Robust's Ranking Index Method for Solving Trapezoidal Fuzzy Assignment Problem

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Abstract- In the present paper, we used Robust's Ranking Method for solving Trapezoidal fuzzy assignment problem then after to get optimal solution we use Hungarian method. In this paper $\widetilde{c}_{ij}$ we considered to be trapezoidal numbers denoted by which are more realistic and general in nature. In this paper first we convert the given trapezoidal fuzzy assignment problem into the crisp assignment problem using the linear programming problem form and then using Robust's ranking method (3) for the trapezoidal fuzzy numbers. For getting the optimal solution we use Hungarian method. Numerical examples show that the fuzzy ranking method offers an effective tool for handling the trapezoidal fuzzy assignment problem. This can be better illustrated with the numerical example. The presented method is very easy to understand and apply to find the optimal fuzzy cost occurring in the real-life situations. There are several papers in the literature in which fuzzy numbers are used for solving real life problems. But in the present paper, i used Robust's Ranking Method and Hungarian method to getting optimal solution of Trapezoidal fuzzy assignment problem.

Key words: Fuzzy sets, fuzzy assignment problem, Triangular fuzzy number, Trapezoidal fuzzy number ranking function.

1.INTRODUCTION

The assignment problem is one of the fundamental combinatorial optimization problems in the branch of optimization or operation research in mathematics. It is a special case of transportation problem in which the objective is to assign a number of origins to the equal number of destinations at the minimum cost (or maximum profit). It includes assignment of people to projects, jobs to machines, workers to jobs and teachers to classes etc., while minimizing the total assignment costs. The main important characteristics of assignment problem are that only one job (or worker) is assigned to one machine (or project). Hence the number of sources is equal to the number of destinations and each requirement and capacity value is exactly one unit.

The assignment problem is also called a 0-1 programming problem and is highly degenerate all the algorithms developed to find optimal solution of transportation problem are applicable to assignment problem. The assignment problem is also known as Hungarian method proposed by kuhn [11], is used for its solution because of its highly degeneracy nature a specially designed algorithm.

To find solutions to assignment problems, various algorithms such as linear programming [2, 3, 8, 13], Hungarian algorithm [11], Neural network [6], genetic algorithm [12] have been developed. Last 50 years, many types of the classical assignment problems are proposed such as Quadratic assignment problem, generalized assignment problem, Bottleneck assignment problem, etc. In present years, fuzzy transportation and fuzzy assignment problems have received are most concentration. Fuzzy sets were introduced by Zadeth [17] as an enter of classical notion of the set. Later many researchers [5, 10] also use fuzzy set theory and fuzzy numbers in different field. After this Buckly [4] used triangular fuzzy numbers in linear programming. Pandian and Natarajan[14] use fuzzy zero-point method in trapezoidal fuzzy numbers for finding a fuzzy optimal solution for a fuzzy transportation problem. Majumdar and Bhunia [12] developed an exclusive genetic algorithm to solve a generalized assignment problem with imprecise cost(s)/time(s). In most of the papers the generalized fuzzy numbers are converted into normal fuzzy numbers through normalization process [9] and then normal fuzzy numbers are used to solve the real-life problems. Kaufmann and Gupta [9] pointed out that there is a serious disadvantage of the normalization process. In which we convert a measurement of an objective value to a valuation of a subjective value, which results in the loss of information. The procedure is mathematically correct, it decreases the amount of information that is available in the original data, and we should avoid it.

In the present paper, we use Robust's ranking method (7) to transform the fuzzy assignment problem to a crisp one so that the conventional solution methods may be applied to solve assignment problem. The fuzzy assignment problem & this method can be tried in project scheduling, maximal flow, transportation problem etc. Robust's ranking method (7) which satisfies the properties of compensation, linearity and additivity. In the paper we have applied Robust's ranking technique (7).

In section 2, we introduce the basic definitions and arithmetic operations of fuzzy numbers. Section 3 numerical example is presented to show the applications of the proposed algorithms and the total optimal fuzzy costs for the proposed algorithms are shown. Finally, the conclusion is given in section 4.

2. PRELIMINARIES

In this section, some basic definitions and arithmetic operations are reviewed.

Definition 2.1

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1], is A={x, μ_A (x): x $\in$ X}, Here : μ_A : X $\rightarrow$ [0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A (X) is called the membership value of x $\in$ X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

Definition 2.2

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one $x \in X$ such that μ_A (X)=1.

Definition: 2.3

The fuzzy set A is convex if and only if, for any x_1, x_2 $\in$ X, the membership function of A satisfies the inequality μ_A

$$\{\lambda x_1 + (1-\lambda)x_2\} \geq \min.\{\mu_A(x_1), \mu_A(x_2)\}, 0 \leq \lambda \leq 1.$$

Definition: 2.4 Triangular fuzzy numbers

A fuzzy number $\tilde{x}$ is a triangular fuzzy number denoted by (*a, b, c ;1*) where *a, b* and *c* are real numbers and its membership function $\mu(X)$ is given below.

$$\mu(X) = \begin{cases} \frac{(x-a)}{(b-a)} & ; \quad \text{for } a \leq x \leq b \\ 1 & ; \quad x = b \\ \frac{(c-x)}{(c-b)} & ; \quad \text{for } b \leq x \leq c \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Definition: 2.4 Trapezoidal fuzzy numbers

A fuzzy number $\tilde{x}$ is a trapezoidal fuzzy number denoted by (*a, b, c,d ;1*) where *a, b, c* and *d* are real numbers and its membership function $\mu(X)$ is given below.

$$\mu(X) = \begin{cases} \frac{(x-a)}{(b-a)} & ; \quad \text{for } a \leq x \leq b \\ 1 & ; \quad x = b \\ \frac{(c-x)}{(d-c)} & ; \quad \text{for } c \leq x \leq d \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Definition 2.5. Arithmetic Operations

Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. Then

(i) $\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(ii) $\tilde{a} - \tilde{b} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4)$

(iii) $k\tilde{a} = (ka_1, ka_2, ka_3, ka_4; w_1)$, for $k \geq 0$;

(iv) $k\tilde{a} = (ka_3, ka_2, ka_1, ka_4)$, for $k < 0$;

Definition 2.6 Fuzzy Assignment Problem

Consider the following fuzzy assignment problem,

(P) Min. $\tilde{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$; [i= 1, 2, 3, …, n; j= 1, 2, 3, …, n]

Subject to the constraints:

(i) $\sum_{i=1}^{n} \tilde{x}_{ij} = 1$; j = 1, 2, …, n. i.e. i^{th} person will do only one work.

(ii) $\sum_{j=1}^{n} \tilde{x}_{ij} = 1$; i= 1, 2, …, n. i.e. j^{th} person will be done only one person.

Where $\tilde{x}_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4)$ = the assignment of facility i to job j such that

$\tilde{x}_{ij}$ = 1; if i^{th} person is assigned j^{th} work

0; if i^{th} person is not assigned the j^{th} work

$\tilde{c}_{ij} = (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4)$ = the cost of assignment of resources i to activity j.

$\tilde{z} = (z_1, z_2, z_3, z_4)$ = min./max. The total cost of the matrix.

Definition: 2.7 k-cut of a trapezoidal fuzzy number

The k-cut of a fuzzy number A(x) is defined as A(k) = {x: $\mu(x) \geq k, k \in [0,1]$}\

Definition: 2.8 Robust's Ranking Index Method

$$R(\tilde{c}) = \int_0^1 0.5(C_k^L, C_k^U)\, dk$$

Where (C_k^L, C_k^U) is the k-level cut of the fuzzy number $\tilde{C}$.

$R(\tilde{c})$ Gives the representative value of the fuzzy number $\tilde{C}$.

The K-Cut of the fuzzy number (10, 20, 30, 40) is

3. NUMERICAL EXAMPLES

To illustrate the proposed algorithm, consider a fuzzy assignment problem with four persons and four works. Fuzzy costs consider here to be the trapezoidal fuzzy number for allocating each person. The fuzzy cost for each person would take to perform each work is given in the effectiveness fuzzy cost matrix as shown in Table (I).

Table-1

Persons/Works	I	II	III	IV
A	(10,20,30,40)	(10,20,40,50)	(10,30,17,40,50)	(10,20,30,40)
B	(10,20,40,50)	(10,30,40,50)	(10, 20,30,40)	(20,30,40,50)
C	(10,20,40,50)	(20,30,40,50)	(20,30,40,50)	(10,20,30,40)
D	(20,30,50,60)	(10,20,30,40)	(20,40,60,80)	(20,30,50,60)

Solution:

Step 1: The fuzzy assignment problem in the example, as shown in Table(I) is a balanced one.

Step 2: Now we calculate R (10, 20, 30, 40) by applying Robust's ranking method. The membership function of the Trapezoidal fuzzy number (10, 20, 30, 40) is

$$\mu(X)=\begin{cases}\frac{(x-10)}{10} & ; \quad \text{for } 10 \le x \le 20\\ 1 & ; \quad x=20\\ \frac{(40-x)}{10} & ; \quad \text{for } 30 \le x \le 40\\ 0 & ; \quad \text{otherwise}\end{cases}$$

The K-Cut of the fuzzy number (10, 20, 30, 40) is

$(C_k^L, C_k^U) = (10k+10, 40-10k)$ for which

$$R(\tilde{c}_{11}) = R(10,20,30,40) = \int_0^1 0.5(C_k^L, C_k^U)dk$$

$$= \int_0^1 0.5(50)dk$$

$= 25$

Similarly, we can find other the Robust's ranking indices for the fuzzy costs. In the similar manner we find the following all Robust's ranking indices are

$R(C_{12}) = 30$, $R(C_{13}) = 32.5$, $R(C_{14}) = 25$, $R(C_{21}) = 30$, $R(C_{22}) = 32.5$, $R(C_{23}) = 25$, $R(C_{24}) = 40$, $R(C_{31}) = 30$, $R(C_{32}) = 40$, $R(C_{33}) = 40$, $R(C_{34}) = 25$, $R(C_{41}) = 40$, $R(C_{42}) = 25$, $R(C_{43}) = 50$, $R(C_{44}) = 40$

We write the above Robust's Ranking indices in the following table form

Table-II: Robust's Rank of table I

Persons/Works	I	II	III	IV
A	25	30	32.5	25
B	30	32.5	25	40
C	30	40	40	25
D	40	25	50	40

The above table is of assignment problem. To get the optimal solution, we use Hungarian Method.

Table-III: Row Minimum

Persons/Works	I	II	III	IV
A	0	5	7.5	0
B	5	7.5	0	15
C	5	15	15	0
D	15	0	25	15

Step 4: Assign zero of Table (III).

Table-IV: Assign Zero

Persons/Works	I	II	III	IV
A	[0]	5	7.5	0
B	5	7.5	[0]	15

C	5	15	15	0
D	15	0	25	15

Step 5: The optimal assignment from Table (IV) is A-I, B-III, C-IV, D-II.

Step 6: From the original fuzzy assignment cost matrix presented in table (I), the optimal fuzzy cost assignment is calculated and presented in Table (V)

Table-V: Optimal fuzzy cost assignment

Optimal assignment	A-I	B-III	C-II	D-IV
A	(10,20,30,40)	(10,20,30,40)	(10,20,30,40)	(10,20,30,40)

Step 7: From the Table-V. The Minimum total fuzzy cost is (40, 80, 120, 160)

4. CONCLUSION

In the paper, we considered the assignment costs as Trapezoidal fuzzy numbers which are more realistic and general in nature. By using Robust's ranking indices the fuzzy assignment problem has been transformed into crisp assignment problem. By using method Numerical examples have the optimal assignment as well as the crisp and fuzzy optimal total cost. By using Robust's ranking methods we get the total cost which is optimal. Thus, we can conclude that the solution of fuzzy problems can be obtained by Robust's ranking methods effectively. This method can also be used in solving other types of problems like, project schedules, transportation problems and network flow .The proposed method for an optimal solution is very simple, easy to understand and apply.

REFERENCE

[1] Avis D., Devroye L., An analysis of a decomposition heuristic for the assignment problem, Operation Research Literature, 3(6), 279-283, 1985.

[2] Balinnski, M. L., Competitive (dual) simplex method for the assignment problem, math. Program, 34(2), 125-141, 1986.

[3] Barr, R. S., Glover, F., Klingman, D., The alternating basis algorithm for assignment problems, Math.Program, 13(1), 1-13, 1977.

[4] Buckly, J. J., Possibilistic linear programming with triangular fuzzy numbers, fuzzy sets and system, 26, 135-138, 1998.

[5] Chen S. H., Operation on fuzzy numbers with function principle, Tamkang Journal of Management Sciences, 6, 13-25, 1985.

[6] Eberhardt S.P., Duad T., Kerns A., Brown T. X., Thakoor A.P., Competitive neural architecture for hardware solution to the assignment problem, Neural Networks, 4($), 431-442, 1931.

[7] Fortemps P. and Roubens M. "Ranking and defuzzification methods-based area compensation" Fuzzy sets and systems Vol.82.PP 319-330,1996

[8] Hung, M. S., Rom, W. O., Solving the assignment problem by relaxation, Operation Research, 28(4), 969-982, 1980.

[9] Kaufmann A., Gupta M.M., Introduction to fuzzy arithmetic: Theory and Applications, Van Nostrand Reinhold, New York, 1985.

[10] Kaufmann A., Introduction to the theory of fuzzy sets, 1, Academic Press, New York, 1976.

[11] Kuhn H. W., The Hungarian method for the assignment problem, Naval Research Logistics Quarterly, 2, 83-97, 1955.

[12] Majumdar J., Bhunia A. K., Elitist genetic algorithm for assignment problem with imprecise goal, European journal of operation research 177, 684-692, 2007.

[13] Mcginnis, L. F., Implementation and testing of a primal-dual algorithm for the assignment problem, Operation Research, 31(2), 277-291, 1983.

[14] Pandian P. and Natarajan G., Anew algorithm for finding a fuzzy optimal solution for fuzzy

transportation problems, Applied mathematical sciences, 4, 79-90, 2010.

[15] Rommelfanger H., Wolf J. and Hanuschek R., Linear programming with fuzzy objectives, Fuzzy Sets and System, 29, 195-206, 1989.

[16] Sharma J. K., Operation research theory and application, Third Edition, 2007.

[17] Zadeh L.A., Fuzzy sets, Information and Control, 8, 3, 338-353, 1965.