GEOMETRIC MEAN LABELING OF LINE GRAPHS

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***Abstract*- In this paper we contribute some new results on Geometric Mean Labeling of graphs. We investigate on some standard graphs that accept Geometric Mean Labeling and proved that the Line Graphs of these Geometric Mean Graphs are also Geometric Mean Graphs. We proved that the Line graphs of Path, Cycle, Star $K_{1,n}$, Comb, $P_n \odot K_{1,2}$ are Geometric Mean Graphs.**

***Index Terms*- Graph, Geometric Mean Graph, Line Graph, Path, Cycle, Star$K_{1,n}$, Comb, $P_n \odot K_{1,2}$**

AMS Subject Classification: 05C78

I. INTRODUCTION

By a graph we mean a finite undirected graph without loops or parallel edges. For all detailed survey of graph labeling, we refer to J.A. Gallian [1]. For all other standard terminology and notations we follow Harary [2].The concept of Mean labeling has been introduced by S.Somasundaram and R.Ponraj [3] in 2004. S.Somasundaram and S.S.Sandhya introduced Harmonic mean labeling [4] in 2012. the basic results on Geometric mean labeling was proved in [5]

In this paper we investigate the Line graphs of some standard Geometric Mean Graphs Path, Cycle, Star $K_{1,n}$, Comb, $P_n \odot K_{1,2}$. We will provide a brief summary of definitions and other information which are necessary for our present investigation.

A Path $\mathbf{P_n}$ is a walk in which all the vertices are distinct. A **Cycle** $\mathbf{C_n}$ is a Closed Path. **A Complete Bipartite** graph $\mathbf{K_{m,n}}$ is a bipartite graph with bipartition $(\mathbf{V_1}, \mathbf{V_2})$ such that every vertex of $\mathbf{V_1}$ is joined to all the vertices of $\mathbf{V_2}$,Where $|\mathbf{V_1}| = \mathbf{m}$ and $|\mathbf{V_2}| = \mathbf{n}$. **A Star** graph is the complete bipartite graph $\mathbf{K_{1,n}}$. The graph obtained by joining a single pendant edge to each vertex of a Path is called a **Comb**. $\boldsymbol{P_n \odot K_{1,2}}$ is a graph obtained by attaching each vertex of P_n to the central vertex of $\boldsymbol{K_{1,2}}$.

Definition 1.1:

A graph **G=(V,E)** with p vertices and q edges is said to be a **Geometric Mean graph** if it is possible to label the vertices $\mathbf{x \in V}$ with distinct labels $\mathbf{f}(\mathbf{x})$ from **1,2,…,q+1** in such a way that when each edge $\mathbf{e = uv}$ is labeled with,

$\mathbf{f}(\mathbf{e = uv}) = \left\lceil \sqrt{f(u)f(v)} \right\rceil$ or $\left\lfloor \sqrt{f(u)\, f(v)} \right\rfloor$

then the edge labels are distinct. In this case **f** is called a **Geometric Mean labeling** of G.

Remark: 1.2

If G is a Geometric Mean graph, then '1' must be a label of one of the vertices of G, Since an edge should get label '1'.

Remark: 1.3

If u gets label '1', then any edge incident with u must get label 1 (or) 2 (or) 3. Hence this vertex must have a degree ≤ 3.

Definition 1.4:

Let G=(V,E) be a non-trivial graph. Now each edge in E can be considered as a set of two elements of V. So E is a non-empty collection of distinct non-empty subsets of v, such that their union is V. So there is a intersection graph Ω(E). Their graph Ω(E) is called the **Line graph** of G and is denoted by **L(G).**

We observe that the vertices of L(G) are the edges of G. Further two vertices of L(G) are adjacent iff their corresponding edges are adjacent in G. Thus the vertices a,b in L(G) are adjacent iff a=uv and b=vw are in G

Theorem 1.5: Any Path $\mathbf{P_n}$ is a Geometric Mean graph.

Theorem 1.6: Any Cycle $\mathbf{C_n}, \boldsymbol{n \geq 3}$ is a Geometric Mean graph.

Theorem 1.7: Star $\mathbf{K_{1,n}}$ is a Geometric Mean graph if and only if $\mathbf{n \leq 5}$.

Theorem 1.8: Any Comb $\mathbf{P_n} \odot \mathbf{K_1}$ is a Geometric Mean graph.

Theorem 1.9: $\mathbf{P_n} \odot \mathbf{K_{1,2}}$ is a Geometric Mean graph.

II. MAIN RESULTS

Theorem: 2.1

Line Graph of Path $\mathbf{P_n}$ is a Geometric Mean graph.

Proof:

Graph G of Path P_n is displayed below.

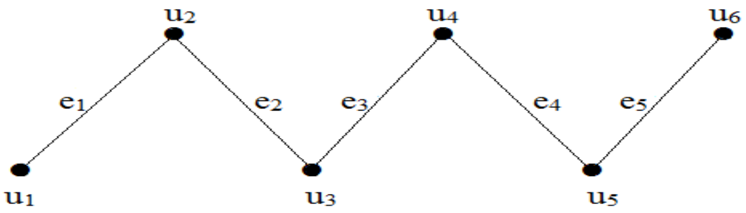

Figure:1

The Line Graph L(G) of Path P_n is shown in figure:2

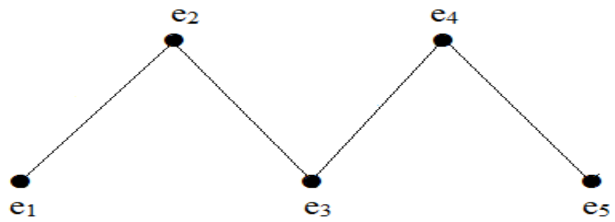

Figure:2

By **Theorem 1.6**, L(G) of Path $\mathbf{P_n}$ is a Geometric Mean Graph.

Theorem: 2.2

Line Graph of Cycle $\mathbf{C_n}$ is a Geometric Mean graph.

Proof:

Graph G of Cycle C_n is displayed below.

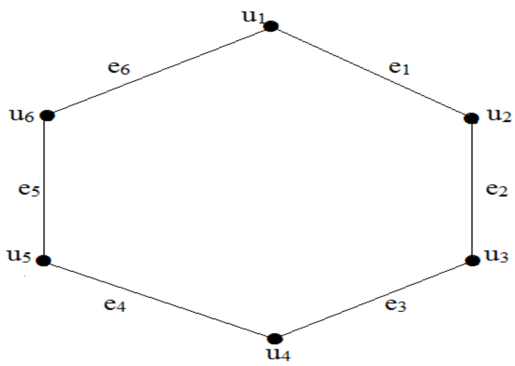

Figure:3

The Line Graph L(G) of Cycle C_n is shown in figure:4

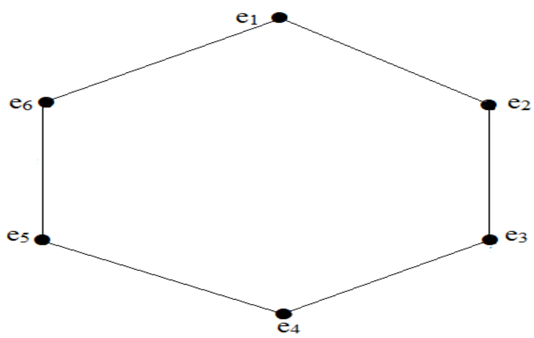

Figure:4

By **Theorem 1.7**, L(G) of Cycle $\mathbf{C_n}$ is a Geometric Mean Graph.

Remark: 2.3

Graph G and Line Graph L(G) of Cycle are isomorphic to each other.

Theorem:2.4

Line Graph of Star $\mathbf{K_{1,n}}$ is a Geometric Mean graph if and only if $\mathbf{n \leq 5}$.

Proof:

Graph G of Star $\mathbf{K_{1,4}}$ is displayed below.

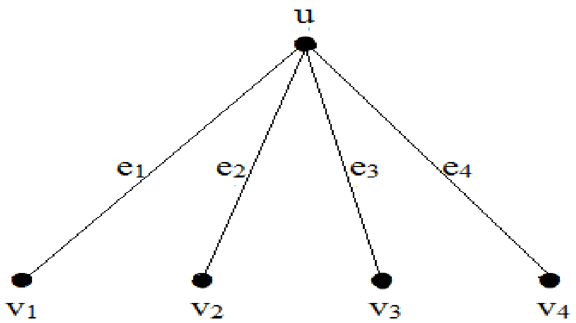

Figure:5

Line Graph L(G) of Star $K_{1,4}$ is shown in figure:6

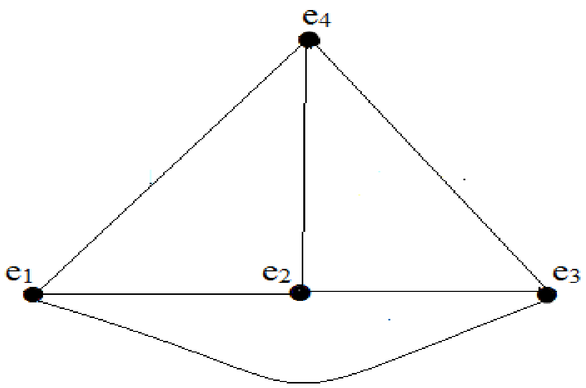

Figure:6

Let u be the central vertex of L(G) and the other vertices be v_1, v_2 and v_3.

Define a function, $f: V(\mathrm{L(G)}) \rightarrow \{1,2,3, \dots\dots, q+1\}$by $f(\mathrm{u}) = 1$

$$f(\mathrm{v_i}) = 3i - 1 , 1 \leq i \leq n-1$$

$$f(\mathrm{v_n}) = \mathrm{f}(\mathrm{v_{n-1}}) + 2$$

Then we get distinct edge labels.

Hence L(G) of Star $K_{1,4}$ is a Geometric Mean graph.

Example 2.5: Geometric Mean labeling of Line graph of **Star $\mathbf{K_{1,4}}$** is shown below.

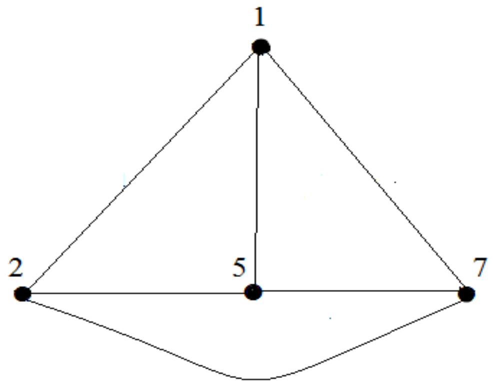

Figure:7

Theorem: 2.6

Line Graph of Comb $\mathbf{P_n} \odot \mathbf{K_1}$ is a Geometric Mean graph.

Proof:

Graph G of Comb $\mathbf{P_5} \odot \mathbf{K_1}$ is displayed below.

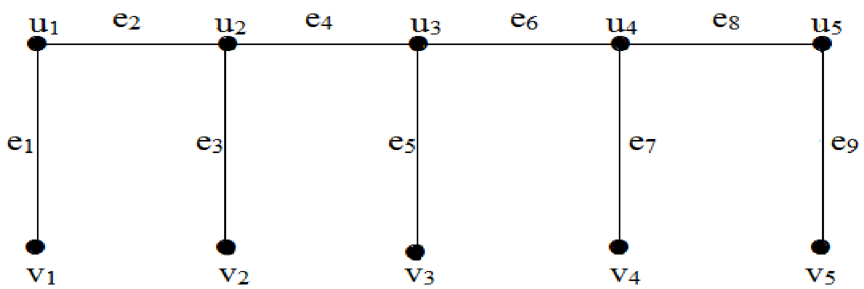

Figure:8

The Line Graph L(G) of Comb $\mathbf{P_5} \odot \mathbf{K_1}$is shown in figure:9

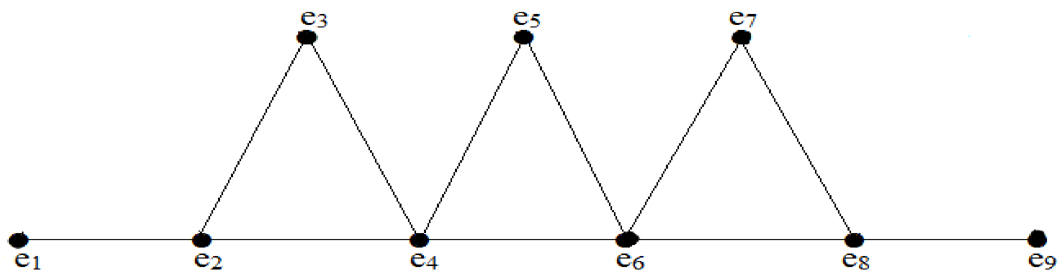

Figure:9

In general, The Line Graph L(G) of Comb $\mathbf{P_n} \odot \mathbf{K_1}$is shown in figure:10

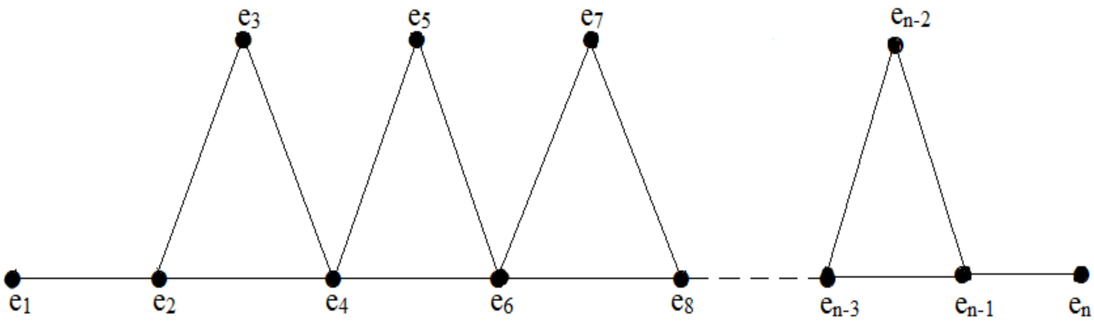

Figure:10

Let L(G) be the Line graph of Comb $\mathbf{P_n} \odot \mathbf{K_1}$. Let u_i, v_i be the vertices of L(G).

Define a function, $f: V(\mathrm{L(G)}) \to \{1,2,3, \ldots\ldots, q+1\}$by $f(\mathrm{u_1}) = 1$

$$f(\mathrm{u_i}) = 3i - 4 \,, 2 \le i \le n-1$$

$$f(\mathrm{u_n}) = \mathrm{f}(\mathrm{u_{n-1}}) + 1$$

$$f(\mathrm{v_i}) = 3i \,, 1 \le i \le n$$

Then we get distinct edge labels.

Hence L(G) of Comb $\mathbf{P_n} \odot \mathbf{K_1}$ is a Geometric Mean graph.

Example 2.7: Geometric Mean labeling of Line Graph of Comb $\boldsymbol{P_5} \odot \boldsymbol{K_1}$ is shown below.

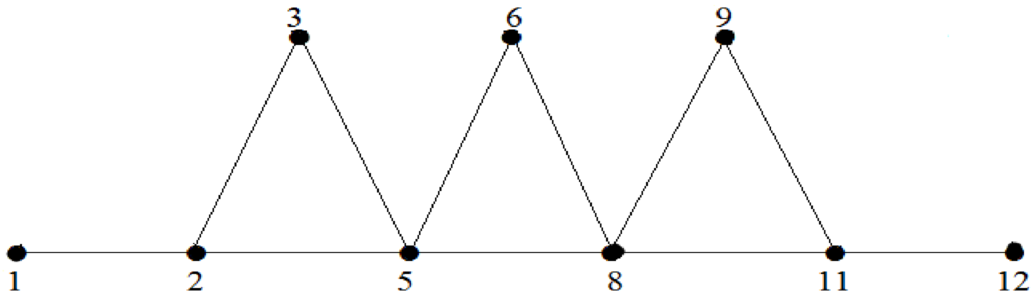

Figure:11

Theorem: 2.8

Line Graph of $\mathbf{P_n} \odot \mathbf{K_{1,2}}$ is a Geometric Mean graph.

Proof:

Graph G of $\mathbf{P_4} \odot \mathbf{K_{1,2}}$ is displayed below.

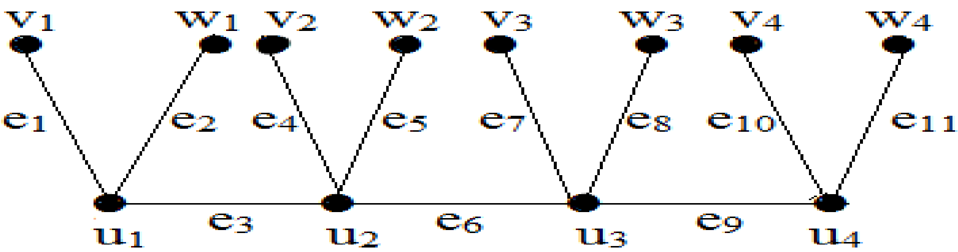

Figure:12

The Line Graph L(G) of $\mathbf{P_4} \odot \mathbf{K_{1,2}}$ is shown in figure:12

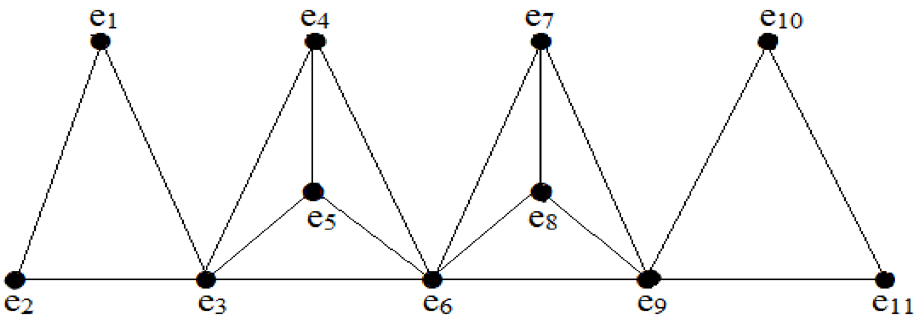

Figure:13

In general, The Line Graph L(G) of Comb $\mathbf{P_n} \odot \mathbf{K_{1,2}}$ is shown in figure:14

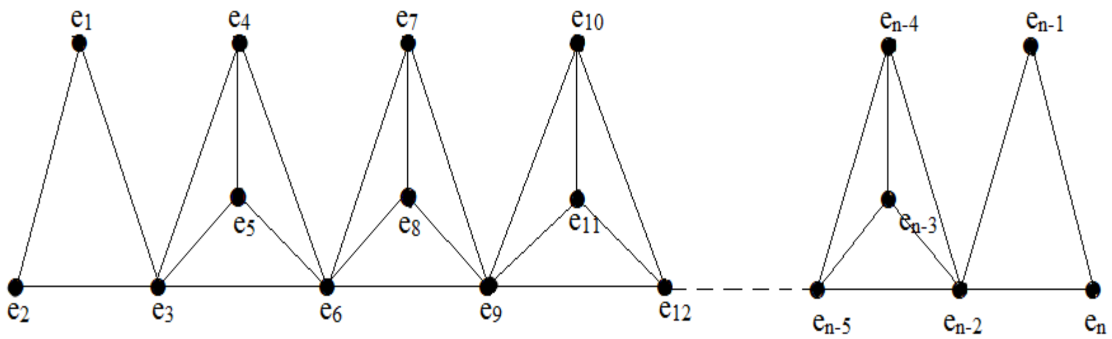

Figure:14

Let L(G) be the Line graph of $\mathbf{P_n} \odot \mathbf{K_{1,2}}$. Let u_i, v_i, w_i be the vertices of L(G).

Define a function, $f: V(\text{L(G)}) \rightarrow \{1,2,3, \ldots\ldots, q+1\}$by $f(\mathrm{u_1}) = 1$

$$f(\mathrm{u_i}) = 7i - 4\,, 2 \leq i \leq n-1$$
$$f(\mathrm{u_n}) = \mathrm{f}(\mathrm{u_{n-1}}) + 2$$
$$f(\mathrm{v_1}) = 2$$
$$f(\mathrm{v_i}) = 7i - 7\,, 2 \leq i \leq n-1$$
$$f(\mathrm{v_n}) = \mathrm{f}(\mathrm{v_{n-1}}) + 2$$
$$f(\mathrm{w_i}) = 7i - 2\,, 1 \leq i \leq n$$

Then we get distinct edge labels.

Hence L(G) of Comb $\mathbf{P_n} \odot \mathbf{K_{1,2}}$ is a Geometric Mean graph.

Example 2.9: Geometric Mean labeling of Line Graph of $\mathbf{P_4} \odot \mathbf{K_{1,2}}$ is shown below.

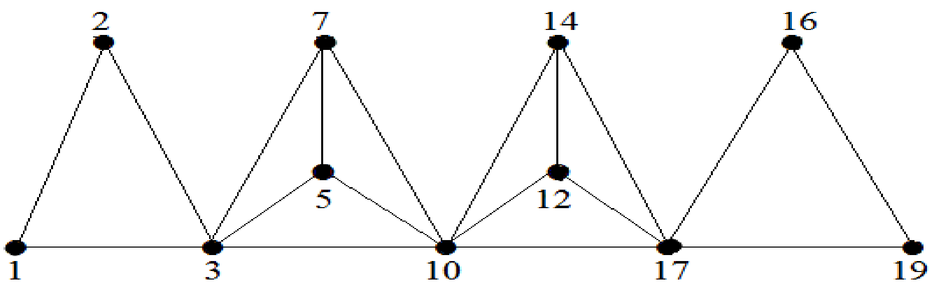

Figure:15

III. CONCLUSION

The Study of Geometric Mean Labeling of Line Graphs is important due to its diversified applications. Line Graphs of all Geometric Mean Graphs are not Geometric Mean graphs. It is very interesting to investigate graphs which admit Geometric Mean Labeling. In this paper, We proved that Line Graph of Path, Cycle, Comb, Star, $\mathbf{P_n} \odot \mathbf{K_{1,2}}$ are Geometric Mean Graphs. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

IV. ACKNOWLEDGEMENT

The authors are thankful to the referee for their valuable comments and suggestions.

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