

Analysis on a Low Complexity Signal Detection Scheme Based on Improved Newton Iteration for Massive MIMO Systems

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Abstract - Massive Multi-Exit Input (MIMO) devices are required to handle the enormous number of matrix inversion operations during signal detection. When dealing with massive MIMO systems, there are a number of alternatives that can be used to avoid an exact matrix inversion. Two types of signal detection techniques are first introduced in this research. The association is then established using a Newton iteration technique. Minimize computing complexity by converting matrix-matrix products into matrix-vector products. This is another way to reduce computing complexity. That numerical simulation can beat Neumann series expansion and Newton method in a few rounds is also demonstrated.

Index Terms - Massive MIMO, signal detection, matrix inversion, Neumann Series, Newton iteration.

1. INTRODUCTION

Data-oriented services have evolved enormously in recent years in communications systems. Massive multiple-output input (mMIMO) is used in 5G wireless communication systems for high data rates, stability, resilience, energy efficiency, and spectrum efficiency (1-3). MIMO can also be employed in systems other than 5G. The base station (BS) must have a large number of antennas to accommodate a large number of users in a single cell[3]. With an increase in the number of elements of an antenna, the complexity increases exponentially. A sophisticated signal processing is thus necessary for designing an efficacious low complexity sensor for the mMIMO uplink(UL) system. The highest probability of ML achieves the lowest bit error rate (BER) is widely known; however, it demands a very high computing cost, which is unwanted to use. For example, the ML detector analyzes 1.84 to 1019 solutions in a mMIMO

system with 64 transmitter antennas in detail to determine the most ineffective result[5].

There are substantial advantages to MIMO compared to regular MIMO in terms of energy economy and power consumption[1]. MIMO is the most popular input multi-output technology. Linear detection approaches, such the MMSE method, have been shown to deliver near-optimal performance [2]. Inverse matrix operations, on the other hand, are required by linear detection approaches.

There have been a great number of papers published in recent years on the complex problem of signal identification in large MIMO systems[2]–[6]. Two primary forms of these algorithms are in general iterative approaches and approximation methods. [1][7] Reversion matrix is approximated using truncated Neumann series extension. The computational complexity and performance of the NSE increase as the number of selected series terms rises. [3] claims that Newton's iteration approach has a higher convergence rate than Neumann's expansion series. [8] and [9]: Using Gaussian signal sources, GMPID analyzes the mean and variances between nodes at the end of each cycle. Furthermore, iterative LMMSE may give the optimal summary capability for all system configurations of multi-user MIMO systems[10]. For large MIMO systems, [11] proposes the MMSE-PIC algorithm using the Neumann expansion approach. Furthermore, to prevent perfect inversion of the matrix, iteration detection methods are applied for MMSE detection. The compromise between computational complexity and performance is a commitment of all algorithms.

2. BACKGROUND

In mMIMO systems, the matrix reversion process is undesired and computational complexity will substantially inflate. When inverting the matrix iteratively, signal estimation can be done in a simpler way. Approximation reverse matrix methods and avoid reverse matrix methods are two types of iterative procedures. Approximate inverting matrix methods like NS and NI are alternate approaches in approximating the reverse of the equalization matrix prior to estimating the signal received. The matrix inversion in the NS method is turned into a multiplication of the matrix-vector that reduces efficiency.

In [17] weighting technique for the NS detection (WNS) is proposed in order to avoid errors when comparing the accurate matrix reversal to the WNS matrix reversal optimum weights of online learning are obtained. As a result of this technique, the performance of large BUARs is improved, i.e. BUAR = 128 16 = 8 and BUAR = 128 32 = 4 O K 3 is required for approximation reverse matrix methods, though. Based on the NI technique, [18] proposes updated detection methods that reduce computer complexity to O (KN), on the BS side, where N is an array of antennas. This is achieved by using a tridiagonal matrix and a modified NSE in [19].

Xilinx Virtex-7 XC7VX690T FPGA is used to test the performance of the suggested algorithm. When the BUAR is large, 128 16 = 8, it obtains a high level of performance. Numerous repetitions are necessary to attain a sufficient performance to approximate matrix inversion algorithms, It adds to the difficulty of computation. Instead, use iterative signal refinement techniques such as matrix inversion using GS, SOR, JA, and RI rather than GS, SOR, JA, and RI. While the first strategy's complexity is usually O.K. 2[20], this approach usually has better performance and is less complex than the first way. It is shown in [21,22] that the GS, SOR, JA, and the RI process detector has a high performance and a minimal complexity.

Initialization is also widely recognized to for excellent performance and low complexity[15]. In [16] the effect in algorithms of the stair matrix based on iterative approaches and the detection algorithms based on the diagonal matrix were examined. In a limited number of iterations, the stair matrix has been shown to achieve satisfactory performance (low complexity)

We explore the association between different detection methods for the first time in this article. When the recommended starting estimate is selected, the estimate outcomes in the iterative methods after Neumann's k-th order expansion is equivalent to k rounds in the Neumann series. In Newton's method, the results of k iterations are seen as 2 k1 iterations. Our augmented Newton iteration technique for huge MIMO systems maximizes its performance based on the posited relationship. In terms of bit error rate (BER) performance and computational complexity, the suggested technique outperforms existing Newton Iteration, MMSE-PIC, and NSE strategies.

3. PROPOSED NEWTON ITERATION METHOD

An improved Newton iteration strategy can be found in this section MMSE on an iterative basis rather than an accurate matrix reversal. Jacobi and Richardson methods are known to be specific examples of iterative approaches. They are also made up of BJ = I-D -1A [12] and BR = I- bis A [13], and k denote the relaxation. For example, if N and K were to converge to infinity, the smallest and largest A's values would be stable[14]

$$\lambda_{\min}(\mathbf{A}) = N\left(1 - \sqrt{K/N}\right)^2, \lambda_{\max}(\mathbf{A}) = N\left(1 + \sqrt{K/N}\right)^2.$$

And, because of the canal hardening effect, A may be represented in this scenario as a diagonal matrix[6]. We got D = D = A = NI, too. The Jacobi iteration matrix BJ's value is hence the correct value.

$$\lambda_{\min}(\mathbf{B}_J) = 1 - \left(1 + \sqrt{K/N}\right)^2, \lambda_{\max}(\mathbf{B}_J) = 1 - \left(1 - \sqrt{K/N}\right)^2.$$

Due to the negative correlation between asymptotic convergence rate and specimen radius, the Richardson method converges quicker than the Jacobi method [12]. A favorable relationship exists between the Newton technique and iterative procedures, according to the derivation made.

$$\begin{aligned} \mathbf{P}_1^{-1} &= \mathbf{P}_0^{-1}(2\mathbf{I}_K - \mathbf{A}\mathbf{P}_0^{-1}) = 2\mathbf{P}_0^{-1} - \mathbf{P}_0^{-1}(\mathbf{P}_0 + \mathbf{Q})\mathbf{P}_0^{-1} \\ &= \mathbf{P}_0^{-1} + (-\mathbf{P}_0^{-1}\mathbf{Q})\mathbf{P}_0^{-1}, \end{aligned}$$

Due to the highly difficulty of Newton's high-order iterations, following a number of Newton iterations the convergence is reached using the Richardson low complexity approach. In conclusion, the proposed implementation of the algorithm is shown in Algorithm 1.

Algorithm 1 Proposed Hybrid Iteration Method

Input: \mathbf{H} , \mathbf{y} , σ^2 , E_s , N , K
Output: $\hat{\mathbf{x}}$: the estimated value of \mathbf{x}
Initialization:
 1: $\mathbf{b} = \mathbf{H}^H \mathbf{y}$, $\omega = \frac{1}{N+K}$, $\mathbf{P}_0^{-1} = \omega \mathbf{I}$, $\mathbf{V} = \sigma^2 \mathbf{P}_0^{-1}$, $\mathbf{x}^{(0)} = \mathbf{P}_0^{-1} \mathbf{b}$;
Iteration:
 2: **for** $i = 1 : n$ **do**
 3: $\mathbf{x}^{(i)} = \mathbf{x}^{(i-1)} + (\mathbf{I} - \mathbf{P}_0^{-1} \mathbf{H}^H \mathbf{H} - \mathbf{V})^{2^{i-1}} \mathbf{x}^{(i-1)}$;
 4: **end for**
 5: **for** $i = (n + 1) : k$ **do**
 6: $\mathbf{x}^{(i)} = \mathbf{x}^{(i-1)} + \omega(\mathbf{b} - \mathbf{H}^H \mathbf{H} \mathbf{x}^{(i-1)})$;
 7: **end for**
Return: $\hat{\mathbf{x}} = \mathbf{x}^{(k)}$.

Now let's look at the advantages for non-linear detectors like SD by employing iterative inverse matrix. Currently two main SD versions are available. First of all, the Schnorr Euchner list[16], which updates the SD radii accordingly, where the search space decreases with every good point, after starting with an unlimited radius, until the optimized solution is achieved. Decoding large/massive MIMO systems would become increasingly difficult with such a technology. SD[11] is the other. which utilizes a fixed radius technique, based on an algorithm based on Fincke-Pohst and compares all sites within the radius search area for the detection of the signal broadcast. This approach is highly sensitive to radius selection. Both techniques have demonstrated near ML performance in the literature. We provide a strategy to lessen SD's complexity in this section.

Methodology

LLR Estimation with Bi-Gaussian Approximation
 Bi-Gaussian Distribution

The symmetrical mixed-Gaussian distribution is referred to in this publication as a bi-Gaussian distribution.

$$p_{BG}(z) = \frac{1}{2\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} + \frac{1}{2\sqrt{2\pi\sigma^2}} e^{-\frac{(z+\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{z^2+\mu^2}{2\sigma^2}} \cosh\left(\frac{\mu z}{\sigma^2}\right).$$

A shifted bi-Gaussian mixture model uses bi-Gaussian distribution to fit the picture intensity histogram. As an alternative to the low-level Gaussian kernel, derivative filters for picture segmentation and enhancement use a bi-Gaussian function. Also included is a bi-gaussian differential entropy analytical expression. As a way to approximate global noise distributions with multiuser interferences, we'll employ (above equation).

This is because all odd moments in a bi-Gaussian PDF are zero. The second (variance) and fourth moments are listed below.

$$\mu_2^{BG} = \mu^2 + \sigma^2,$$

$$\mu_4^{BG} = \mu^4 + 3\sigma^4 + 6\mu^2\sigma^2.$$

LLR Calculation

By employing techniques based on the bi-Gaussian approximation, the LLR of each code bit can be determined.

Estimate the second and fourth moments of the signal using sample averaging y_1 and y_2 and solve the bi-Gaussian approximation parameters and 2 through (12) for each codeword (13).

To calculate the LLR of the I-th code bit using approximated parameters 2, substitute as follows:

$$\lambda_i = \ln \frac{p_{BG}(y_i - 1)}{p_{BG}(y_i + 1)}$$

$$= \frac{2(y_i - \mu)}{\sigma^2} + \ln \frac{1 + \exp(\frac{2\mu(1-y_i)}{\sigma^2})}{1 + \exp(\frac{-2\mu(1+y_i)}{\sigma^2})}.$$

Complexity Analysis

Multiplications and divisions are two examples of mathematical operations has a significant influence on the complexity of a problem. There's also a correlation between a high number of repetitions and a high It should be noted that the complexity of the computation is influenced by beginning vector It takes K real divisions and $3(K + 1)$ real multiplications to invert a stair matrix (S-1). It takes $4K$ 2 $2K$ real multiplications to initialize the JA approach with the stair matrix. $4nK^2$ real multiplications are needed for the GS technique. The suggested algorithm requires a real number of multiplications, which is $K^2(1 + 4n) + K + 3$. A limited number of iterations is used in the suggested method, which reduces complexity. Multiplications are included in Table 1.

Method	Complexity
NS	$(n - 2)K^3 + NK^2 + NK$
GS	$4nK^2$
JA	$n(4K^2 - 2K)$
Proposed Algorithm	$K^2(1 + 4n) + K - 3$

Table 1. Intricacy of the suggested algorithm, NS, GS, and JA.

4.SIMULATION RESULTS

It is possible to simulate several large MIMO system topologies in order to evaluate the performance of the

augmented Newton technique in terms of BER. A 64QAM modulation technique is used in simulations of huge MIMO systems when $N = 64 \times 16$ and $N + K = 128 \times 16$. To do this we'll use the 64800-byte-coded-LDPC-code. Also included is a comparison of the MMSE detection technique with other algorithms. Iterations or expansion order is indicated by k .

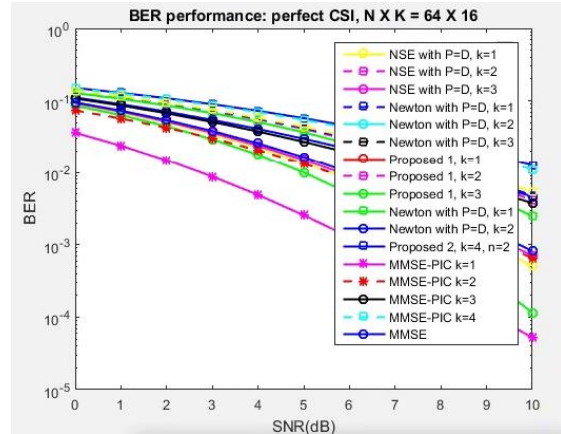


Fig. 1: Performance of the BER: perfect According to the CSI formula, $N \times K = 64 \times 16$.

NSE methodology, Newton method, and MMSE-PIC are depicted in Fig. 1. At a lower ratio of BS antennas per user, however, the NSE method's performance appears to plateau. After three cycles, the performance of the suggested Newton technique approaches that of the MMSE algorithm. The proposed methods outperform other methods iteration-for-iteration.

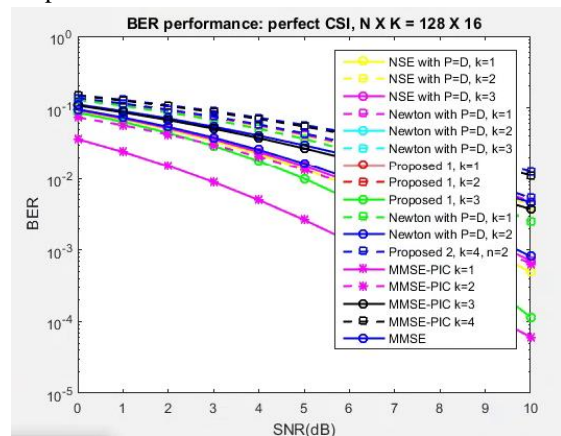


Fig. 2: BER performance: perfect CSI, $N \times K = 128 \times 16$

Since the number of antennas has increased in Fig.2, Figure 2 shows that the performance of all algorithms has improved dramatically. FIGURE 2 shows that the bit error rate reduces with increasing iterations (or NSE order) for all algorithms. Newton iteration can, as a result, match the performance of the MMSE method

after just three iterations. In terms of performance, the conventional Newton approach is still far below MMSE. Finally, after four cycles of the hybrid iteration approach, it achieves the MMSE with a lower level of complexity. As a result of their superior performance, the proposed approaches require less complexity or SNR to get the same performance.

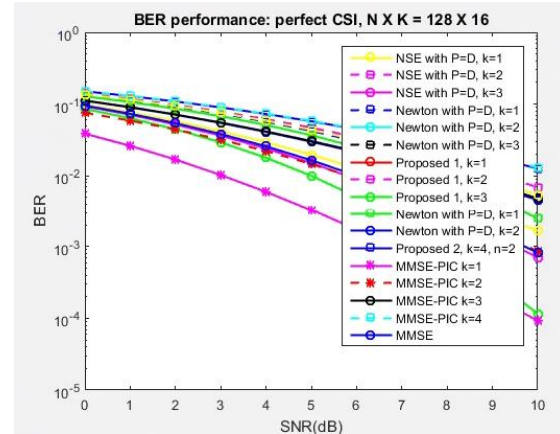
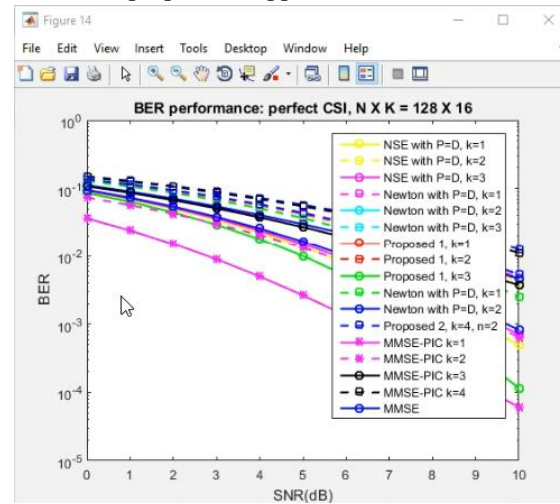
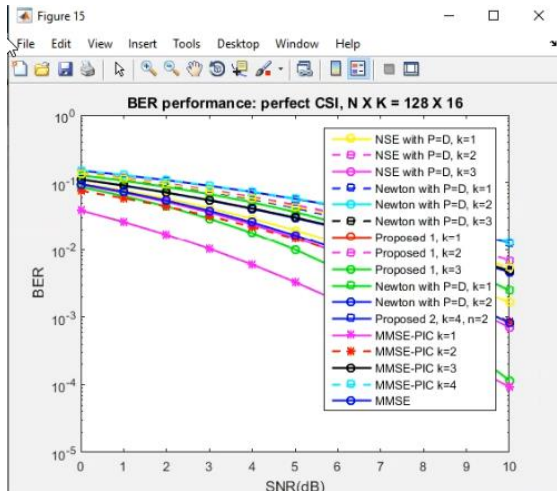


Fig. 3: BER performance: perfect CSI, $N \times K = 128 \times 16$

For example, when $N \times K = 128 \times 16$ and utilizing the least squares method, the advantage of the proposed method is still evident. On the other hand, the performance of all detection algorithms is poor compared to Fig.2, which has a comparable configuration, but the performance gap between them remains constant. This means that the Newton technique's complexity is significantly lower than the existing Newton method, even if its performance after 3 rounds is comparable to that of the proposed-2 approach after 4 iterations.



For channel decoding, two approximate techniques of computing log-likelihood ratios (LLRs) are also obtained, as well as an optimal relaxation parameter.



As a result of analysis and simulation, the suggested approach outperforms other low-complexity signal detection techniques that are commonly used. A modest number of iterations is all that is required for the suggested method to converge quickly and attain performance that is very similar to the MMSE technique.

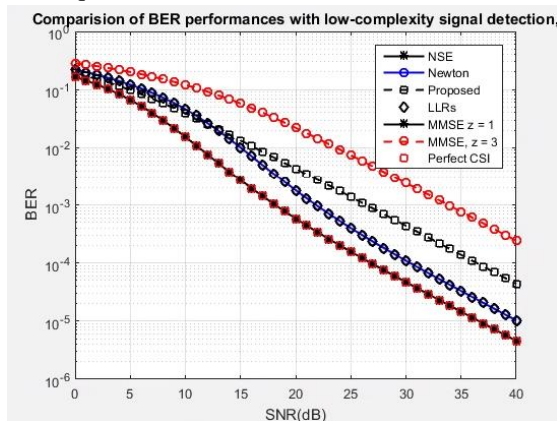


Fig 4: Comparison of BER performances with low complexity signal detection.

When compared to other standard signal detection techniques with approximate matrix inversion, the suggested algorithm has a better BER and computational complexity. A few rounds into the modified Kaczmarz algorithm, and the performance is quite comparable to that of the MMSE method, $O(K^2)$. To identify signals in an uplink massive MIMO system, the approach can be employed as a candidate scheme with a low level of complexity.

5.CONCLUSION

The results of the simulations reveal that in order to calculate the ZF and MMSE solutions, iterative

inversion methods have achieved the same performance as the exact reverse of adequate number of iterations. If we extend the approach to complicated detectors such as SD, we show that the value of BR computed by iterative methods is less than the BR achieved by the correct procedure. We have illustrated the benefits of using an approximate reverse matrix for detectors in large / huge MIMO systems. A quantized ZF-MME solution has been developed by determining the maximum error to be allowed inverse. In addition, the computer complexity is decreased by an order of magnitude by the adoption of matrix-vector products. The BER performance is greatly improved, while maintaining minimal complexity, in comparison to NSE and the current Newton technique.

Simulations demonstrate that the suggested technique outperforms other common signal detection techniques using approximate matrix inversion in terms of BER and computational cost. As soon as a few iterations are completed, the modified Kaczmarz algorithm achieves a level of performance that is comparable to that of the MMSE method, whilst the order of complexity remains the same (K^2). Signal detection using this technique in massive MIMO uplink systems is a low-complexity candidate solution.

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