Onset of Multi-diffusive Convection in a Rotating Dusty Porous Layer: A Brinkman Model

Kapil Kumar[1], Rajan Singh[2], B. K. Singh[3]
[1]*Department of Applied Mathematics, Government Polytechnic, Pilibhit, 262001, India*
[2,3]*Department of Mathematics, IFTM University, Moradabad, 244001, India*

***Abstract* - The onset of multi-diffusive convection problem is analysed theoretically to include the effects of suspended particles and rotation through a porous medium. In the present paper, Brinkman model is considered for the porous medium. The variations in fluid density are due to the variation in $(n+1)$ stratifying components having different thermal and solute diffusivities. Linear stability analysis procedure along with normal mode method is employed to obtain a dispersion relation in terms of thermal and solute Rayleigh number. Further, the case of stationary convection (when the growth rate vanishes) is also discussed and a dispersion relationship between thermal and solute Rayleigh numbers is obtained to study the effect of various embedded parameters. The critical thermal and solute Rayleigh numbers can be obtained with the help of critical dimensionless wave number x_c for varying values of physical parameters.**

***Index Terms* - Multi-diffusive Convection, Suspended Particles, Rotation, Brinkman Porous Medium.**

1.INTRODUCTION

Thermal convective instability of a horizontal layer of fluid heated from below has several applications in geophysics, earth’s science, oceanography and extensive reviews of this subject can be found in Chandrasekhar (1981). Rayleigh (1916) laid the foundation of the linear instability theory using small infinitesimal perturbations. When two or more stratifying components (e.g. heat and salt diffusing at different rates) are present then the convective phenomenon is termed as Double-diffusive or Multi-diffusive convection having extensive physical applications in ocean water, magmas, contaminant transport and underground water flow. The flow through a porous medium has been of fundamental importance in geothermal reservoirs, solidification, geothermal power resources, astrophysics, chemical processing industry, petroleum industry, recovery of crude oil from earth’s interior. A detailed study of convection through a porous layer can be found in Nield and Bejan (2006). The numerical and analytical treatment of the double-diffusive and multi-diffusive convection saturating a porous layer is reviewed in the references Huppert and Turner (1981), Turner (1985), Terrones and Pearlstein (1989), Tracey (1996), Straughan and Tracey (1999), Radko (2013), Rionero (2013a, b), Prakash et al. (2016), Kumar et al. (2017).

Convective instability in a rotating frame has numerous applications in rotating machinery, food processing industry, centrifugal casting of metals and in thermal power plants (to generate electricity by rotation of turbine blades). Rudraiah et al. (1986) considered the effect of rotation on linear and non-linear double-diffusive convective problem saturating a porous layer.

In geophysical context, the fluid is often not pure but may instead be permeated with dust particles. These suspended particles have scientific relevance in geophysics, chemical engineering and astrophysics (McDonnel [1978]). Scanlon and Segel (1973) considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure fluid was supplemented by that of the particles.

The intention of the present paper is to analyse theoretically the onset of thermal convection in a multi-diffusive fluid layer in the presence of suspended dust particles, uniform vertical rotation saturating a porous medium. Most research outcomes for porous medium flows are based on the Darcy model which gives appropriate results at small Reynolds number. Therefore, Darcy-Brinkman model is employed for porous medium which is considered physically more realistic than the usual Darcy model and also gave satisfactorily result at large Reynolds number and for high porosity porous medium by incorporating the inertial and viscous effects in addition to the usual Darcy model. The research on multi-component fluid layer through porous medium has notable geophysical relevance in real life and is increasing with the number of salts dissolved in it.

2.PROBLEM FORMULATION AND LINEAR STABILITY ANALYSIS

Consider an infinite horizontal Boussinesq fluid layer permeated with dust particles lying in the region $0 \le z \le d$ through a Darcy-Brinkman porous medium under the effect of a uniform vertical rotation $\mathbf{\Omega}(0,0,\Omega_z)$. Both the boundaries are maintained at uniform temperatures $T_l\left(>T_u\right)$ and T_u and uniform n concentrations $C_l^{\,1}\left(>C_u^{\,1}\right),\, C_l^{\,2}\left(>C_u^{\,2}\right),........,C_l^{\,n}\left(>C_u^{\,n}\right)$ and $C_u^{\,1},C_u^{\,2},......,C_u^{\,n}$ with gravity acting in vertical downward direction (Fig. 1).

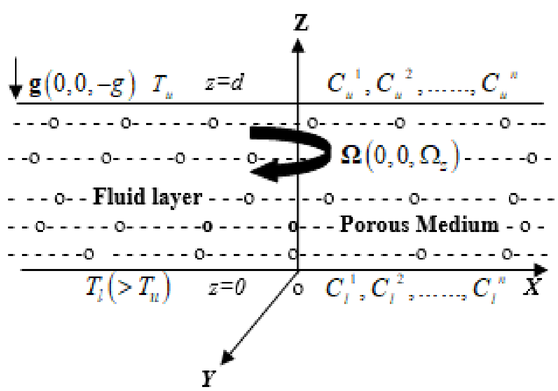

Fig.1. Geometrical sketch of the physical problem

The governing equations of motion and continuity for an incompressible Oberbeck-Boussinesq (1903) fluid layer saturating a Darcy-Brinkman porous medium (1947a, b) are as:

$$\frac{1}{\epsilon}\left[\frac{\partial v}{\partial t}+\frac{1}{\epsilon}\left(v\frac{\partial v}{\partial x}\right)\right]=\left[\begin{array}{l}-\frac{1}{\rho_0}\left(\nabla p\right)-\frac{\upsilon}{k_1}v+\tilde{\upsilon}_{ef}\left(\nabla^2 v\right)+\\ \left(1+\frac{\delta\rho}{\rho_0}+\frac{\delta\rho_1}{\rho_0}+\frac{\delta\rho_2}{\rho_0}+.......+\frac{\delta\rho_n}{\rho_0}\right)\mathbf{g}\\ +\frac{K'N_0}{\rho_0\,\epsilon}\left(v_d-v\right)+\frac{2}{\epsilon}\left(v\times\mathbf{\Omega}\right)\end{array}\right] \quad (1)$$

$$\nabla.v=0 \quad (2)$$

where, $t,\rho_0,\rho,\in,p,\upsilon,\tilde{\upsilon}_{ef},v,v_d,k_1,N_0$ *and* $\mathbf{g}$ denote, respectively, the time, the reference density, fluid density, effective porosity, pressure, kinematic viscosity, effective kinematic viscosity, fluid velocity components, particles velocity, effective permeability, number density of suspended particles and the gravitational acceleration vector. The term $K'=6\pi\rho\upsilon\delta$ $(\delta$ being particle radius$)$, is the Stoke's drag co-efficient.

The presence of suspended particles adds an extra force term, in equation of motion, proportional to velocity difference between particles and fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. Inter-particle reactions are ignored as the distances between the particles are assumed to be quite large compared with their diameters.

The governing equations of motion and continuity for the particles (ignoring the pressure, magnetic field and gravity) are as:

$$mN_0\left[\frac{\partial v_d}{\partial t}+\frac{1}{\in}\left(v_d.\nabla\right)v_d\right]=K'N_0\left(v-v_d\right) \quad (3)$$

$$\in\frac{\partial N_0}{\partial t}+\nabla.\left(N_0 v_d\right)=0 \quad (4)$$

where mN_0 is the mass of particles per unit volume.

The equations for temperature field and solute concentrations are as:

$$\left[\in\rho_0 c_v+\rho_s c_s\left(1-\in\right)\right]\frac{\partial T}{\partial t}+\rho_0 c_v\left(v.\nabla\right)T$$
$$+mN_0 c_{pt}\left(\in\frac{\partial}{\partial t}+v_d.\nabla\right)T=k_T\nabla^2 T \quad (5)$$

where, $\rho_s, c_s, c_v, c_{pt}, T$ *and* k_T denote, respectively, the density of solid material, heat capacity of solid material, the specific heat at constant volume, heat capacity of suspended particles, the temperature and the coefficient of heat conduction.

$$\left[\in \rho_0 c_v{}^{\lambda} + \rho_s c_s{}^{\lambda}(1-\in)\right]\frac{\partial C^{\lambda}}{\partial t} + \rho_0 c_v{}^{\lambda}(v.\nabla)C^{\lambda}$$

$$+mN_0 c_{pt}{}^{\lambda}\left(\in\frac{\partial}{\partial t} + v_d.\nabla\right)C^{\lambda} = k_{C^{\lambda}}\nabla^2 C^{\lambda} \;(\lambda = 1,2,...,n) \quad (6)$$

The symbols $c_s{}^{\lambda}, c_v{}^{\lambda}, c_{pt}{}^{\lambda}, C^{\lambda}$ *and* $k_C{}^{\lambda}\;(\lambda = 1,2,,...,n)$ denote the analogous n solute components.

The density is taken as a linear function of temperature field and salt concentrations as:

$$\rho = \rho_0\left[1 + \alpha_T(T_l - T_u) - \sum_{\lambda=1}^{n}\alpha_{C^{\lambda}}\left(C_l{}^{\lambda} - C_u{}^{\lambda}\right)\right] \quad (7)$$

where, $T_l, T_u, \alpha_T, \alpha_{C_\lambda}, C_l{}^{\lambda}$ *and* $C_u{}^{\lambda}\;(\lambda = 1,2,.....n)$ denote, respectively, the temperature at lower boundary, temperature at upper boundary, coefficient of thermal expansion, coefficients of solute expansion, concentration components at lower and upper boundaries.

The basic state is assumed to be stationary and therefore, for determining the stability/instability of the system linear stability analysis procedure followed by normal mode method is adopted by introducing small infinitesimal perturbations in the basic variables. The basic state of the system is defined as:

$$v = (0,0,0),\; v_d = (0,0,0), T = T_0 - \beta_T z,\; \Omega = [0,0,\Omega],$$

$$\rho = \rho_0[1 + \alpha\beta_T z],\; p = p_0 - g\rho_0 z\left(1 + \frac{\alpha\beta_T z}{2}\right),\; N_0,\; C^{\lambda}. \quad (8)$$

Let the perturbations in the basic variables given in (8) are defined as:

$$v = (u,v,w), v_d = (l,r,s), \theta, \Omega\left(\Omega_x, \Omega_y, \Omega_z\right), \delta\rho, \delta p, N, \gamma^{\lambda}. \quad (9)$$

So, the resulting linearized perturbation equations after eliminating the pressure gradient term are as:

$$\frac{1}{\in}\frac{\partial}{\partial t}\left(\nabla^2 w\right) = \begin{bmatrix} -\frac{\upsilon}{k_1}\left(\nabla^2 w\right) + \tilde{\upsilon}_{eff}\left(\nabla^4 w\right) + g\alpha_T \nabla_1{}^2\theta \\ -g\nabla_1{}^2\sum_{\lambda=1}^{n}\alpha_{C^{\lambda}}\gamma^{\lambda} - \frac{mN_0}{\rho_0 \in \left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)} \\ \frac{\partial}{\partial t}\left(\nabla^2 w\right) - \frac{2\Omega}{\in}\left(\frac{\partial\zeta}{\partial z}\right) \end{bmatrix} \quad (10)$$

$$\frac{1}{\in}\left(\frac{\partial\zeta}{\partial t}\right) = \begin{bmatrix} -\frac{\upsilon}{k_1}\zeta + \tilde{\upsilon}_{eff}\left(\nabla^2\zeta\right) - \frac{mN_0}{\rho_0 \in \left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)} \\ \left(\frac{\partial\zeta}{\partial t}\right) + \frac{2\Omega}{\in}\left(\frac{\partial w}{\partial z}\right) \end{bmatrix} \quad (12)$$

$$\left[(E + b\in)\frac{\partial}{\partial t} - \kappa_T\nabla^2\right]\theta = \beta_T\left[1 + \frac{b}{\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)}\right]w \quad (13)$$

$$\left[\left(E^{\lambda} + b^{\lambda}\in\right)\frac{\partial}{\partial t} - \kappa_{C^{\lambda}}\nabla^2\right]\gamma^{\lambda} = \beta_{C^{\lambda}}\left[1 + \frac{b^{\lambda}}{\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1\right)}\right]w \quad (14)$$

The change in density $\delta\rho$ due to temperature variation θ and concentration variations $\gamma^{\lambda}\,(\lambda = 1,2,....,n)$, is given by

$$\delta\rho = -\left(\alpha_T\theta - \sum_{\lambda=1}^{n}\alpha_{C^{\lambda}}\gamma^{\lambda}\right)\rho_0 \quad (14)$$

where, in equations (10)-(13), $\kappa_T = \frac{k_T}{\rho_0 c_v}, \kappa_{C^{\lambda}} = \frac{k_{C^{\lambda}}}{\rho_0 c_v{}^{\lambda}}, w,\; s, \zeta\left(= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right), \nabla_1{}^2$ *and* ∇^2 denote, respectively, the thermal diffusivity, the solute diffusivity, vertical component of fluid velocity, vertical component of suspended particles velocity, vertical component of vorticity, horizontal Laplacian operator and Laplacian operator, with

$$E = \in + (1-\in)\left(\frac{\rho_s c_s}{\rho_0 c_v}\right), b = \frac{mN_0 c_{pt}}{\rho_0 c_v},$$

$$E^{\lambda} = \in + (1-\in)\left(\frac{\rho_s c_s{}^{\lambda}}{\rho_0 c_v{}^{\lambda}}\right) \text{ and } b^{\lambda} = \frac{mN_0 c_{pt}{}^{\lambda}}{\rho_0 c_v{}^{\lambda}}.$$

3. NORMAL MODE METHOD AND DISPERSION RELATION

A normal mode representation is assumed in various physical disturbances with a dependence on x, y *and* t of the form:

$$\left[w,\theta,\zeta,\gamma^{\lambda}\right]=\left[W(z),\Theta(z),Z(z),\Phi^{\lambda}(z)\right]\exp\left(ik_x x+ik_y y+nt\right) \tag{15}$$

where, k_x and k_y are the wave numbers along x and y directions, respectively.

Using expression (15), the non-dimensional form of Eqs. (10) - (13) (after dropping the asterisk for convenience) are as:

$$\left[\frac{\sigma}{\in}\left\{1+\frac{M}{(1+\tau_1\sigma)}\right\}-\frac{D_A}{P_l}\left(D^2-a^2\right)+\frac{1}{P_l}\right]\left(D^2-a^2\right)W+\frac{ga^2d^2}{\upsilon}\left[\alpha_T\Theta-\sum_{\lambda=1}^{n}\alpha_{C^{\lambda}}\Phi^{\lambda}\right]+\frac{2\Omega d^3}{\in\upsilon}DZ=0 \tag{16}$$

$$\left[\frac{\sigma}{\in}\left(1+\frac{M}{(1+\tau_1\sigma)}\right)-\frac{D_A}{P_l}\left(D^2-a^2\right)+\frac{1}{P_l}\right]Z+\frac{2\Omega d}{\in\upsilon}DW=0 \tag{17}$$

$$\left[\left(D^2-a^2\right)-p_1E_1\sigma\right]\Theta=-\left(\frac{\beta_T d^2}{\kappa_T}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W \tag{18}$$

$$\left[\left(D^2-a^2\right)-q^{\lambda}E_1^{\lambda}\sigma\right]\Theta^{\lambda}=-\left(\frac{\beta_{C^{\lambda}}d^2}{\kappa_{C^{\lambda}}}\right)\left(\frac{B^{\lambda}+\tau_1\sigma}{1+\tau_1\sigma}\right)W \tag{19}$$

The above perturbation equations (16)-(19) are non-dimensionalized using the following scalings:

$$z^*=\left(\frac{z}{d}\right),\ k=\left(\frac{a}{d}\right),\ \sigma=\frac{nd^2}{\upsilon},\tau=\frac{m}{K'},\tau_1=\frac{\tau\upsilon}{d^2},B=1+b,$$

$$B^{\lambda}=1+b^{\lambda},N_0=\frac{\rho_0 M}{m},D_A=\left(\frac{\tilde{\mu}_{ef}P_l}{\mu}\right),E_1=E+b\in,$$

$$E_1^{\lambda}=E^{\lambda}+b^{\lambda}\in,\ P_l=\frac{k_1}{d^2},\ p_1=\frac{\upsilon}{\kappa_T},q^{\lambda}=\frac{\upsilon}{\kappa_{C^{\lambda}}}.$$

where, P_l is the dimensionless medium permeability, p_1 is the thermal Prandtl number, $q^{\lambda}(\lambda=1,2,...,n)$ are the n Schmidt numbers, β_T is the adverse temperature gradient, $\beta_{C^{\lambda}}(\lambda=1,2,...,n)$ are the n solute concentration gradients, $k^2=\left(k_x{}^2+k_y{}^2\right)$ is a wave number and n is the frequency of the harmonic disturbance and $D=\left(\frac{d}{dz}\right)$.

The boundary conditions (for the case of two free boundaries are defined as:

$$W=D^2W=DZ=\Theta=\Phi^{\lambda}\left(\lambda=1,2,..,n\right)=0\ at\ z=0 \text{ and d.} \tag{20}$$

Eliminating $\Theta(z),\ \Phi^{\lambda}(\mathbf{z})$ and $\mathbf{Z}(\mathbf{z})$ from equations (16)–(19), a dispersion relation in W is obtained as:

$$\left[\frac{\sigma}{\in}\left\{1+\frac{M}{(1+\tau_1\sigma)}\right\}-\frac{D_A}{P_l}\left(D^2-a^2\right)+\frac{1}{P_l}\right]\left(D^2-a^2\right)W-\frac{R_Ta^2\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)}{\left[\left(D^2-a^2\right)-p_1E_1\sigma\right]}W+\frac{R_{C^{\lambda}}a^2\left(\frac{B^{\lambda}+\tau_1\sigma}{1+\tau_1\sigma}\right)}{\left[\left(D^2-a^2\right)-q^{\lambda}E_1{}^{\lambda}\sigma\right]}W-\frac{T_A}{\in^2\left[\frac{\sigma}{\in}\left\{1+\frac{M}{(1+\tau_1\sigma)}\right\}-\frac{D_A}{P_l}\left(D^2-a^2\right)+\frac{1}{P_l}\right]}D^2W=0 \tag{21}$$

where, $R_T=\dfrac{g\alpha_T\beta_T d^4}{\upsilon\kappa_T}$ (thermal Rayleigh number), $R_{C^{\lambda}}=\dfrac{g\alpha_{C^{\lambda}}\beta_{C^{\lambda}}d^4}{\upsilon\kappa_{C^{\lambda}}}$ (solute Rayleigh numbers), $T_A=\dfrac{4\Omega^2d^4}{\upsilon^2}$ (Taylor number)

4.THE STATIONARY CONVECTION

For stationary state $(\sigma=0)$, Eq. (21) yields an expression of the form:

$$\left[1-D_A\left(D^2-a^2\right)\right]^2\left(D^2-a^2\right)^2W-\left[R_TB-\sum_{\lambda=1}^{n}R_{C^{\lambda}}B^{\lambda}\right]\left[1-D_A\left(D^2-a^2\right)\right]a^2P_lW-\frac{T_AP_l^2}{\in^2}\left(D^2-a^2\right)D^2W=0 \tag{22}$$

Since all the even derivatives of W vanishes, so considering an appropriate solution for W of the form:

$W=W_0\sin l\pi z,\ (W_0\neq 0,\ l=1,2,3....)$

Equation (22) yields:

$$R_T{}^{\dagger}=\frac{1}{B}\left[\sum_{\lambda=1}^{n}R^{\dagger}{}_{C^{\lambda}}\ B^{\lambda}+\frac{\left\{1+D_{A_1}(1+x)\right\}(1+x)^2}{Px}-\frac{T_{A_1}P(1+x)}{\in^2\ x\left\{1+D_{A_1}(1+x)\right\}}\right] \tag{23}$$

where, the following notations are assumed as:

$R_T{}^{\dagger} = \frac{R_T}{\pi^4}, R^{\dagger}{}_{C^{\lambda}} = \frac{R_{C^{\lambda}}}{l^4\pi^4}, \ x = \frac{a^2}{l^2\pi^2}, \ P_l = \frac{P}{l^2\pi^2},$
$T_{A_1} = \frac{T_A}{l^4\pi^4}, \ D_{A_1} = \frac{D_A}{l^2\pi^2}.$

Minimizing Eq. (23) with respect to $x \left(i.e. \frac{\partial R_T{}^{\dagger}}{\partial x} = 0 \right)$ yields a fifth-degree equation in x as:

$$\alpha_1 x^5 + \alpha_2 x^4 + \alpha_3 x^3 + \alpha_4 x^2 + \alpha_5 x + \alpha_6 = 0 \qquad (24)$$

where,

$\alpha_1 = \left(2 \in^2 D_{A_1}{}^3\right), \alpha_2 = \left(7 \in^2 D_{A_1}{}^3 + 5 \in^2 D_{A_1}{}^2\right),$

$\alpha_3 = \left(8 \in^2 D_{A_1}{}^3 + 12 \in^2 D_{A_1}{}^2 + 4 \in^2 D_{A_1}\right),$

$\alpha_4 = \left(2 \in^2 D_{A_1}{}^3 + 6 \in^2 D_{A_1}{}^2 + 5 \in^2 D_{A_1} + T_{A_1} P^2 D_{A_1} + \in^2\right),$

$\alpha_5 = \left(-2 \in^2 D_{A_1}{}^3 - 4 \in^2 D_{A_1}{}^2 - 2 \in^2 D_{A_1} + 2T_{A_1} P^2 D_{A_1}\right),$

$\alpha_6 = \left(- \in^2 D_{A_1}{}^3 - 3 \in^2 D_{A_1}{}^2 - 3 \in^2 D_{A_1} - \in^2 + T_{A_1} P^2 D_{A_1} + T_{A_1} P^2\right).$

The critical dimensionless wave number x_c for varying values of parameters can be obtained from Eq. (24) and then the critical thermal and solute Rayleigh numbers can be deduced from Eq. (23).

Equation (23) represents a relationship between thermal and solute Rayleigh numbers in terms of various embedded parameters. The effect of these parameters (suspended particles, medium permeability, medium porosity, Taylor number, Darcy-Brinkman) on thermal Rayleigh number can be examined analytically from the following derivatives $\frac{dR_T{}^{\dagger}}{dB}, \frac{dR_T{}^{\dagger}}{dB^{\lambda}}, \frac{dR_T{}^{\dagger}}{dP}, \frac{dR_T{}^{\dagger}}{d\in}, \frac{dR_T{}^{\dagger}}{dT_{A_1}}$ and $\frac{dR_T{}^{\dagger}}{dD_{A_1}}$.

CONCLUSION

A linear stability analysis followed by normal mode method is taken into account to discuss the effect of uniform vertical rotation and suspended particles on the onset of multi-diffusive convection through a Darcy-Brinkman porous medium and a dispersion relation is obtained in terms of thermal and solute Rayleigh numbers. Further, the case of stationary convection is also discussed and a relationship between thermal and solute Rayleigh numbers is obtained to study the effect of various embedded parameters. The critical thermal and solute Rayleigh numbers can be obtained with the help of critical dimensionless wave number for varying values of physical parameters.

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