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Study of Connectivity and Complexity of Architecture for Parallel and Distributed System by Using Topological Linear Space

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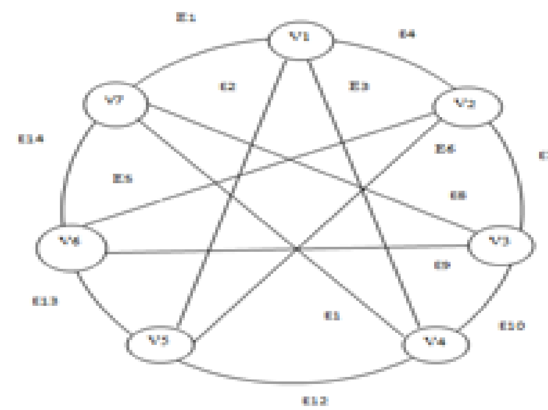
***Abstract* - In this paper we have analyzed the connectivity matrix pattern of Perfect Difference Network and complete graph .we have used the concept of graph theory to study of mathematical structure of PDN and complete graph .Many important properties are proved in the form of lemmas which may be provide benefits for the further research in the matrix patterns of different data structures.**

***Index Terms* - Perfect difference Network, Complete Graph, Connectivity Matrix, Regular Graph, Density, Symmetric Matrix, Square Matrix.**

1.INTRODUCTION

Perfect difference sets was first discussed by Veblen in 1919 and by J. Singer in 1938. They have used the concept of finite projective Geometry for explanation of PDS. They used points & line for formation of PDS to projective plane [1, 2, 3]. Graph theory as a tool plays an important role in analyzing the Parallel and distributed system. The graph depicting the processing elements and a line connecting a pair of node acting as a communication medium between them represents an edge[8]. This paper we have studied two architectures Perfect Difference network and complete graph on the basis of various connectivity matrices and derived some important topological properties in the from of lemmas. In Perfect Difference Networks connectivity diameter is 2 [3, 4, 5] while complete graph connectivity diameter is 1.PDN interconnection network is represented via graph theoretic mathematical model with the vertex as processors and edges as a network communication links. We have to compare these two architectures in the form of vertex to vertex and edge to edge connectivity matrix. We explored that both architectures reveal symmetrical matrix properties and conclude that as number of connections grow the complexity grows along.

2.STUDY OF CONNECTIVITY AND COMPLEXITY OF COMPLETE GRAPH AND PERFECT DIFFERENCE NETWORK



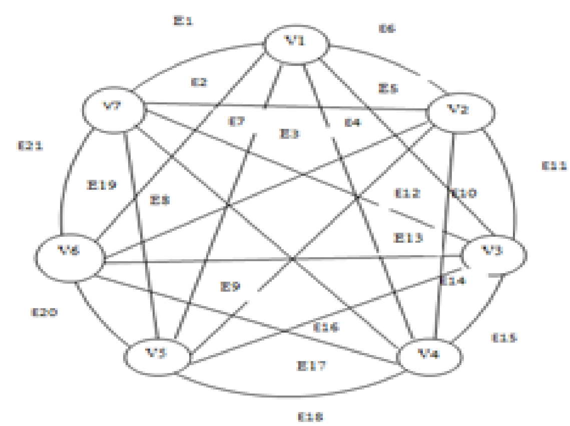


Fig 2.1: PDN for 7 Nodes (δ=2) Fig 2.2: Complete Graph for 7 Nodes (δ=2)

Lemma1: Difference of total number of communication path between Perfect difference network and complete graph is $-\delta^3 + 2\delta^2 + 2\delta + 3$.
Proof: As we know the value of δ is always prime or power of prime in PDN.
Let $\delta = 2$ (fig 2.1)

Then the communication link of one node in PDN = 2δ

The total number of communication link of PDN $=2\delta(\delta^2+\delta+1)$

So that the total number of communication path in PDN= $2\delta(\delta^2+\delta+1)/2$

$=\delta(\delta^2+\delta+1)$

$=\delta^3+\delta^2+\delta$

And the total number of communication path in Complete Graph $=3(\delta^2+\delta+1)$

Therefore

The difference of communication path between complete graph and PDN $=(3\delta^2+3\delta+3)-(\delta^3+\delta^2+\delta)$

$=3\delta^2+3\delta+3-\delta^3-\delta^2-\delta$

$=-\delta^3+2\delta^2+2\delta$+3 hence proved

2.1 Vertex to Vertex Connectivity Matrix-

Vertex –vertex connectivity matrix directly gives edge connectivity set for each vertex from matrix. It also gives vertex set for each edge where as vertex –vertex matrix we can derive from the graph by using vertex-vertex matrix set for each edge the beauty of vertex –vertex connectivity is that it gives bidirectional vertex-vertex pairs for each edge.

M1 =

	v1	V2	V3	V4	V5	V6	V7	Connectivity percent
V1	0	1	0	1	1	0	1	(4/7)*100=57.14%
V2	1	0	1	0	1	1	0	(4/7)*100=57.14%
V3	0	1	0	1	0	1	1	(4/7)*100=57.14%
V4	1	0	1	0	1	0	1	(4/7)*100=57.14%
V5	1	1	0	1	0	1	0	(4/7)*100=57.14%
V6	0	1	1	0	1	0	1	(4/7)*100=57.14%
V7	1	0	1	1	0	1	0	(4/7)*100=57.14%
Connectivity percent	(4/7)*100 =57.14%	(4/7)*100 =57.14%	(4/7)*100 =57.14%	(4/7)*100 =57.14%	(4/7)*100 =57.14%	(4/7)*100 =57.14 %	(4/7)*100 =57.14%	57.14%

Table1: Vertex to Vertex Connectivity of PDN for δ=2

M2 =

|	V1	V2	V3	V4	V5	V6	V7	Connectivity percent
V1	0	1	1	1	1	1	1	(6/7)*100=85.71%
V2	1	0	1	1	1	1	1	(6/7)*100=85.71%
V3	1	1	0	1	1	1	1	(6/7)*100=85.71%
V4	1	1	1	0	1	1	1	(6/7)*100=85.71%
V5	1	1	1	1	0	1	1	(6/7)*100=85.71%
V6	1	1	1	1	1	0	1	(6/7)*100=85.71%
V7	1	1	1	1	1	1	0	(6/7)*100=85.71%
Connectivity percent	85.71%	85.71%	85.71%	85.71%	85.71%	85.71%	85.71%	85.71%

Table2: Vertex to Vertex Connectivity of Complete Graph for δ=2

The following observation on vertex-to-vertex connectivity matrix of PDN and complete graph made:

1. Both connectivity matrices show properties of the symmetric matrix.
2. The percent of connectivity of each vertex or node shows well connected node or vertex.

In PDN, connectivity matrix each row and column vector has exactly four 1's which signify that every node of PDN has four communication link incident on it.

Node v1= 0 1 0 1 1 0 1

Here we can see that (directly from matrix) v1 is connected to {v2, v4, v5, v7} through direct communication link.

Similarly v2 = {v1, v 3, v5, v6} etc.

In complete graph, connectivity matrix each row and column vector has exactly six 1's, which signify that every node of complete graph is connected through six communication links.

Node v1= 0 1 1 1 1 1 1

Here we can found directly from matrix v1 is connected to {v2, v3, v4, v5, v6, v7} through direct communication link.

3. Both connectivity matrix shows reverse patterns of matrix value is same that is reverse connectivity will be establish therefore OR operation gives tautology means maximum connectivity. On the other hand same pattern between nodes will give one directional connectivity and AND operation not given tautology means minimum connectivity.

Lemma2: A regular graph is derived by removing the property of PDN in complete graph.

Proof: Connectivity matrix of PDN (M1) has four 1's in each vector. Number of 1 in each vector constitutes robust connection between nodes.

Let Node v1 = 0 10 1 1 0 1

And from table2 connectivity matrix of complete Graph (M2) has six 1's in each node. Number of 1 in each vector constitutes robust connection between nodes.

Let Node V1 = 0 1 1 1 1 1 1

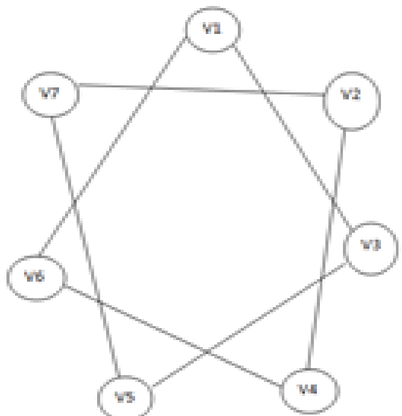
If we subtract v1 vector of PDN to v1 vector of complete Graph then we obtain new v1 vector is

V1 = 0 0 1 0 0 1 0

Similarly we can apply same operation node to node and finally obtain a new matrix that is deigning a new interconnection network.

M2-M1=

	v1	V2	V3	V4	V5	V6	V7
V1	0	0	1	0	0	1	0
V2	0	0	0	1	0	0	1
V3	1	0	0	0	1	0	0
V4	0	1	0	0	0	1	0
V5	0	0	1	0	0	0	1
V6	1	0	0	1	0	0	0
V7	0	1	0	0	1	0	0



Regular Graph

Now using the value of third connectivity matrix we can show that if Perfect Difference network connectivity matrix values are subtracted with complete graph connectivity matrix then resultant matrix designs a regular graph because of in connectivity matrix every row or column vector contain exactly two 1's i.e. every node has degree 2 and it is also symmetric matrix pattern contain.

Lemma3: The Dense Connectivity of the complete graph shows the Tautology for each node.

Proof: A dense connectivity in a graph is the number of edges is close to the maximal number of edges. The complete graph is shows dense connectivity because all nodes are strongly connected with each other.

We have taken the value of nodes from matrix M2 and apply OR logical operation then we found tautology between nodes

Let V1= 0 1 1 1 1 1 1 and V2= 1 0 1 1 1 1 1

Then

V1 ∨ V2 = 1 1 1 1 1 1 1 (Tautology)

Similarly

V1∨ V3 = (0 1 1 1 1 1 1) ∨ (1 1 0 1 1 1 1) = 1 1 1 1 1 1 1 (Tautology)

V1∨ V4 = (01 1 1 1 1 1) ∨ (1 1 1 0 1 1 1) = 1 1 1 1 1 1 1 (Tautology)

V1∨ V5 = (01 1 1 1 1 1) ∨ (1 1 1 1 0 1 1) = 1 1 1 1 1 1 1 (Tautology)

V1∨ V6 (01 1 1 1 1 1) ∨ (1 1 1 1 1 0 1) = 1 1 1 1 1 1 1 (Tautology)

V1∨ V7 = (01 1 1 1 1 1) ∨ (1 1 1 1 1 1 0) = 1 1 1 1 1 1 1 (Tautology)

Hence proved

Now we are proved that at every node of the complete graph shows tautology. Because matrix shows reverse pattern of matrix values which is same that is reverse connectivity will be establish.

2.2 Edge to edge connectivity Matrix-

We know that edge to edge connectivity gives spanning graph between the nodes /vertex .It also gives edge set of each vertex. By calculating edge set for vertex we can find out degree of each vertex. Edge to edge connectivity gives the degree and set theoretic notations are required to use for interpretation of these matrix.

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	connectivity Percent
E1	1	1	1	1	0	0	0	1	0	0	1	0	0	1	7/14*100=50%
E2	1	1	1	1	0	1	0	0	0	0	0	1	1	0	7/14*100=50%
E3	1	1	1	1	0	0	0	0	0	1	1	1	0	0	7/14*100=50%
E4	1	1	1	1	1	1	1	0	0	0	0	0	0	0	7/14*100=50%
E5	0	0	0	1	1	1	1	0	1	0	0	0	1	1	7/14*100=50%
E6	0	1	0	1	1	1	1	0	0	0	0	1	1	0	7/14*100=50%
E7	0	0	0	1	1	1	1	1	1	0	0	0	0	0	7/14*100=50%
E8	1	0	0	0	0	0	1	1	1	1	1	0	0	1	7/14*100=50%
E9	0	0	0	0	1	0	1	1	1	1	0	0	1	1	7/14*100=50%
E10	0	0	1	0	0	0	1	1	1	1	1	1	0	0	7/14*100=50%
E11	1	0	1	0	0	0	0	1	0	1	1	1	0	1	7/14*100=50%
E12	0	1	1	0	0	1	0	0	0	1	1	1	1	0	7/14*100=50%
E13	0	1	0	0	1	1	0	0	1	0	0	1	1	1	7/14*100=50%
E14	1	0	0	0	1	0	0	1	1	0	1	0	1	1	7/14*100=50%
connectivity Percent	50%	50%	50%	50%	50%	50%	50%	50%	50%	50%	50%	50%	50%	50%	50%

Table3: Edge to Edge connectivity of PDN for 7 nodes

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15	E16	E17	E18	E19	E20	E21	Connectivity percent
E1	1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	1	0	0	1	1	1	11/21*100= 52.38%
E2	1	1	1	1	1	1	0	1	0	0	0	0	1	0	0	0	1	0	0	1	1	52.38%
E3	1	1	1	1	1	1	0	0	1	0	0	0	0	1	0	0	0	1	1	1	0	52.38%
E4	1	1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	52.38%
E5	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	52.38%
E6	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	52.38%
E7	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	1	0	1	52.38%
E8	0	1	0	0	0	1	1	1	1	1	1	0	1	0	0	0	1	0	0	1	1	52.38%
E9	0	0	1	0	0	1	1	1	1	1	1	0	0	1	0	0	0	1	1	1	0	52.38%
E10	0	0	0	1	0	1	1	1	1	1	1	0	0	0	1	1	1	1	0	0	0	52.38%
E11	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	52.38%
E12	1	0	0	0	1	0	1	0	0	0	1	1	1	1	1	1	0	0	1	0	1	52.38%
E13	0	1	0	0	1	0	0	1	0	0	1	1	1	1	1	0	1	0	0	1	1	52.38%
E14	0	0	1	0	1	0	0	0	1	0	1	1	1	1	1	0	0	1	1	1	0	52.38%
E15	0	0	0	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	52.38%
E16	1	0	0	1	0	0	1	0	0	1	0	1	0	0	1	1	1	1	1	0	1	52.38%
E17	0	0	1	1	0	0	0	1	0	1	0	0	1	0	1	1	1	1	0	1	1	52.38%
E18	0	0	1	1	0	0	0	0	1	1	0	0	0	1	1	1	1	1	1	1	0	52.38%
E19	1	0	1	0	0	0	1	0	1	0	0	1	0	0	0	1	0	1	1	1	1	52.38%
E20	0	1	1	0	0	0	0	1	1	0	0	0	1	1	0	0	1	1	1	1	1	52.38%
E21	1	1	0	0	0	0	1	1	0	0	0	1	1	0	0	1	1	1	1	1	0	52.38%
Connectivity percent	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%	52.38%

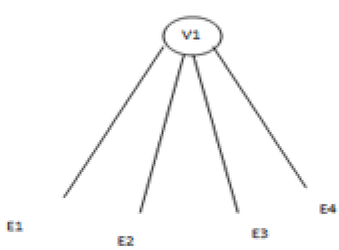
Table4: Edge to Edge connectivity of Complete Graph for 7 nodes

Edge to edge connectivity matrix of both interconnection networks is square matrix but not a symmetric matrix.

Lemma3: Edge to Edge connectivity gives spanning graph between the nodes /vertex.

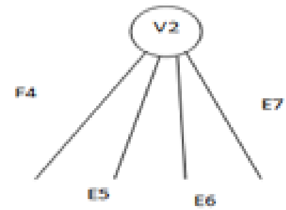
Proof: A spanning graph of a connected graph can also be defined as a maximal set of edges of graph that contains no cycle or as a minimal set of edges that connect all vertices. Using the connectivity matrix we can found that all spanning graph between nodes.

Node1: E1E2=E2E1=V1, E1E3=E3E1=V1, E1E4=E4E1=V1, E2E3=E3E2=V1, E2E4=E4E2=V1, E3E4=E4E3=V1



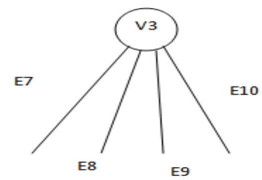
Spanning graph of node V1

Node2: E4E5=E5E4=V2, E4E6=E6E4=V2, E4E7=E7E4=V2, E5E6=E6E5=V2, E5E6=E6E5=V2, E6E7=E7E6=V2



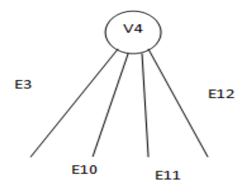
Spanning graph of node V2

Node 3: E7E8=E8E7=V3, E7E9=E9E7=V3, E7E10=E10E7=V3, E8E9=E9E8=V3, E8E10=E10E8=V3, E9E10=E10E9=V3



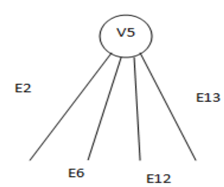
Spanning graph of node V3

Node 4: E10E3=E3E10=V4, E10E11=E11E10=V4, E10E12=E12E10=V4, E11E12=E12E11=V4, E12E3=E3E12=V4, E11E3=E3E11=V4



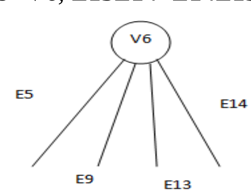
Spanning graph of node V4

Node 5: E2E6=E6E2=V5, E2E12=E12E2=V5, E2E13=E13E2=V5, E6E12=E12E6=V5, E6E13=E13E6=V5, E12E13=E13E12=V5



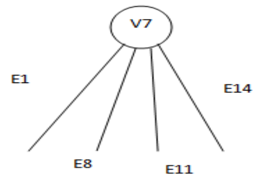
Spanning graph of node V5

Node 6: E5E9=E9E5=V6, E5E13=E13E5=V6, E5E14=E14E5=V6, E9 E13=E13E9=V6, E9E14=E14E9=V6, E13E14=E14E13=V6



Spanning graph of node V6

Node 7: E1E8=E8E1=V7, E1E11=E11E1=V7, E1E14=E14E1=V7, E8E11=E11E8=V7, E8E14=E14E8=V7, E11E14=E14E11=V7



Spanning graph of node V7

Now using the spanning graph we have found degree of each node as follows:

d (V1) = 4
d (V2) = 4
d (V3) = 4
d (V4) = 4
d (V5) = 4
d (V6) = 4
d (V7) = 4

Then the total degree of PDN = 42

3.CONCLUSIONS

Parallel systems in interconnection network have focused on how to improve connectivity for understanding & describing topologies of architecture. In this paper we evaluate how a connectivity based approach has initiate new understanding of structural relationship that specify topological properties for overcoming existing constraints and improving the complexity of Interconnection Network .The percent of connectivity between each edge and percent of connectivity between each vertex is shown in the Table 1 and Table2 we total percent of connectivity of edges and total percent of connectivity of vertex are always found same in each vectors of vertex and edges of the graph in connectivity matrix. With the help of edge-to-edge matrix we derived, we can find spanning graphs that provide us information about the degree of each nodes/vertexes. In vertex-to-vertex connectivity matrix we found that regular graph property if we removed the property of PDN in complete graphs.

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