{A}_{ij}^{TzL}$ of every of A is as in the Following manner:

$\tilde{A}_{ij}^{TzL} = \tilde{0}^{TzL}$ for all $i \neq j$ and $\tilde{A}_{ii}^{TzL} = |\hat{1}| = \tilde{1}^{TzL}$.

Hence $adjA = (\tilde{A}_{ji}^{TzL})$ is also an unit-equivalent TrFM of order n.

Property 5.1.10:

If A is an unit TrFM of order n, then $adjA$ is also an unit TrFM of order n .

Proof:

Let $A = (\tilde{a}_{ij}^{TzL})$ be an unit TrFM of order n. Then all the entries in the principal diagonal $\tilde{a}_{ii}^{TzL} = 1$ and the remaining entries outside the principal diagonal are 0.

Hence the cofactor $\tilde{A}_{ij}^{TzL}$ of every of A is as in the following manner:

$\tilde{A}_{ij}^{TzL} = \tilde{0}^{TzL}$ for all $i \neq j$ and $\tilde{A}_{ii}^{TzL} = |1|$ of order $n - 1$. .

We know that $|I| = 1$. Therefore $\tilde{A}_{ii}^{TzL} = 1$.

Hence $adjA = (\tilde{A}_{ji}^{TzL})$ is also an unit TrFM of order n.

VI NUMERICAL EXAMPLES

If $A = \begin{bmatrix} (-1,2,4,7) & (-1,1,2,6) \\ (2,3,4,7) & (-2,-1,3,4) \end{bmatrix}$

$|A| = [(-1,2,4,7) - (4,6,8,14)]$
$= [(-15,-6,-2,3)]$

Now $\hat{R}(|A|) = -\frac{20}{4}$

$\hat{R}(|A|) = -5$.

Also ,

$adj\ A = \begin{bmatrix} (-2,-1,3,4) & (-6,-2,-1,1) \\ (-7,-4,-3,-2) & (-1,2,4,7) \end{bmatrix}$

$A(adj\ A) =$

$$\begin{bmatrix} (-1,2,4,7) & (-1,1,2,6) \\ (2,3,4,7) & (-2,-1,3,4) \end{bmatrix}\begin{bmatrix} (-2,-1,3,4) & (-6,-2,-1,1) \\ (-7,-4,-3,-2) & (-1,2,4,7) \end{bmatrix}$$

$$= \begin{bmatrix} [(-1,2,4,7)+(-24,-8,-4,4)] & [(-14,-8,-4,2)+(-3,3,6,18)] \\ [(2,3,4,7)+(-16,-12,4,8)] & [(-14,-8,-6,-4)+(-6,-3,9,12)] \end{bmatrix}$$

$$= \begin{bmatrix} (-25,-6,0,11) & (-17,-5,2,20) \\ (-14,-9,8,15) & (-20,-11,3,8) \end{bmatrix}$$

$$c = \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$$

$$= -5\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

i.e. $A(adj\ A) = \hat{R}(|A|)I_2$

$(adj\, A)A$

$$= \begin{bmatrix} (-2,-1,3,4) & (-6,-2,-1,1) \\ (-7,-4,-3,-2) & (-1,2,4,7) \end{bmatrix}\begin{bmatrix} (-1,2,4,7) & (-1,1,2,6) \\ (2,3,4,7) & (-2,-1,3,4) \end{bmatrix}$$

$$= \begin{bmatrix} [(-6,-3,9,12)+(-24,-8,-4,4)] & [(-4,-2,6,8)+(-6,-2,-1,1)] \\ [(-21,-12,-9,-6)+(-4,8,16,28)] & [(-14,-8,-6,-4)+(-1,2,4,7)] \end{bmatrix}$$

$$= \begin{bmatrix} (-30,-11,5,16) & (-10,-4,5,9) \\ (-25,-4,7,22) & (-15,-6,-2,3) \end{bmatrix}$$

$$c = \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$$

$$= -5\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

i.e., $(adj\ A)\ A = \hat{R}(|A|)I_2$

VII. CONCLUSION

In this article, We have concentrate the notion of the adjoint of trapezoidal fuzzy matrices are defined and also some special properties of adjoint of TrFMs are proved. Using theAdjoint of Trapezoidal fuzzy Matrix work can be extended to the another domain of inverse of matrices is discuss in future. Also the theories of the discussed TrFMs may be utilized in further works.

REFERENCES

[1] Dubasis.D and Prade.H., Operations on fuzzy numbers. International journal of systems, vol.9(6), 1978, 613-626.

[2] Dwyer.P.S., Fuzzy sets information and control, 8, 338-353

[3] Hisdal.E., The IF THEN ELSE statement and International-valued Fuzzy sets of Higher type. Int.J.Man-Machine studie 15(1981),PP 384-455.

[4] Hasimato.H., Convergence of powers of fuzzy transitive matrix, Fuzzy Sets and Systems, 9(1983)., PP.153-160.

[5] Jhon., Type-2 Fuzzy sets, an Appraisal of Theory and Application, Int.J.Fuzziness Knowledge- Based Systems 6(6) (1998)., PP 563-576.

[6] Kim.J.B., Inverse of Boolean Matrices, Bull.Inst.Math.Acod, Science 12(2)(1984), PP 125-128.

[7] Kim.J.B, Determinant theory for fuzzy and Boolean matrices, Congressus numerantium, 1988, 273-276.

[8] Kim.J.B., A Baartmans Determina Theory for Fuzzy Matrices, Fuzzy Sets and Systems, 29(1989), PP 349-356.

[9] Kandel.A, Fuzzy Mathematical Techniques with Applications, Addition Wisley, Tokyo,1996

[10] Mamoni Dhar., A Note on Determinant and Adjoint of Fuzzy Square Matrix, I.J.Intelligent Systems and Applications, Vol 05., 2013., PP 58-67.

[11] Overhinniko.S.V., Structure of fuzzy relations, Fuzzy Sets and Systems, Vol 6, 1981, PP 169-195.

[12] Shyamal`A.K and Pal.M., Two new operators on fuzzy matrices, J.Applied Mathematics and computing 13, 2004, 91- 107.

[13] Shyamal`A.K and Pal.M., Distance between fuzzy matrices and its applications, Actasiencia Indica, XXXI- M(1),2005, 199-204.

[14] Shyamal.A.K and Pal.M., Triangular fuzzy matrices Iranian journal of fuzzy systems,volume- 4, No-1, 2007, 75-87.

[15] Stephen.D and Anbalagan.A., Fuzzy Programming based on Type-2 Generalized Fuzzy Numbers, International Journal of Mathematical Sciences and Engineering Applications, Vol.5, No.IV, 317-329.

[16] Stephen.D and Latha.K A Study On Adjoint of Type-2 Triangular Fuzzy Matrices, International Journal of Current Research .,Vol-6.,Issue-06., PP-7064-7072.

[17] Thomson.M.G., Convergence of power of the fuzzy matrix, J.Math, Anal.Appl, 57, 1997, 476-2480.

[18] Zadeh.L.A., Fuzzy sets Information and control., 8, 1965, 338-353.

[19] Zadeh.L.A., Fuzzy set as a basis for a theory of possibility, fuzzy sets and systems,1,1978,3- 28.