A Mathematical Imperfect Production Inventory Problem for Perishable Items Considering Two Different Warehouses under Fuzzy Environment

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Abstract **- This paper presents a mathematical framework to obtain a production model for perishable items with a learning effect in production cost. The study considers different demand rates at different. This model is along with the concept of two warehouses, where one is its own warehouse (OW) and the other is a rented warehouse (RW). The demand rate for RW is strictly increasing function of time which is exponential, and selling price of inventory in own warehouse in such a way that the producer gets the benefit and can build the according to his profit, also production rate is dependent on demand. Hethe re concept of shortage is also considered. In this model, we analyzed the changes in the total cost by taking item price and production time as a decision variable. The optimal solution has presented with the use of the centroid method with q- fuzzy number. A numerical examples and sensitivity analyses of some parameters are provided to examine the impact on the optimal total cost of the system.**

Index Terms **- Two-warehouses, imperfect production, shortage, deterioration, learning effect.**

1.INTRODUCTION

The most important thing in the production inventory model is how the inventory is produced. The unrealistic assumption of the economic production quantity (EPQ) model is that a machine is always in an 'in control' state and the product produced from it is of perfect and good quality. It would be unfair to say that 100% of the items will be perfect products because there are many reasons, it is not always possible to produce perfect items. Therefore, for many years, many researchers have published their research papers regarding the Imperfect Product in inventory model, in which. Portius [1] was the first, who investigated that whenever a large quantity of production has to be done, the machine gets out of control, due to which, along with perfect items, machine produces some amount of imperfect quality items. He attempted to improve production systems by reducing setup costs. Daya [2] presented an integrated inventory model for the assessment of EPQ and preventive maintenance level under imperfect production process having deterioration with increasing hazard rate. Chung and Hou [3] proposed a model to determine the optimal run time for a deteriorating production system. Shortages are allowed, which are fully backlogged and the demand rate is constant and deterministic. The time until the production process shift is arbitrarily distributed and shows that a unique optimal production run time exists to reduce total costs. Chung et al. [4] presented a limited storage production model for imperfect quality items. The basic assumption for the development of the proposed model is that defective goods are sold together in a batch at a discounted price. Maddah et al. [5] introduced the EOQ model having two states markovian process, the first of which hypothesized that imperfect quality is removed from inventory at no cost and the second inventory model assumes that items of imperfect quality are shipped together. Sana [6] introduced a production inventory model having an imperfect production process by considering the fact that if machine is out-of-control state, then certain percent of total lot-size to be defective. He also assumes that the production rate is time dependent, and the unit production cost is a function of production rate and product reliability parameter. Total cost is maximized by Euler–Lagrange's method under the effect of inflation and time value of money. Singh and Urvashi [7] proposed an inventory model for imperfect items having fuzzy demand rate, where production rate are volume flexible in nature. Simultaneously, it is hypothesized that, along with the production facility, there are separate management units for each commodity. Jaber et al [8] presented two EPQ models in which the first model hypothesizes that imperfect goods are sent for repair on which a markup margin is paid, while the second model assumes that imperfect items are replaced by good ones at a higher cost from a local supplier. Pal et al. [9] presented a two-echelon production inventory model over two cycles involving single supplier and single retailer. In the first cycle, the retailer sells only the good product with the original price and in the second, a cash discount is provided by him on selling the products. The defective items are repaired to sell in the markets. Jaggi et al. [10] considered an imperfect production model having two-warehouse facility for deteriorating items. First they keep the inventory into own warehouse after that in the rented warehouse, the preservation facilities of rented warehouse is better than the own warehouse. When the screening process is in process, the products which are found to be of perfect quality, to meet the demand, the screening rate is considered to be higher than the demand rate. Sarkar and Saren [11] described an inspection policy for imperfect items by performing product inspection policy by using type one and type two error. They assume that if the system is in the out-of-control, this state remains same until the production is run. Defective items are separated through the product inspection and are reworked at some cost. Total cost is minimized by the product inspection policy and production-run length.

There are many products in the market, where price fluctuation creates a lot of uncertainty. Fuzzy set theory mainly deals with quantitative analysis of uncertainty and imprecision. Therefore, fuzzy set theory plays an important role to develop an inventory management in better way. In the last few years. The main function of fuzzy set theory is how to deals with objects related to uncertainty. A lot of factors in market create uncertainty in reality. Therefore, the fuzzy set theory is playing a very important role for inventory research by developing models using this concept. The use of fuzzy models in inventory theory indicates a very important experiment. Lot of researchers have developed an inventory model with the help of fuzzy set theory. Halim et al. [12] described an EPQ model for imperfect items under fuzzy environment. Jaggi et al. [13] analysed an inventory model for time varying demand of with deteriorating items and shortage in which the total cost was discussed in fuzzy sense. Demand of items depends on various factors viz. price, advertisement, quality of products etc. Sharmila and Uthayakumar [14] formulated the model for the deteriorating items with exponential demand in which deteriorating cost, holding cost, shortage cost and purchasing cost are taken as triangular fuzzy number. De and Beg [15] explained the triangular dense fuzzy sets and gave different method of solving the problem based on optimization. Saha [16] developed a fuzzy model in which the demand was price dependent and he used the singed distance method to defuzzify the cost function.

Deterioration refers to the decay of a substance. Foods, medicines, chemicals, electric appliances, etc. can deteriorate during the period of storage. The deterioration of any substance after a certain time period is a natural property, so researchers considered it necessary to develop an inventory model keeping this factor in mind. In recent years, many researchers have developed properties related to perishable items of objects in inventory modeling. At present, there are lot of products in the market which get spoiled very quickly, so deterioration is playing an important role in inventory research. Rau et al. [17] have worked on supply chain system for deteriorating items, in which production and demand rate are constant also shortage are not allowed in any part of this chain. Singh and Singh [18] developed a two-level supply chain model for Weibull distribution deteriorating items and prepared the entire model in fuzzy theory and inflationary environment by giving importance to the imperfect production process, where demand was exponential and production rate is scalar multiple of demand. Rani et al. [19] introduced the inventory model for deteriorating items through green supply chain with variable demand in inflationary environment. Panda et al. [20] discussed an inventory model for deteriorating goods with storage stock problem and stock price dependent demand rate. Singh et al. [21] introduced two level of storage model for deteriorating items with stock dependent demand Shaikh et al. [22] developed an inventory model for deteriorating items with ramp type demand under trade credit and preservation technology.

Generally, whenever suppliers offer discounts on bulk purchases, the inventory manager may purchase more goods than the actual capacity of own warehouse. In this situation, the major problem that arises in front of inventory manager is the availability of sufficient space to store the goods. Often, instead of building a new warehouse, hiring a nearby warehouse is more beneficial from a financial point of view. Due to the better preservation features in rented warehouse, inventory costs, which include holding costs and deterioration costs, are generally higher than in an owned warehouse. Therefore, in order to reduce the holding cost and deterioration cost, the rental warehouse should be as close to the showroom as possible, and the goods kept in the rented warehouse should be consumed first. Hartley [23] was the first author who presented a two -warehouse inventory model for determining the optimal order quantity. Pakkala and Achary [24] presented a deteriorating item inventory model with two storage facility having finite replenishment rate. Due to the differences in preservation facilities in both the warehouses, the rate of deterioration is also different, but they assume that this model is also applicable even if rate of deterioration is same in both the warehouses. By assuming the constant production rate, uniform demand and with allowable shortage they formulate the model for continuous release pattern. Bhunia and Maiti [25] discussed the issue of deterministic EOQ model with two storage facility having different level of deterioration. This model assumes the demand as a linear function of time and the replenishment rate is taken as infinite. Shortages are allowed and excess demand over supply is backlogged. Ishii and Nose [26] proposed an inventory model for a single perishable item consisting of two types of customers with different selling price and the shortages of items in a two-warehouse inventory system. Zhou [27] considered a multi warehouse inventory model consisting of single own warehouse and Multiple rented warehouses. The demand of the item is varying with time and shortages are also allowed in this model which are partially backlogged. This model also assumes that replenishment rate is infinite and lead time is zero. Yang [28] considered an inventory model for deteriorating item with shortage having two storage system under the consideration of inflation. The inventory costs in RW are higher than those in OW. Wee et al. [29] presented an inventory model for the items having Weibull distribution deterioration and partially backlogging shortage for a two-warehouse inventory system. This model is also discussed under inflationary environment and approach of discount cash flow is taken in this model. Teng et al [30] gives a note on inventory model with limited storage capacity under the effect of two level of trade credit policy and derived without derivatives. Jaggi et al [31] develop an inventory model for deteriorating items having two storage facility with price sensitive demand and considered optimal shipment policy. The model optimises the order quantity and selling price of the item. Dem and Singh [32] investigated a two- warehouse inventory model for deteriorating items, which follows the time dependent demand pattern. This model considered the production of imperfect items, shortages are allowed which are partially backlogged. Due to the complexities of solution Heuritstic approach taken to obtain the solution. Yang [33] presented a two-warehouse inventory model having weibull distribution deterioration with partially backlogging shortages. The whole model is studied under the effect of inflation and with the help of this model they tried to formulate the replenishment policy which minimize the total cost per unit time. Bhunia et al [34] deals with a two- warehouse inventory model for single deteriorating items under the permissible delay in payment. The preservation facilities are different in both the warehouses. Demand is known and constant, shortages are allowed and are partially backlogged. Tiwari et al [35] discuss the impact of trade credit and inflation on the basis of ordering policies proposed by the retailer for deteriorating items with partially backlogging shortage in a two -warehouse inventory system. Kumar et al. [36] developed an inventory model for the items having variable deterioration rate and stock dependent demand and partially backlogging shortages.

2. ASSUMPTIONS AND NOTATION

2.1 Assumption

1. Model is developed for single item.

2. Initial inventory level is zero and Lead time is zero.
3. Shortage is allowed in this model.
4. The OW has a fixed capacity while the RW has unlimited capacity.
5. Production rate L is dependent upon demand rate of corresponding warehouses. i.e. L= KD_i where i=1,2

6 Demand in RW is exponential which is represented by D_1=ae^{bt} and for OW it is depend on selling price which is represented by $D_2 = ap^{-b}$ where p is selling price, and a, b are constant.

2.2 Notation

- P: production rate(unit);
- W: capacity of own warehouse;(unit);
- α: deterioration in own warehouse (OW); 0<α<1;
- β: deterioration in rented warehouse (RW); 0<β<1;
- D_1: demand rate in own warehouse (OW);
- D_2 demand rate in rented warehouse (RW);

a, b : constant parameters of demand;

- $\left(c_P + \frac{c_0}{n^\delta}\right)$: production cost with learning effect;
- c_P : production cost($/ day);
- δ: backlogging parameter;
- f: setup cost($/setup);
- $h_{r:}$ present worth of holding cost in rented warehouse RW ($/unit);
- h_o : present worth of holding cost in rented warehouse OW ($/unit);
- d: present worth of deterioration cost ($/unit);
- s: present worth of shortage cost ($/unit);
- $I_1(t)$: inventory level in OW at a time t with t ϵ [0, t_1];
- $I_2(t)$: inventory level in RW at a time t with t ϵ [t_1, t_2];
- $I_3(t)$: inventory level in RW at a time t with t ϵ [t_2, t_3];
- $I_4(t)$: inventory level in OW at a time t with t ϵ [t_1, t_3];
- $I_5(t)$: inventory level in OW at a time t with t ϵ [t_3, t_4];
- $I_6(t)$: inventory level in OW at a time t with t ϵ [t_4, t_5];
- $I_7(t)$: inventory level in OW at a time t with t ϵ [t_5,T].

3 MATHEMATICAL FORMULATIONS OF MODEL

Both the warehouses are being used during the production cycle. Initially the level of inventory is t=0 and the production starts from t = 0 and goes up to W units. After t=t1 whatever be the production quantity, it goes to the RW. After that the production is stopped, the level of the inventory decreases in the rented warehouse till the time t=t2 and after that it reaches zero because of the mixed effect of demand and deteriorating on the inventory at t=t3, in OW the level of inventory comes to decrease due to deterioration at t= t1 and then falls below W at t= t3. The remaining inventory in OW will be fully exhausted at t= t4 owing to demand and deteriorating, the level of inventory become zero. At this time the shortage start developing and at time t5 it reaches to maximum shortage level, at this time fresh production start to clear the backlog by the time T. The inventory position can be represented as follows:

$$\frac{dI_1(t)}{dt} = \text{L-}D_1\text{-}\alpha I_1(t). \qquad 0\le t \le t_1 \qquad (1)$$

$$\frac{dI_2(t)}{dt} = \text{L-}D_2\text{- }\beta I_2(t). \qquad t_1 \le t \le t_2 \qquad (2)$$

$$\frac{dI_3(t)}{dt} = \text{-}D_2\text{- }\beta I_3(t). \qquad t_2 \le t \le t_3 \qquad (3)$$

$$\frac{dI_4(t)}{dt} = \text{-}\alpha I_4(t). \qquad t_1 \le t \le t_3 \qquad (4)$$

$$\frac{dI_5(t)}{dt} = \text{-}D_1\text{-}\alpha I_5(t). \qquad t_3 \le t \le t_4 \qquad (5)$$

$$\frac{dI_6(t)}{dt} = \text{-}D_1 \qquad t_4 \le t \le t_5 \qquad (6)$$

$$\frac{dI_7(t)}{dt} = L\text{ -}D_1 \qquad t_5 \le t \le T \qquad (7)$$

Solution of equation (1) to (7) by using the boundary conditions $I_1(0) = 0$, $I_2(t_1) = 0$, $I_3(t_3) = 0$, $I_4(t_1) = W$, $I_5(t_4) = 0$, $I_6(t_4) = 0$, $I_7(T)$ =0 are represented below:

$$I_1(\text{t}) = \frac{(K-1)a}{(b+\alpha)} \left[e^{bt} - e^{-\alpha t}\right] \qquad (8)$$

$$I_2(\text{t}) = \frac{(K-1)ap^{-b}}{\beta}\left[1 - e^{\beta(t_1-t)}\right] \qquad (9)$$

$$I_3(t) = \frac{ap^{-b}}{\beta}\left[e^{\beta(t_3-t)} - 1\right] \qquad (10)$$

$$I_4(\text{t}) = \text{W}e^{\alpha(t_1-t)} \qquad (11)$$

$$I_5(\text{t}) = \frac{a}{(b+\alpha)}\left[e^{bt_4}e^{\alpha(t_4-t)} - e^{bt}\right] \qquad (12)$$

$$I_6(\text{t}) = \frac{a}{b}\left[e^{b(t_4-t)}\right] \qquad (13)$$

$$I_7(\text{t}) = \frac{(K-1)a}{b}\left[e^{b(t-T)}\right] \qquad (14)$$

The producer bears the material setup cost, holding cost for OW and RW, deterioration cost and production cost. The total cost function is given by the following components:

Production cost:

Production cost is the cost involved in producing the items in the production plant which includes material cost, labour cost and energy cost, therefore production cost for in this model is

$$\text{P.C.} = \left(c_P + \frac{c_0}{n^{\delta}}\right)\int_0^{t_2} L\, dt$$

$$\text{P.C.} = \text{Ka}\left(c_P + \frac{c_0}{n^{\delta}}\right)\left[t_1 + \frac{bt_1^{\,2}}{2} + \frac{b^2 t_1^{\,3}}{6} + p^{-b}(t_2 - t_1)\right] \quad (15)$$

Holding cost for OW:

First of all, inventory is kept in OW till then different types of expenses are incurred to keep the inventory safe in OW which includes maintenance of inventory etc. therefore holding cost for OW is:

$$\text{H.C}_1 = h_W\left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{t_3} I_4(t)dt + \int_{t_3}^{t_4} I_5(t)dt\right]$$

$$\text{H.C}_1 = h_W\begin{bmatrix}\frac{(K-1)a}{b+\alpha}\left(\frac{bt_1^{\,2}}{2} + \frac{b^2 t_1^{\,3}}{6} + \frac{\alpha t_1^{\,2}}{2}\right) + W\left\{(t_3 - t_1) - \frac{\alpha}{2}(t_3 - t_1)^2\right\} + \\ \frac{a}{(b+\alpha)}\left\{a(t_4 - t_3)^2 + \alpha(1 + bt_4)(t_4 - t_3)^2 + \frac{b^2}{2}\left(\frac{2}{3}t_4^{\,3} + \frac{1}{3}t_3^{\,3} - t_4^{\,2}t_3\right)\right\}\end{bmatrix} \quad (16)$$

Holding cost for RW:

Extra inventory is kept in RW. Holding cost involved in carefully storage and maintenance of inventory including hardware equipment, material handling equipment, IT software applications etc. Finally the total holding cost for producer in RW is:

$$\text{H.C}_2 = h_R\left[\int_{t_1}^{t_2} I_2(t)dt + \int_{t_2}^{t_3} I_3(t)dt\right]$$

$$\text{H.C}_2 = h_R a p^{-b}\left[(K-1)\left\{\frac{(t_1-t_2)^2}{2} + \frac{\beta}{6}(t_1 - t_2)^3\right\} + \left\{\frac{(t_3-t_2)^2}{2} + \frac{\beta}{6}(t_3 - t_2)^3\right\}\right] \quad (17)$$

Deterioration cost:

Deterioration cost introduced when the item is damaged. During the manufacturing many items cannot reproduce after deterioration, which became useless so the deteriorating cost for this model is:

$$\text{D.C} = d\begin{bmatrix}\int_0^{t_1} \alpha t\, I_1(t)dt + \int_{t_1}^{t_2} \beta t\, I_2(t)dt + \int_{t_2}^{t_3} \beta t\, I_3(t)dt + \\ \int_{t_1}^{t_3} \alpha t\, I_4(t)dt + \int_{t_3}^{t_4} \alpha t\, I_5(t)\,\text{dt}\end{bmatrix}$$

$$\text{D.C} = d\begin{bmatrix}\alpha(K-1)a\left[\frac{t_1^{\,3}}{3} + \frac{b^2 t_1^{\,4}}{8(b+\alpha)}\right] - \beta(K-1)ap^{-b}\left[\frac{t_1 t_2^{\,2}}{2} - \frac{t_2^{\,3}}{3} - \frac{t_1^{\,3}}{6}\right] + \\ ap^{-b}\beta\left[\frac{t_2^{\,3}}{3} + \frac{t_3^{\,3}}{6} - \frac{t_3 t_2^{\,2}}{2}\right] + \frac{\alpha W}{2}\left[t_3^{\,2} - t_1^{\,2}\right] + \\ \frac{a\alpha}{(b+\alpha)}\left[\frac{b}{6}t_4^{\,3} + \frac{b^2}{8}t_4^{\,4} - \frac{b}{2}t_3^{\,2}t_4 - \frac{b^2}{4}t_3^{\,2}t_4^{\,2} + \frac{b}{3}t_3^{\,3} + \frac{b^2}{8}t_3^{\,4}\right]\end{bmatrix}$$

(18)

Shortage cost:

Sometimes due to increasing demand or as per the extreme requirement of the customer, it is natural for the producer to lack inventory. Due to which the producer has to bear many types of expenses for re-supplying that inventory. This involves a shortage cost in this model. Present worth of shortage cost for producer is:

$$\text{S.C} = s\left[\int_{t_4}^{t_5} I_6(t)dt + \int_{t_5}^{T} I_7(t)dt\right]$$

$$= s\begin{bmatrix}\frac{a}{b}\left[(t_4 - t_5) + \frac{b}{2}(t_4 - t_5)^2 + \frac{b^2}{6}(t_4 - t_5)^3\right] + \\ \frac{(K-1)a}{b}\left[(t_5 - T) + \frac{b}{2}(t_5 - T)^2 + \frac{b^2}{6}(t_5 - T)^3\right]\end{bmatrix} \quad (19)$$

Setup cost:

Before producing the items in the production plant to setup the production system or to setup machine for preparing the production, labour, etc., setup cost is involved which is as:

$$\text{A.S} = f \quad (20)$$

Total relevant cost of the system is the sum of production cost, holding cost in OW and RW, deterioration cost setup cost and shortage cost which is represented by:

$$\text{T.C} = \frac{1}{T}\left[P.C + H.C + D.C + A.S + S.C\right]$$

$$= \frac{1}{T}\begin{bmatrix}\text{Ka}\left(c_P + \frac{c_0}{n^{\delta}}\right)\left[t_1 + \frac{bt_1^{\,2}}{2} + \frac{b^2 t_1^{\,3}}{6} + p^{-b}(t_2 - t_1)\right] + \\ h_0\begin{bmatrix}\frac{(K-1)a}{b+\alpha}\left(\frac{bt_1^{\,2}}{2} + \frac{b^2 t_1^{\,3}}{6} + \frac{\alpha t_1^{\,2}}{2}\right) + W\left\{(t_3 - t_1) - \frac{\alpha}{2}(t_3 - t_1)^2\right\} + \\ \frac{a}{(b+\alpha)}\left\{a(t_4 - t_3)^2 + \alpha(1 + bt_4)(t_4 - t_3)^2 + \frac{b^2}{2}\left(\frac{2}{3}t_4^{\,3} + \frac{1}{3}t_3^{\,3} - t_4^{\,2}t_3\right)\right\}\end{bmatrix} + \\ h_r\, ap^{-b}\left[(K-1)\left\{\left(\frac{t_1-t_2}{2}\right)^2 + \frac{\beta}{6}(t_1 - t_2)^3\right\} + \left\{\frac{(t_3-t_2)^2}{2} + \frac{\beta}{6}(t_3 - t_2)^3\right\}\right] + \\ d\begin{bmatrix}\alpha(K-1)a\left[\frac{t_1^{\,3}}{3} + \frac{b^2 t_1^{\,4}}{8(b+\alpha)}\right] - \beta(K-1)ap^{-b}\left[\frac{t_1 t_2^{\,2}}{2} - \frac{t_2^{\,3}}{3} - \frac{t_1^{\,3}}{6}\right] + \\ ap^{-b}\beta\left[\frac{t_2^{\,3}}{3} + \frac{t_3^{\,3}}{6} - \frac{t_3 t_2^{\,2}}{2}\right] + \frac{\alpha W}{2}\left[t_3^{\,2} - t_1^{\,2}\right] + \\ \frac{a\alpha}{(b+\alpha)}\left[\frac{b}{6}t_4^{\,3} + \frac{b^2}{8}t_4^{\,4} - \frac{b}{2}t_3^{\,2}t_4 - \frac{b^2}{4}t_3^{\,2}t_4^{\,2} + \frac{b}{3}t_3^{\,3} + \frac{b^2}{8}t_3^{\,4}\right]\end{bmatrix} + f + \\ s\begin{bmatrix}\frac{a}{b}\left[(t_4 - t_5) + \frac{b}{2}(t_4 - t_5)^2 + \frac{b^2}{6}(t_4 - t_5)^3\right] + \\ \frac{(K-1)a}{b}\left[(t_5 - T) + \frac{b}{2}(t_5 - T)^2 + \frac{b^2}{6}(t_5 - T)^3\right]\end{bmatrix}\end{bmatrix}$$

(21)

4. FUZZY MODEL

In order to discuss the fuzzy model in this supply chain, there is need of some important definitions, which are as follows:

Definition 4.1.

A fuzzy set $\tilde{\beta}$ on the interval $(-\infty, \infty)$ is called a fuzzy point if its membership function (MF) is

$$\mu_{\tilde{\beta}}(y) = \begin{cases} 1, & y = \beta \\ 0, & y \neq \beta \end{cases}$$

Where β is the support point of fuzzy set

Definition 4.2.

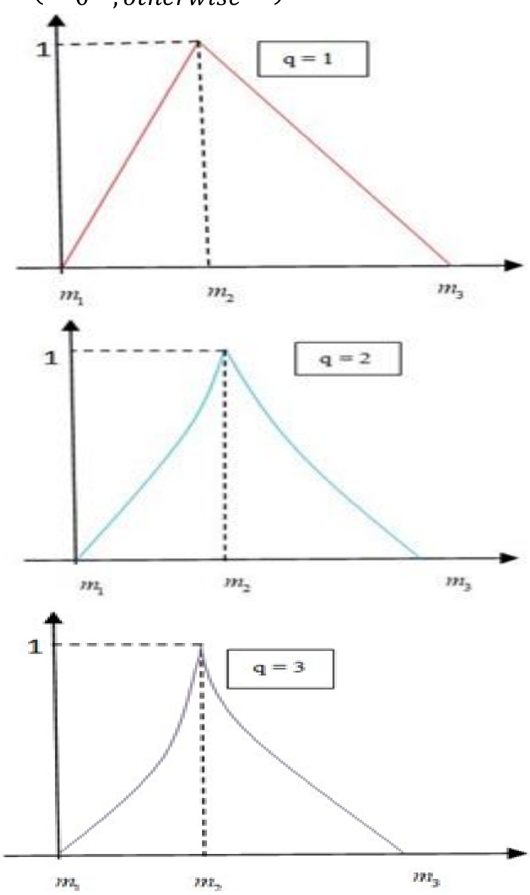
A fuzzy set $[U_\beta, V_\beta]$ where $0 \leq \beta \leq 1$. U, V ϵ R and U$< V$, is called a level of fuzzy interval if its MF is

$$\mu_{[U_\beta, V_\beta]}(x) = \begin{cases} \beta, & U \leq y \leq V \\ 0, & otherwise \end{cases}$$

Definition 4.3.

A q-fuzzy number $\widetilde{M} = (m_1, m_2, m_3; q)$ where $\xi < \sigma < \rho$ and ξ, σ, ρ ϵ R, has membership function is given by:

$$\mu_M = \begin{cases} \frac{y - m_1}{m_2 - m_1}, & m_1 \leq y \leq m_2 \\ \frac{m_3 - y}{m_3 - m_2}, & m_2 \leq y \leq m_3 \\ 0, & otherwise \end{cases}$$



Definition 4.4. If $\widetilde{M}$ be a q-fuzzy number then defuzzification of $\widetilde{M}$ with Centroid method is defined as

$$\frac{\int x\, \mu_M(x)dx}{\int \mu_M(x)dx} = (m_1 - m_2 + m_3) + \frac{q+1}{q+2}(2m_2 - m_1 - m_3)$$

5. SOLUTION PROCEDURE AND NUMERICAL EXAMPLE

5.1 Solution procedure

The total annual cost has two variables p and t_1. To minimize the total annual cost the optimal values of p and t_1 can be obtained by solving the following equations simultaneously.

$$\frac{\partial TC}{\partial t_1} = 0 \qquad (22)$$

And $$\frac{\partial TC}{\partial p} = 0 \qquad (23)$$

Provided they satisfied the following conditions

$$\left.\begin{matrix} \frac{\partial^2 TC\,(t_1,p)}{\partial t_1^{\,2}} > 0\,, \qquad \frac{\partial^2 TC(t_1,p)}{\partial p^2} > 0 \\ \left(\frac{\partial^2 TC\,(t_1,p)}{\partial t_1^{\,2}}\right)\left(\frac{\partial^2 TC(t_1,p)}{\partial p^2}\right) - \left(\frac{\partial^2 TC(t_1,p)}{\partial t_1 \partial p}\right)^2 > 0 \end{matrix}\right\} \quad (24)$$

To minimize the objective function, the optimal solution of t_1 and p can be obtained from the equations (22) and (24) for both the cases.

5.2 Numerical analysis

In the practical way the following value of various parameters are given for this model:

a=3, b = 8, β= 0.03, α= 0.01, c_0 = 1.3, h_0 = 1.9\$/unit, δ = 0.11, c_p =2.2 $\$/month$, s= 0.8\$/unit, t_3 =20month, k= 2.5, n=2, t_4= 25 month, W= 200unit, d= 1.2\$/unit, h_r= 2\$/unit, t_5 =40 month, f=20000\$/setup, t_2= 15 month, T= 50 month. And $\tilde{f}$ =(14000, 18000, 24000) be a q-fuzzy number.

The optimal solution for total cost with corresponding optimal time is given in Table 1. The solution procedure provided in Section 5 is applied to evaluate these values.

Table 1

Model	Crisp	Fuzzy
t_2	1.53 month	1.53 month
p	66.51	66.51
Total cost	1510.09	1480.09

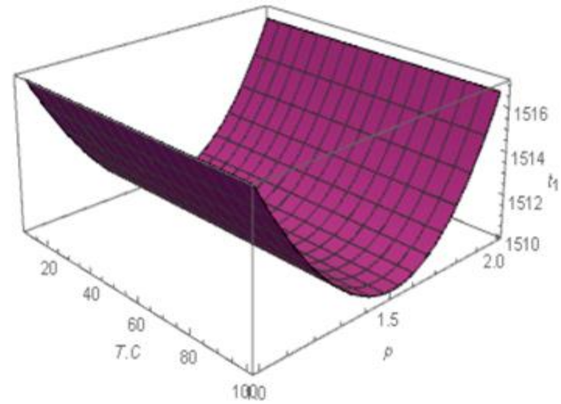


Fig. 1. Convexity of total cost in crisp model

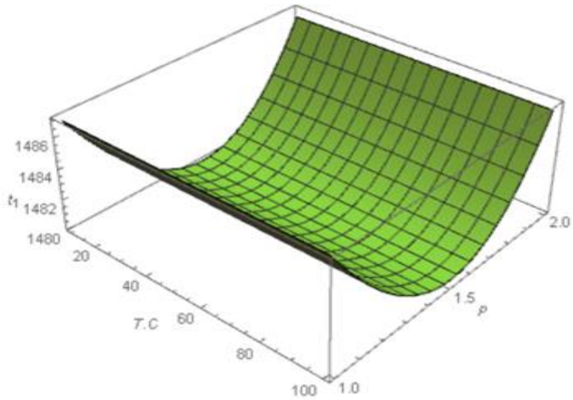


Fig. 2. Convexity of total cost in fuzzy model

6. SENSITIVITY ANALYSIS

Parameter	change in parameter	Crisp total cost	%change in crisp total cost	Fuzzy total cost	%change in fuzzy total cost
W	500	825.4	-45.34	795.4	-46.26
	1000	1167.7	-22.67	1137.7	-23.12
	2000	1852.4	+22.67	1822.4	+23.12
	2500	2194.7	+45.34	2164.7	+46.25
c_p	0.5	1497.5	-0.83	1467.5	-0.85
	2	1508.6	-0.09	1478.6	-0.10
	2.5	1512.3	+0.14	1482.3	+0.15
	3	1516.01	+0.39	1486.01	+0.60
c_o	0.5	1504.6	-0.36	1474.6	-0.37
	1	1508.04	-0.135	1478	-0.14
	1.5	1511.5	+0.09	1481.4	+0.08
	2	1514.89	+0.32	1484.8	+0.31
d	1	1492.3	--1.17	1462.3	-1.2
	2	1581	+469	1551	+4.8
	3	1669.6	+10.56	1639.6	+10.7
	4	1758.2	+16.43	1728.2	+16.76
s	0.5	1602.1	+6.09	1572.1	+6.21
	1	1448.7	-4.06	1418.7	-4.14
	1.5	1295.3	-14.22	1265.3	-14.5
	2	1142.0	-24.37	1112.03	-24.86
h_r	1	1574.5	+4.26	1480.09	0
	1.5	1574.5	+4.26	1480.09	0
	2.5	1574.5	+4.26	1480.09	0
	4	1574.5	+4.26	1480.09	0
h_o	0.5	608.25	-59.72	578.2	-60.9
	1.5	1252.4	-17.06	1222.4	-17.41
	2	1574.5	+4.26	1544.5	+4.35
	2.5	1896.6	+25.59	1866.6	+26.2
λ	0.01	1510.7	+0.04	1479.6	-0.03
	0.2	1509.5	-0.03	1479.3	-0.05
	0.25	1509.2	-0.05	1479.2	-0.06
	0.3	1508.9	-0.07	1478.9	-0.08

7. GRAPHICAL REPRESENTATION AND OBSERVATION

7.1 Graphical representation:

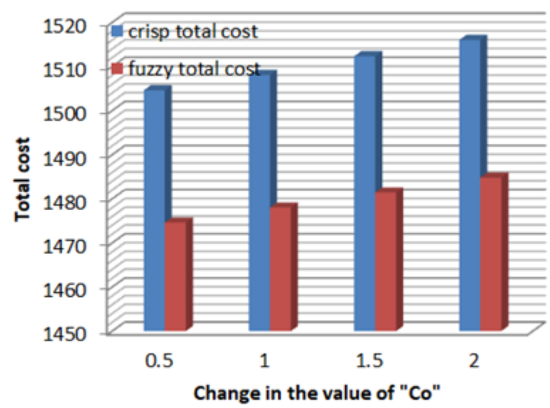


Fig. 3

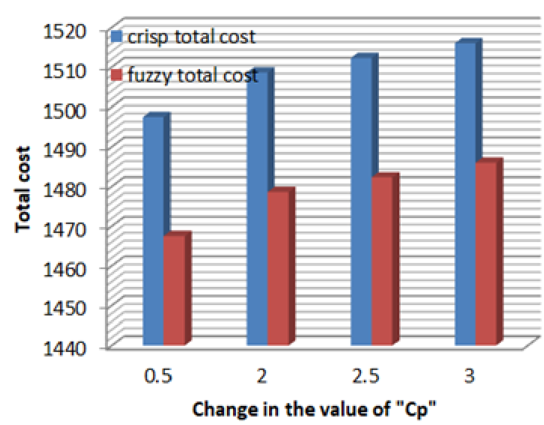


Fig. 4

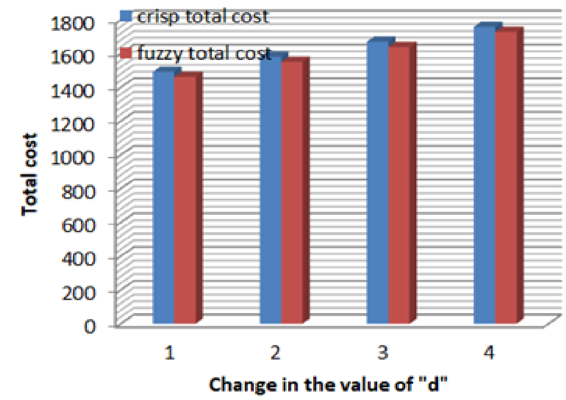


Fig. 5

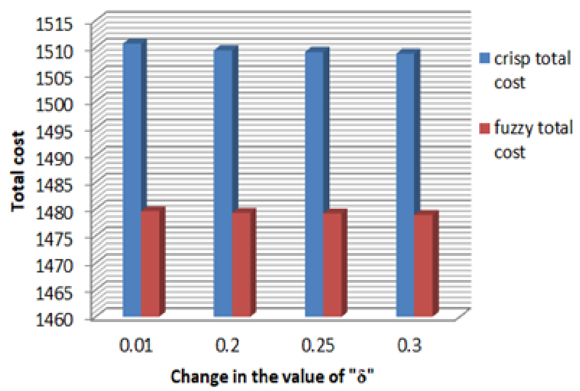


Fig. 6

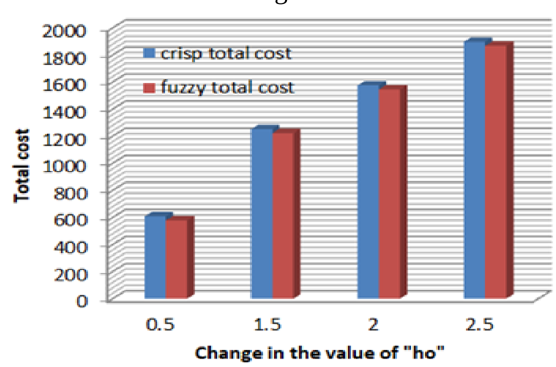


Fig. 7

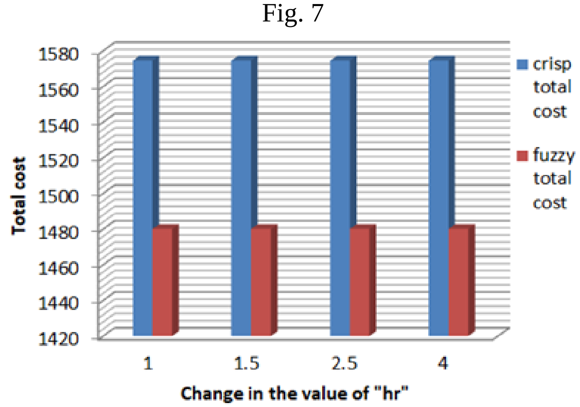


Fig. 8

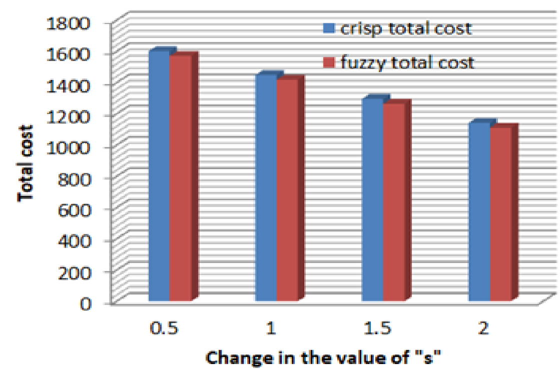


Fig. 9

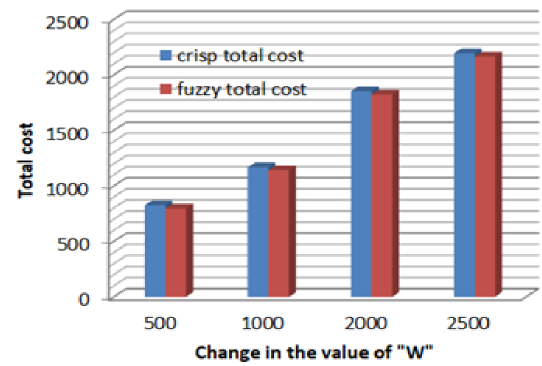


Fig. 10

7.2 Observation:

1. From the table of sensitivity analysis it was found that. As the value of parameter W increases, the value of total cost is increasing continuously in both the crisp and fuzzy conditions.
2. As the values of parameters c_p , c_o , d, h_r are increasing, then the value of total cost is increasing continuously in both crisp and fuzzy solutions.
3. As the value of parameters s, λ are increases, the value of total cost is found to decrease in both crisp and fuzzy conditions.
4. Conversely, if the value of parameter h_r is being changed, then there is no change in the value of total cost in both crisp and fuzzy solutions.

8. CONCLUSION

In this research, the imperfect production inventory model has been developed under shortage, learning and fuzzy environment. In this model it has been analysed the effect of imperfect production process on total cost and the goods which have been damaged during the imperfect production process cannot be reused. In this model along with the concept of two warehouses one is own warehouse (OW) and other one is rented warehouse (RW). From the sensitivity analysis of this model it was concluded that if the the value of parameter W increases, the value of total cost is increasing continuously in both the crisp and fuzzy conditions, and also the values of parameters c_p, c_o, d, h_r are increasing, then the value of total cost is increasing continuously in both crisp and fuzzy solutions.This model is not valid when the demand for OW is not depend on selling price, also production rate is free from demand rate, The model can be extended with time dependent demand and variable production rate, carbon emission reduction under preservation technology with cloudy fuzzy and replenishment policy for deteriorating ítems.

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