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ECCENTRICITY BASED INDICES OF CYCLE GRAPHS

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Abstract

Eccentricity based indices have been successfully employed in facilitating the study of physio-chemical properties of biochemically interesting compounds. In this paper we compute some eccentricity based indices namely total eccentricity, total eccentric connectivity polynomial, eccentric connectivity index and its polynomial, Zagreb eccentricity indices of cycles graph and its middle graphs, central graphs, total graphs, Mycielski graph of cycle graphs, complement of cycles.

Keywords: total eccentricity, total eccentric connectivity polynomial, eccentric connectivity index and its polynomial, Zagreb eccentricity indices.

AMS Subject Classification: 05C07, 05C12, 05C38, 05C76.

1 Introduction

Topological Indices have found applications in characterizing the physio-chemical properties of various molecules. These indices are used in measuring the boiling and melting point of molecules and their further analysis. Eccentricity based indices are useful parameters in prediction of accentric factor of the molecules with high correlation coefficient. They are been studied for analysing physico-chemical, biological, toxicological, pharmacologic characteristics of chemical compounds.

S. M. Kang et al. computed the various distance and eccentricity based invariants of windmill graphs [7], K. Pattabiraman and et al., studied on eccentricity related indices of cartesian product of graphs [9]. J. Zheng et al., has derived eccentricity based topological indices, polynomials of poly ethylene amido amine(PETAA) Dendrimers [16], M.D.Farhani et al., has derived the exact value of eccentric connectivity and connective eccentric indices of generalized Petersen graphs [3], M. Veena and P. Padmapriya has computed few eccentricity based indices for crown graphs, gear, friendship, helm, flower graphs and their line graphs [8]. S. Sujitha and A. E. Surya has made a study on eccentric-distance sum of cycles and related graphs [14].

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In this paper, we compute some eccentricity based indices namely total eccentricity, total eccentric connectivity polynomial, eccentric connectivity index and its polynomial, Zagreb eccentricity indices of cycles graph and its middle graphs, central graphs, total graphs, Mycielski graph of cycle graphs, complement of cycles.

2 Definitions and Preliminaries

Let G be a connected simple graph where *V(G)* is vertex set, *E(G)* is edge set. For any two vertices u, v in G distance between them is denoted by *d(u,v)*, it is the length of the shortest path in G. For a vertex v of G its eccentricity is denoted by *ecc(v)*, it is the maximum distance from vertex u to vertex v in G and *deg(v)* is the number of edges incident with v.

The total eccentricity index of G was introduced by R. Farooq et al. in [10] as

$$\zeta(G) = \sum_{v \in V(G)} ecc(v) \tag{1}$$

The total eccentricity connectivity polynomial was introduced by A. R. Ashrafi et al. in [1] as

$$TECP(G) = \sum_{v \in V(G)} y^{ecc(v)} \tag{2}$$

The First and Second Zagreb eccentricity indices was introduced by Ghorbani et al. in [5] and D. Vukicevic et al. in [15] and defined as

$$E_1(G) = \sum_{v \in V(G)} (ecc(v))^2 \tag{3}$$

$$E_2(G) = \sum_{u,v \in E(G)} ecc(u)\ ecc(v) \tag{4}$$

The eccentric connectivity index of a graph G, was proposed by Sharma, Goswami and Madan [13] as

$$\xi^c(G) = \sum_{v \in V(G)} deg(v)\ ecc(v) \tag{5}$$

The eccentric connectivity polynomial was introduced by Ghorbhani and Hemmasi [4] as

$$ECP(G, y) = \sum_{v \in V(G)} deg(v)\ y^{ecc(v)} \tag{6}$$

Definition 2.1 Let $M(G)$ be the Middle graph of G, whose vertex set is $V(G) \cup E(G)$. $M(G)$ will have two adjacent vertices u, v if (i) these $uv \in E(G)$ and u, v are adjacent in G. (ii) $u \in V(G)$,

$v \in E(G)$ and u, v are incident in G.

Definition 2.2 $T(G)$ is the total graph of graph G with the vertex set $V(G) \cup E(G)$. Two vertices u, v are adjacent in $T(G)$ if (i) $u, v \in V(G)$, if u and v are adjacent in G. (ii) $u, v \in E(G)$, u and v are adjacent in G (iii) $u \in V(G)$, $v \in E(G)$ and u, v are incident vertices.

Definition 2.3 The central graph $C(G)$ of a graph G is obtained from G by adding an extra vertex on each edge of G and then joining each pair of vertices of the original graph which were not adjacent in the graph G.

Definition 2.4 Let G be a graph, G' is the complement of a graph G with same vertex set of G where edges of G' between two vertices u and v if they were not adjacent in the graph.

Definition 2.5 Let G be a graph with $V(G) = \{v_1, v_2, \ldots, v_n\}$ the Mycielski graph $\mu(G)$ of a graph G has vertex set $V(\mu(G)) = \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\} \cup \{w\}$ and edge set $E(\mu(G)) = E(G) \cup \{u_i u_j, v_i v_j \in E(G)\} \cup \{u_i w : 1 \leq i \leq n\}$.

Definition 2.6 Let G be a graph with $V(G) = \{v_1, v_2, \ldots, v_n\}$ the Mycielski graph $\mu(G)$ of a graph G has vertex set $V(\mu(G)) = \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\} \cup \{w\}$ and edge set $E(\mu(G)) = E(G) \cup \{u_i u_j, v_i v_j \in E(G)\} \cup \{u_i w : 1 \leq i \leq n\}$.
In this section, we compute the above mentioned eccentricity based indices for the cycle graph, complement of cycle graph, Middle graph of cycle graph, central graph of cycle graph, total graph of cycle graph, Mycielski graph of cycle graph.

3 Results

Theorem 3.1. *let C_n be a cycle graph then*
(i) If n is even (1) $\zeta(C_n) = n^2$
(2) $TECP(C_n, y) = ny^{\frac{n}{2}}$
(3) $E_1(C_n) = \frac{n^3}{4}$
(4) $E_2(C_n) = \frac{n^3}{4}$
(5) $\xi^c(C_n) = n^2$
(6) $ECP(C_n, y) = 2ny^{\frac{n}{2}}$
(ii) If n is odd
(1) $\zeta(C_n) = \frac{n(n-1)}{2}$
(2) $TECP(C_n, y) = ny^{\frac{n-1}{2}}$

(3) $E_1(C_n) = \frac{n(n-1)^2}{4}$
(4) $E_2(C_n) = \frac{n(n-1)^2}{4}$
(5) $\xi^c(C_n) = n(n-1)$
(6) $ECP(C_n, y) = 2ny^{\frac{n-1}{2}}$

Proof. The cycle is 2-regular graph having n vertices and n edges. The eccentricity of every vertex in C_n is $\frac{n^2}{2}$ if n is even integer and the eccentricity of every vertex in C_n is $\frac{n(n-1)}{2}$ if n is odd integer. From the mentioned formulae in (1), (2), (3), (4), (5), (6) we obtain the results on straightforward computation. □

Theorem 3.2. *let C_n be a cycle graph then , C'_n be the complement graph of the C_n then for all $n \geq 5$*
(1) $\zeta(C'_n) = 2n$
(2) $TECP(C'_n, y) = ny^2$
(3) $E_1(C'_n) = 4n$
(4) $E_2(C'_n) = 2n(n-3)$
(5) $\xi^c(C'_n) = 2n(n-3)$
(6) $ECP(C'_n, y) = n(n-3)y^2$.

Proof. Let C'_n be the complement of a cycle graph C_n, the complement of C_3, C_4 are disconnected graphs. Hence, we compute eccentricity indices for $n \geq 5$. The number of vertices in C'_n is n and the number of edges is $\frac{n(n-3)}{2}$. The eccentricity of every vertex in C'_n graph is 2 for all n and degree of the each vertex is $(n-3)$. Substituting these data in (1), (2), (3), (4), (5), (6) we obtain the results. □

Theorem 3.3. *Let C_n be a cycle graph, $M(C_n)$ represents the middle graph of C_n graph. Then for*
(i) n is even.
$(1)\zeta((C_n)) = n(n+1)$
(2) $TECP(M(C_n, y)) = n(y^{\frac{n}{2}} + y^{(\frac{n}{2}+1)})$
(3) $E_1(M(C_n)) = \frac{n^3}{2} + n^2 + n$
(4) $E_2(M(C_n)) = \frac{3n^3}{4} + n^2$
(5) $\xi^c(M(C_n) = 3n^2 + 2n$
(6) $ECP(M(C_n, y)) = 2ny^{\frac{n}{2}+1} + 4ny^{\frac{n}{2}}$
(ii) n is odd
(1) $\zeta(M(C_n)) = n(n+1)$
(2) $TECP(M(C_n, y)) = 2ny^{\frac{n+1}{2}}$
(3) $E_1(M(C_n)) = \frac{n(n+1)^2}{2}$
(4) $E_2(M(C_n)) = \frac{3}{4}n(n+1)^2$
(5) $\xi^c(M(C_n) = 3n(n+1)$
(6) $ECP(M(C_n, y)) = 6ny^{\frac{n+1}{2}}$.

Proof. The middle graph of cycle graph $M(C_n)$ is the graph with vertex set $V = U \cup W$ in which set $U = \{u_i\}$ has n vertices and set $W = \{w_i\}$ has n vertices where $i = 1, 2, 3, \ldots, n$ and the edge set in the form of $E = \{u_i w_i, w_i w_{i+1}, \forall i = 1, 2, 3, \ldots, n\}$. The middle graph of cycle graph $M(C_n)$ has $2n$ vertices and $3n$ edges. The eccentricity of a vertex u_i of U in $M(C_n)$ is $\frac{n}{2} + 1$, the eccentricity of a vertex w_i of W in $M(C_n)$ is $\frac{n}{2}$ for n is even. The eccentricity of a vertex u_i of U and w_i of W in $M(C_n)$ is $\frac{n+1}{2}$ for n is odd.

Table 1: The vertex partition of the $M(C_n)$ with their degrees, eccentricities, frequencies for n is even and n is odd is as follows:

n is even				n is odd			
Type	frequency	eccentricity	degree	Type	frequency	eccentricity	degree
u_i	n	$\frac{n}{2} + 1$	2	u_i	n	$\frac{n+1}{2}$	2
w_i	n	$\frac{n}{2}$	4	w_i	n	$\frac{n+1}{2}$	4

Table 2: The edge partition of the $M(C_n)$ with their eccentricities, frequencies for n is even and odd is as follows:

n is even			n is odd		
Type	frequency	eccentricity	Type	frequency	eccentricity
$u_i w_i$	$2n$	$(\frac{n}{2} + 1, \frac{n}{2})$	$u_i w_i$	$2n$	$(\frac{n+1}{2}, \frac{n+1}{2})$
$w_i w_{i+1}$	n	$(\frac{n}{2}, \frac{n}{2})$	$w_i w_{i+1}$	n	$(\frac{n+1}{2}, \frac{n+1}{2})$

Using the data from **Tables 1, 2** and applying in the formulae $(1), (2), (3), (4), (5), (6)$ we deduce the above mentioned results. □

Theorem 3.4. *Let C_n be a cycle graph and $C(C_n)$ represent the central graph of C_n. Then for $n \geq 4$ we have the following*

(1) $\zeta(C(C_n)) = 5n$

(2) $TECP(C(C_n), y) = n(y^3 + y^2)$

(3) $E_1(C(C_n)) = 13n$

(4) $E_2(C(C_n)) = 2n^2 + 6n$

(5) $\xi^c(C(C_n) = 2n^2 + 4n$

(6) $ECP(C(C_n), y) = 2ny^3 + n(n-1)y^2$.

Proof. The central graph of cycle graph $C(C_n)$ is a graph with the vertex set $V = U \cup W$ where set $U = \{u_i\}$ has n vertices and set $W = \{w_i\}$ has n vertices where $i = 1, 2, 3, \ldots, n$ and the edge set in the form of $E = \{u_i u_{i+1}, u_i w_i, \forall i = 1, 2, 3, \ldots, n\}$. The central graph of a cycle graph $C(C_n)$ has $2n$ vertices and $\frac{n(n+1)}{2}$ edges. Based on the nature of $C(C_3)$ and other central graph of cycle graphs we compute the eccentricity based indices of $C(C_3)$ is $\zeta(C(C_3)) = 12$, $TECP(C(C_3), y) = y^3$, $E_1(C(C_3)) = 54$, $E_2(C(C_3)) = 54$, $\xi_c(C(C_3) = 36$, $ECP(C(C_3), y) = 12y^3$. We generalize the computations for $n \geq 4$. The eccentricity of the vertex u_i of U in $C(C_n)$ is 2 and eccentricity of the vertex w_i of W in $C(C_n)$ is 3.

Table 3: The vertex and edge partition of the $C(C_n)$ for $n \geq 4$ is as follows:

Vertex partition				Edge partition		
type	frequency	eccentricity	degree	type	edges	eccentricity
u_i	n	2	$n-1$	$u_i u_{i+1}$	$\frac{n(n-3)}{2}$	$(2, 2)$
w_i	n	3	2	$u_i w_i$	$2n$	$(2, 3)$

Using the data from **Table** 3 and applying in the formulae $(1), (2), (3), (4), (5), (6)$ we get the mentioned results. □

Theorem 3.5. *Let C_n be a cycle graph and $T(C_n)$ represents the total graph of C_n.*
Then for (i) n is even
(1) $\zeta(T(C_n)) = n^2$
(2) $TECP(T(C_n), y) = 2ny^{\frac{n}{2}}$
(3) $E_1(T(C_n)) = \frac{n^3}{2}$
(4) $E_2(T(C_n)) = n^3$
(5) $\xi^c(T(C_n) = 4n^2$
(6) $ECP(T(C_n), y) = 8ny^{\frac{n}{2}}$
(ii) n is odd
(1) $\zeta(T(C_n)) = n(n+1)$
(2) $TECP(T(C_n), y) = 2ny^{\frac{n+1}{2}}$
(3) $E_1(T(C_n)) = \frac{n(n+1)^2}{2}$
(4) $E_2(T(C_n)) = n(n+1)^2$
(5) $\xi^c(T(C_n) = 4n(n+1)$
(6) $ECP(T(C_n), y) = 8ny^{\frac{n+1}{2}}$.

Proof. The total graph $T(C_n)$ has the vertex set $V = U \cup W$ in which set $U = \{u_i\}$ has n vertices and set $W = \{w_i\}$ has n vertices where $i = 1, 2, 3, \ldots, n$ and the edge set in the form of $E = \{u_i u_{i+1}, u_i w_i, u_i w_{i+1}, w_i u_{i+1}, w_i w_{i+1}, \forall i = 1, 2, 3, \ldots, n\}$. $T(C_n)$ is a 4-regular graph having $2n$ vertices and $4n$ edges.

Table 4: Vertex partition of $T(C_n)$ with degrees, eccentricities for n is even and odd as follows:

n is even				n is odd			
type	frequency	eccentricity	degree	type	frequency	eccentricity	degree
u_i	n	$\frac{n}{2}$	4	u_i	n	$\frac{n+1}{2}$	4
w_i	n	$\frac{n}{2}$	4	w_i	n	$\frac{n+1}{2}$	4

Table 5: Edge partition of $T(C_n)$ with degrees, eccentricities for n is even and odd as follows:

n is even			n is odd		
type	edge	eccentricity	type	edge	eccentricity
u_iu_{i+1}	n	$(\frac{n}{2}, \frac{n}{2})$	u_iu_{i+1}	$2n$	$(\frac{n+1}{2}, \frac{n+1}{2})$
w_iw_{i+1}	n	$(\frac{n}{2}, \frac{n}{2})$	w_iw_{i+1}	n	$(\frac{n+1}{2}, \frac{n+1}{2})$
u_iw_i	$2n$	$(\frac{n}{2}, \frac{n}{2})$	u_iw_i	$2n$	$(\frac{n+1}{2}, \frac{n+1}{2})$

Using the data from **Tables** $4, 5$ and applying in $(1), (2), (3), (4), (5), (6)$ we get the mentioned results. $\square$

Theorem 3.6. *Let C_n be a cycle graph and $\mu(C_n)$ represents the Mycielski graph of C_n. Then we have the following cases.*

Case(1): $n \leq 7$

(i) n is even

(1) $\zeta(\mu(C_n)) = n^2 + 2$

(2) $TECP(\mu(C_n), y) = 2ny^{\frac{n}{2}} + y^2$

(3) $E_1(\mu(C_n)) = \frac{n^3}{2} + 4$

(4) $E_2(\mu(C_n)) = \frac{3}{4}n^3 + n^2$

(5) $\xi^c(\mu(C_n) = \frac{7}{2}n^2 + 2n$

(6) $ECP(\mu(C_n), y) = 7ny^{\frac{n}{2}} + ny^2$

(ii) n is odd

(1) $\zeta(\mu(C_n)) = n(n-1) + 2$

(2) $TECP(\mu(C_n), y) = 2ny^{\frac{n-1}{2}} + y^2$

(3) $E_1(\mu(C_n)) = \frac{n(n-1)^2}{2} + 4$

(4) $E_2(\mu(C_n)) = \frac{3}{4}n^3 - \frac{1}{2}n^2 - \frac{1}{4}n$

(5) $\xi^c(\mu(C_n) = \frac{7}{2}n^2 - \frac{7}{2}n + 2n$

(6) $ECP(\mu(C_n), y) = 7ny^{\frac{n-1}{2}} + ny^2$

Case(2): *for* $n \geq 8$

(1) $\zeta(\mu(C_n)) = 7n + 2$

(2) $TECP(\mu(C_n), y) = ny^4 + ny^3 + y^2$

(3) $E_1(\mu(C_n)) = 25n + 4$

(4) $E_2(\mu(C_n)) = 46n$

(5) $\xi^c(\mu(C_n) = 27n$

(6) $ECP(\mu(C_n), y) = 4ny^4 + 3ny^3 + ny^2$.

Proof. Let C_n be a cycle graph, the Mycielski graph of C_n denoted by $\mu(C_n)$ has vertex set $V = U \cup W \cup \{z\}$ where set $U = \{u_i\}$ has n vertices and set $W = \{w_i\}$ has n vertices where $i = 1, 2, 3, \ldots, n$ and the edge set in the form of $E = \{u_i u_{i+1}, u_i w_i, u_i w_{i+1}, u_i z, \forall i = 1, 2, 3, \ldots, n\}$. $\mu(C_n)$ has $2n + 1$ vertices and $4n$ edges.

Case(1):

Table 6**:** The vertex partition of $\mu(C_n)$ for $n \leq 7$ with degrees and eccentricities are as follows:

n is even				n is odd			
type	frequency	eccentricity	degree	type	frequency	eccentricity	degree
u_i	n	$\frac{n}{2}$	4	u_i	n	$\frac{n-1}{2}$	4
w_i	n	$\frac{n}{2}$	3	w_i	n	$\frac{n-1}{2}$	3
z	1	2	n	z	1	2	n

Table 7**:** The edge partition of $\mu(C_n)$ for $n \leq 7$ with eccentricity as follows:

n is even			n is odd		
type	edge	eccentricity	type	edge	eccentricity
$u_i u_j$	n	$(\frac{n}{2}, \frac{n}{2})$	$u_i u_j$	$2n$	$(\frac{n-1}{2}, \frac{n-1}{2})$
$u_i w_j$	$2n$	$(\frac{n}{2}, \frac{n}{2})$	$u_i w_j$	$2n$	$(\frac{n-1}{2}, \frac{n-1}{2})$
$u_i z$	n	$(\frac{n}{2}, 2)$	$u_i z$	n	$(\frac{n-1}{2}, 2)$

Using the data from **Tables** 6, 7 and applying in (1), (2), (3), (4), (5), (6) we get the mentioned results.

Case(2): $n \geq 8$

Table 8**:** The vertex and edge partition of $\mu(C_n)$ for $n \geq 8$ as follows:

Vertex partition				Edge partition		
type	frequency	eccentricity	degree	type	edges	eccentricity
u_i	n	4	4	$u_i u_j$	n	$(4, 4)$
w_i	n	3	3	$u_i w_i$	$2n$	$(4, 3)$
z	1	2	n	$w_i z$	n	$(3, 2)$

Using the data from **Table** 8 and applying in (1), (2), (3), (4), (5), (6) we get the mentioned results. □

4 conclusion

Eccentricity and Distance based indices can be studied for the middle graphs, central graphs, total graphs, Mycielski graphs of various other graphs.

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