Homomorphism and anti homomorphism in Intuitionistic fuzzy ideal of M*Γ* group in near rings

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***Abstract* - In this paper, we study the effects of homomorphism and anti homomorphism on the domain and codomain of Intuitionistic fuzzy ideal of M*Γ* group in near rings are explained by few theorems.**

***Index Terms* - Intuitionistic fuzzy ideals of M*Γ* group in near rings, homomorphism and anti homomorphism.**

1.INTRODUCTION

Atanassov K. T introduced intuitionistic fuzzy sets in 1986. This is as an extension of fuzzy sets which was introduced by Zadeh L. A in 1965.The abstract concept of near rings developed by Pilz G., later expanded into fuzzy near rings and intuitionistic fuzzy near rings. Jun Y. B studied fuzzy *Γ* rings in 1992 and fuzzy M*Γ* group elaborately in 1995. Kim S. D analyzed fuzzy ideals of near rings in 1996. Later the characteristic of intuitionistic fuzzy ideals in *Γ* rings are discussed by Palaniappan N in 2010. Sathyanarayana. B studied fuzzy ideals over near rings along wih their properties and represented it as a graph. Intuitionistic fuzzy ideals of M*Γ* group was introduced. Their homomorphisms with properties and effects are discussed in this paper. Saravanan. V defined and explained homomorphism and anti-homomorphism in intuitionistic fuzzy sub semi ring of a semi ring.

2. PRELIMINARIES

2.1 Definition:

Let (N*, +) be a group and Γ be a non-empty set the N* is called a Γ near ring if there exists a function from N* x Γ x N* → N* satisfying

1. $(n_1 + n_2)\, \alpha_1\, n_3 = n_1\, \alpha_1\, n_3 + n_2\, \alpha_1\, n_3$
2. $(n_1\, \alpha_1\, n_2)\, \alpha_2\, n_3 = n_1\, \alpha_1\, (n_2\, \alpha_2\, n_3)$ for all $n_1, n_2, n_3 \in$ N* and $\alpha_1, \alpha_2 \in \Gamma$.

2.2 Definition:

Let N* be a zero symmetric gamma near ring and μ* defined from N* to [0, 1] is said to be a fuzzy ideal of N* if it satisfied

1. $\mu^* (n_1 + n_2) \geq \min (\mu^* (n_1), \mu^* (n_2))$
2. $\mu^* (-n_1) \geq \mu^* (n_1)$
3. $\mu^* (n_1) = \mu^* (n_2 + n_1 - n_2)$
4. $\mu^* (n_1\, \alpha\, n_2) \geq \mu^* (n_1)$ and
5. $\mu^* (n_1\, \alpha\, (n_2 + n_3) - n_1\, \alpha\, n_2) \geq \mu^* (n_3)$ for all $n_1, n_2, n_3 \in N$ and $\alpha \in \Gamma$.

2.3 Definition:

A fuzzy mapping μ*: G* → [0, 1] is said to be a fuzzy ideal of G* if it satisfies

1. $\mu^* (n_1 + n_2) \geq \min (\mu^* (n_1), \mu^* (n_2))$
2. $\mu^* (n_2 + n_1 - n_2) \geq \mu^* (n_2)$
3. $\mu^* (n_1) = \mu^* (-n_1)$
4. $\mu^* (a\, \alpha\, (n_1 + n_2) - a\, \alpha\, n_1) \geq \mu^* (n_2)$ for all $n_1, n_2 \in$ G*, a ϵ N* and $\alpha \in \Gamma$.

Remark:

If μ* satisfies (i), (ii) and (iii) condition then μ* is a fuzzy normal MΓ subgroup of G*.

2.4 Definition:

An intuitionistic fuzzy set I (μ_I, γ_I) of the near ring N* is called an intuitionistic fuzzy ideal of N* if for all $n_1, n_2, n \in$ N*

1. $\mu_I (n_1 + n_2) \geq \min (\mu_I (n_1), \mu_I (n_2))$
2. $\mu_I (nn_1) \geq \mu_I (n_1)$
3. $\mu_I (n_2 + n_1 - n_2) \geq \mu_I (n_2)$
4. $\mu_I (n (n_1 + n_2) - nn_1) \geq \mu_I (n_2)$
5. $\gamma_I (n_1 - n_2) \leq \max (\gamma_I (n_1), \gamma_I (n_2))$
6. $\gamma_I (nn_1) \leq \gamma_I (n_1)$
7. $\gamma_I (n_2 + n_1 - n_2) \leq \gamma_I (n_1)$
8. $\gamma_I (n (n_1 + n_2) - nn_1) \leq \gamma_I (n_2)$

2.5 Definition:

If I is said to be an intuitionistic fuzzy ideal of G*in N* if μ_I: G*→ [0,1] and

γ_I: G*→ [0,1] satisfying the following properties.

a. $\mu_I (x + y) \geq \min \{\mu_I (x), \mu_I (y)\}$

b. $\mu_I (x + y - x) \geq \mu_I (y)$

c. $\mu_I (x) = \mu_I (-x)$

d. $\mu_I (n\alpha (a + x) - n \alpha a) \geq \mu_I (x)$

e. $\gamma_I (x + y) \leq \max \{\gamma_I (x), \gamma_I (y)\}$

f. $\gamma_I (x + y - x) \leq \gamma_I (y)$

g. $\gamma_I (x) = \gamma_I (- x)$

h. $\gamma_I (n \alpha (a + x) - n \alpha a) \leq \gamma_I (x)$ for all $n \in N^*$, $\alpha \in \Gamma$, a, x, y $\in$ I.

3. HOMOMORPHISM AND ANTI-HOMOMORRPHISM

3.1 Theorem:

Let (N_1^*, +, .) and (N_2^*, +, .) be any two near rings. Then the homomorphism image of an IFI of N_1^* is an IFI of N_2^*.

Proof:

Let f: $N_1^* \to N_2^*$ be a homomorphism. Then

$f (x_1 + x_2) = f (x_1) + f (x_2)$ and $f (x_1x_2) = f (x_1) f (x_2)$ for all x_1, x_2 in N_1^*.

Let $I_2 = f (I_1)$ where I_1 is an IFI of N_1^*.

Then $f (x_1)$, $f (x_2)$ are in N_2^*.

1. Consider $\mu_{I2} (f (x_1) + f (x_2)) = \mu_{I1}\ x_1 + x_2)$

$\geq \min \{\mu_{I1} (x_1), \mu_{I1} (x_2)\}$

$= \min \{\mu_{I1} f (x_1), \mu_{I2} f (x_2)\}$

$\mu_{I2} (f (x_1) + f (x_2)) \geq \min \{\mu_{I2} f (x_1), \mu_{I2} f (x_2)\}$

2. Consider $\mu_{I2} (f (x_1) + f (x_2) - f (x_1)) = \mu_{I1} (x_1 + x_2 - x_1)$

$\geq \mu_{I1} (x_2)$

$= \mu_{I2} f (x_2)$

$\mu_{I2} (f (x_1) + f (x_2) - f (x_1)) \geq \mu_{I2} f (x_2)$.

3. Consider $\mu_{I2} (f (x_1)) = \mu_{I1} (x_1)$

$= \mu_{I1} (-x_1)$

$= \mu_{I2} (-f (x_1))$

$\mu_{I2} (f (x_1)) = \mu_{I2} (-f (x_1))$

4. Consider $\mu_{I2} (f (n) \alpha (f (x_1) + f (x_2)) - f (n) \alpha f(x_1)$

$\mu_{I1} (n \alpha (x_1 + x_2) - n\alpha x_1)$

$\geq \mu_{I1} (x_2)$

$= \mu_{I2} (f (x_2))$

$\Longrightarrow \mu_{I2} (f (n) \alpha (f (x_1) + f (x_2)) \geq \mu_{I2} (f (x_2))$.

5. Consider $\gamma_{I2} (f (x_1) + f (x_2)) = \gamma_{I1} (x_1 + x_2)$

$\leq \max \{\gamma_{I1} (x_1), \gamma_{I1} (x_2)\}$

$= \max \{\gamma_{I1} f (x_1), \gamma_{I2} f (x_2)\}$

$\Longrightarrow \gamma_{I2} (f (x_1) + f (x_2)) \leq \max \{\gamma_{I2} f (x_1), \gamma_{I2} f (x_2)\}$

6. Consider $\gamma_{I2} (f (x_1) + f (x_2) - f (x_1)) = \gamma_{I1} (x_1 + x_2 - x_1)$

$\leq \gamma_{I1} (x_2)$

$= \gamma_{I2} f (x_2)$

$\Longrightarrow \gamma_{I2} (f (x_1) + f (x_2) - f (x_1)) \geq \gamma_{I2} f (x_2)$.

7. Consider $\gamma_{I2} (f (x_1)) = \gamma_{I1} (x_1)$

$= \gamma_{I1} (-x_1)$

$= \gamma_{I2} (-f (x_1))$

$\Longrightarrow \gamma_{I2} (f (x_1)) = \gamma_{I2} (-f (x_1))$

8. Consider $\gamma_{I2} (f (n) \alpha (f (x_1) + f (x_2)) - f (n) \alpha f(x_1)$

$= \gamma_{I1} (n \alpha (x_1 + x_2) - n\alpha x_1)$

$\leq \gamma_{I1} (x_2)$

$= \gamma_{I2} (f (x_2))$

$\Longrightarrow \gamma_{I2} (f (n) \alpha (f (x_1) + f (x_2)) \geq \gamma_{I2} (f (x_2))$.

Hence I_2 is an IFI of N_2^*.

3.2 Theorem:

Let (N_1^*,+, .) and (N_2^*, +,.) be any two near rings. The homomorphic preimage of an intuitionistic fuzzy ideal of N_2^* is an intuitionistic fuzzy ideal of N_1^*.

Proof:

Let f: $N_1^* \to N_2^*$ be a homeomorphism. Then

$f (x_1 + x_2) = f (x_1) + f (x_2)$ and $f (x_1x_2) = f (x_1) f (x_2)$ for all x_1, x_2 in N_1^*.

Let $I_2 = f (I_1)$ where I_2 is an IFI of N_2^*.

1. Consider $\mu_{I1} (x_1 + x_2) = \mu_{I2} (f (x_1) + f (x_2))$

$\geq \min \{\mu_{I2} (f (x_1), \mu_{I2} (f (x_2))\}$

$= \min \{\mu_{I1} (x_1), \mu_{I1} (x_2)\}$

$\mu_{I1} (x_1 + x_2) \geq \min \{\mu_{I1} (x_1), \mu_{I1} (x_2)\}$

2. Consider $\mu_{I1} (x_1 + x_2 - x_1) = \mu_{I2} [f (x_1) + f (x_2) - f (x_1)]$

$\geq \mu_{I2} (f (x_2))$

$= \mu_{I1} (x_2)$

$\Longrightarrow \mu_{I1} (x_1 + x_2 - x_1) \geq \mu_{I1} (x_2)$

3. Consider $\mu_{I1} (-x_1) = \mu_{I2} (-f (x_1))$

$= \mu_{I2} (+f (x_1))$

$= \mu_{I1} (x_1)$

$\mu_{I1} (x_1) = \mu_{I1} (-x_1)$

4. Consider $\mu_{I1} (n \alpha (x_1 + x_2) - n\alpha x_1)$

$= \mu_{I2} (f (n) \alpha (f (x_1) + f (x_2)) - f (n) \alpha f (x_1))$

$\geq \mu_{I2} (f (x_2))$

$= \mu_{I1} (x_2)$.

5. Consider $\gamma_{I1} (x_1 + x_2) = \gamma_{I2} (f (x_1) + f (x_2))$

$\leq \max \{\gamma_{I2} (f (x_1), \gamma_{I2} (f (x_2))\}$

$= \max \{\gamma_{I1} (x_1), \gamma_{I1} (x_2)\}$

$\Longrightarrow \gamma_{I1} (x_1 + x_2) \leq \max \{\gamma_{I1} (x_1), \gamma_{I1} (x_2)\}$

6. Consider $\gamma_{I1} (x_1 + x_2 - x_1) = \gamma_{I2} [f (x_1) + f (x_2) - f (x_1)]$

$\leq \gamma_{I2} (f (x_2))$

$= \gamma_{I1} (x_2)$

$\gamma_{I1} (x_1 + x_2 - x_1) \leq \gamma_{I1} (x_2)$

7. Consider $\gamma_{I1} (-x_1) = \gamma_{I2} (-f (x_1))$

$= \gamma_{I2}$ (+f (x_1))
$= \gamma_{I1}$ (x_1)
$\Longrightarrow \gamma_{I1}$ (x_1) = γ_{I1} (-x_1)
8. Consider γ_{I1} (n α (x_1 + x_2) – nαx_1)
= γ_{I2} (f (n) α (f (x_1) + f (x_2)) – f (n) α f (x_1))
≥ γ_{I2} (f (x_2))
= γ_{I1} (x_2).
$\Longrightarrow \gamma_{I1}$ (n α (x_1 + x_2) – nαx_1) ≤ γ_{I1} (nαx_2)
Therefore f (I_1) = I_2 is an IFI. ⟹ I_1 is an IFI.

3.3 Theorem:
Let (N_1^*,+, .) and (N_2^*, +,.) be any two near rings. The anti homomorphic image of an IFI of N_1^* is an IFI of N_2^*.
Proof:
Let f: $N_1^* \rightarrow N_2^*$ be a homeomorphism. Then
f (x_1 + x_2) = f (x_1) + f (x_2) and f (x_1x_2) = f (x_1) f (x_2) for all x_1, x_2 in N_1^*.
Let I_2 = f (I_1) where I_1 is an IFI of N_1^*.
Let f (x_1), f (x_2) ∈ N_2^* for x_1, x_2 ∈ N_1^*.
1. Consider μ_{I2} (f (x_1) + f (x_2)) = μ_{I2} [f (x_2 + x_1)]
= μ_{I1} (x_1 + x_2)
≥ min {μ_{I1} (x_2), μ_{I1} (x_2)}
= min {μ_{I1} (x_1), μ_{I2} (x_2)}
= min {μ_{I2} (f (x_1)), μ_{I2} (f (x_2))}
μ_{I2} (f (x_1) + f (x_2)) ≥ min {μ_{I2} (f (x_1), μ_{I2} (f (x_2))}
2. Consider μ_{I2} (f (x_1) + f (x_2) – f (x_1))
= μ_{I2} [f (x_2 + x_1) + f (-x_1)]
= μ_{I2} [f (-x_1 + x_2 + x_1)]
= μ_{I1} (-x_1 + x_2 + x_1)
≥ μ_{I1} (x_2) = μ_{I2} (f (x_2))
⟹ μ_{I2} [f (x_1) + f (x_2) – f (x_2)] ≥ μ_{I2} (f (x_2))
3. Consider μ_{I2} (f (x_1)) = - μ_{I1} (x_1)
= μ_{I1} (-x_1)
= μ_{I2} (- f (x_1))
⟹ μ_{I2} (- f (x_1)) = - μ_{I2} (f (x_1))
4. Consider μ_{I2} [f (n) α (f (x_1) + f (x_2)) – f (n) α f (x_1)]
= μ_{I2} [f (n) α f (x_2 + x_1) – f (n) α f (x_1)]
= μ_{I1} [n α (x_2 + x_1) – nαx_1]
≥ μ_{I1} (x_2)
= μ_{I2} (f (x_2))
⟹ μ_{I2} [f (n) α (f (x_1) + f (x_2)) – f (n) α f (x_1)] ≥ μ_{I2} (f (x_2))
5. Consider γ_{I2} (f (x_1) + f (x_2)) = γ_{I2} [f (x_2 + x_1)]
= γ_{I1} (x_1 + x_2)
≤ max {γ_{I1} (x_2), γ_{I1} (x_2)}
= max {γ_{I1} (x_1), γ_{I2} (x_2)}
= max {γ_{I2} (f (x_1)), γ_{I2} (f (x_2))}
⟹ γ_{I2} (f (x_1) + f (x_2)) ≤ max {γ_{I2} (f (x_1), γ_{I2} (f (x_2))}
6. Consider γ_{I2} (f (x_1) + f (x_2) – f (x_1))
= γ_{I1} [f (x_2 + x_1) + f (-x_1)]
= γ_{I1} [f (-x_1 + x_2 + x_1)]
= γ_{I1} (-x_1 + x_2 + x_1)
≥ γ_{I1} (x_2) = γ_{I2} (f (x_2))
⟹ γ_{I2} [f (x_1) + f (x_2) – f (x_2)] ≥ γ_{I2} (f (x_2))
7. Consider -γ_{I2} (f (x_1)) = - γ_{I1} (x_1)
= γ_{I1} (-x_1)
= γ_{I2} (- f (x_1))
⟹ γ_{I2} (- f (x_1)) = - γ_{I2} (f (x_1))
8. Consider γ_{I2} [f (n) α (f (x_1) + f (x_2)) – f (n) α f (x_1)]
= γ_{I2} [f (n) α f (x_2 + x_1) – f (n) α f (x_1)]
= γ_{I1} [n α (x_2 + x_1) – nαx_1]
≤ γ_{I1} (x_2)
= γ_{I2} (f (x_2))
⟹ γ_{I2} [f (n) α (f (x_1) + f (x_2)) – f (n) α f (x_1)] ≤ γ_{I2} (f (x_2))
Hence all the axioms of the IFI of MΓ group in near rings are satisfied by I_2.
⟹ I_2 is an IFI of N_2^*.

3.4 Theorem:
Let (N_1^*,+, .) and (N_2^*, +,.) be any two near rings. The anti homomorphic pre image of an IFI of N_2^* is an IFI of N_1^*.
Proof:
Let f: $N_1^* \rightarrow N_2^*$ be a homeomorphism. Then
f (x_1 + x_2) = f (x_1) + f (x_2) and f (x_1x_2) = f (x_1) f (x_2) for all x_1, x_2 in N_1^*.
Let I_2 = f (I_1) where I_2 is an IFI of N_2^*.
1. μ_{I1} (x_1 + x_2) = μ_{I2} [f (x_1 + x_2)]
≥ min μ_{I2} {f (x_2), f (x_1)}
= min {μ_{I1} (x_2), μ_{I1} (x_1)}
μ_{I1} (x_1 + x_2) ≥ min {μ_{I1} (x_1), μ_{I1} (x_2)}
2. μ_{I1} (x_1 + x_2 - x_1) = μ_{I2} [f (x_1 + x_2 – x_1)]
= μ_{I2} [f (-x_1) + f (x_1 + x_2)]
= μ_{I2} [-f (x_1) + f (x_2) + f (x_1)]
≥ μ_{I2} [f (x_2)] = μ_{I1} (x_2)
⟹ μ_{I1} (x_1 + x_2 – x_1) ≥ μ_{I1} (x_2)
3. - μ_{I1} (x_1) = - μ_{I2} (f (x_1))
= μ_{I2} [- f (x_1)]
= μ_{I1} (- x_1)
μ_{I1} (- x_1) = - μ_{I1} (x_1)
4. μ_{I1} (nα (x_1 + x_2) – nαx_1) = μ_{I2} [f (n) α f (x_1 + x_2) – f (n) α f (x_1)]
= μ_{I2} [f (n) α [f (x_2) + f (x_1)] – f (n) α f (x_1)]
≥ μ_{I2} [f (x_2)]
= μ_{I1} (x_2)

$\Longrightarrow \mu_{I1}$ (nα ($x_1 + x_2$) – nαx_1) ≥ μ_{I1} (x_2)

5. γ_{I1} ($x_1 + x_2$) = γ_{I2} [f ($x_1 + x_2$)]
≤ max γ_{I2} {f (x_2), f (x_1)}
= max {γ_{I1} (x_2), γ_{I1} (x_1)}
γ_{I1} ($x_1 + x_2$) ≤ max {γ_{I1} (x_1), γ_{I1} (x_2)}

6. γ_{I1} ($x_1 + x_2 - x_1$) = γ_{I2} [f ($x_1 + x_2 - x_1$)]
= γ_{I2} [f (-x_1) + f ($x_1 + x_2$)]
= γ_{I2} [-f (x_1) + f (x_2) + f (x_1)]
≤ γ_{I2} [f (x_2)] = μ_{I1} (x_2)
γ_{I1} ($x_1 + x_2 - x_1$) ≤ γ_{I1} (x_2)

7. - γ_{I1} (x_1) = - γ_{I2} (f (x_1))
= γ_{I2} [- f (x_1)]
= γ_{I1} (- x_1)
γ_{I1} (- x_1) = - γ_{I1} (x_1)

8. γ_{I1} (nα ($x_1 + x_2$) – nαx_1) = γ_{I2} [f (n) α f ($x_1 + x_2$) – f (n) α f (x_1)]
= γ_{I2} [f (n) α [f (x_2) + f (x_1)] – f (n) α f (x_1)]
≥ γ_{I2} [f (x_2)]
= γ_{I1} (x_2)
γ_{I1} (nα ($x_1 + x_2$) – nαx_1) ≤ γ_{I1} (x_2)

Hence all the conditions of IFI are satisfied by I_1.
the preimage of IFI by an anti-homomorphism is also an IFI.

4.CONCLUSION

The effects of homomorphism and anti homomorphism on the domain and codomain of Intuitionistic fuzzy ideal of M*Γ* group in near rings are studied by few theorems.

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