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Remarks on the Paper Entitled Observations on The Cone $15x^2-32y^2=7z^2$

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Abstract— This paper aims at determining non-zero distinct integer solutions satisfying the homogeneous cone represented by the ternary quadratic equation $15x^2-32y^2=7z^2$.

Indexed Terms— Ternary quadratic, homogeneous quadratic, homogeneous cone, integer solutions.

I. INTRODUCTION

The quadratic Diophantine equations with three unknowns offers an unlimited field for research because of their variety [1-3]. For an extensive review of various problems on ternary quadratic Diophantine equations representing specific 3 dimensional surfaces, one may refer [4-13,15]. While making a collection of ternary quadratic quadratic equations ,the paper [14] came to our reference wherein the author has given some sets of integer solutions to the cone given by $15x^2-32y^2=7z^2$ .In this communication, we exhibit other sets of non-zero distinct integer solutions satisfying the above homogeneous cone .

Method of Analysis:

Consider the homogeneous cone represented by the ternary quadratic equation

$$15x^2-32y^2=7z^2 \quad (1)$$

we present below different illustrations of solving (1) and thus, obtain different sets of integer solutions to (1)

Illustration 1:

Introducing the linear transformations

$$x=2X \ , \ \ y=\alpha+7\beta, z=2\alpha-16\beta \quad (2)$$

in (1),it is written as

$$X^2=\alpha^2+56\beta^2 \quad (3)$$

which is satisfied by

$$\beta=2rs, \alpha=56r^2-s^2, X=56r^2+s^2 \quad (4)$$

In view of (2),the corresponding integer solutions to (1) are given by

$$z=2(56r^2-s^2-16rs), y=56r^2-s^2+14rs, x=2(56r^2+s^2) \quad (5)$$

Illustration 2:

Write (3) as the system of double equations as in Table 1 below:

Table 1: system of double equations

System	1	2	3	4	5	6	7	8	9	10
$X+\alpha$	$28\beta^2$	$14\beta^2$	$7\beta^2$	$4\beta^2$	$2\beta^2$	β^2	56β	28β	14β	8β
$X-\alpha$	2	4	8	14	28	56	β	2β	4β	7β

Solving each of the above system of equations ,the values of X, α, β are obtained .In view of (2), the corresponding solutions to (1) are found. For simplicity and brevity, the solutions thus obtained are given below:

Solutions from system 1:

$$x=28\beta^2+2, y=14\beta^2+7\beta-1, z=28\beta^2-16\beta-2$$

Solutions from system 2:

$$x = 14\beta^2 + 4, y = 7\beta^2 + 7\beta - 2, z = 14\beta^2 - 16\beta - 4$$

Solutions from system 3:

$$x = 28k^2 + 8, y = 14k^2 + 14\beta - 4, z = 28k^2 - 32k - 8$$

Solutions from system 4:

$$x = 4\beta^2 + 14, y = 2\beta^2 + 7\beta - 7, z = 4\beta^2 - 16\beta - 14$$

Solutions from system 5:

$$x = 2\beta^2 + 28, y = \beta^2 + 7\beta - 14, z = 2\beta^2 - 16\beta - 28$$

Solutions from system 6:

$$x = 4k^2 + 56, y = 2k^2 + 14k - 28, z = 4k^2 - 32k - 56$$

Solutions from system 7:

$$x = 114k, y = 269k - 28, z = 78k$$

Solutions from system 8:

$$x = 30k, y = 20k, z = 10k$$

Solutions from system 9:

$$x = 18k, y = 12k, z = -6k$$

Solutions from system 10:

$$x = 30k, y = 15k, z = -30k$$

Illustration 3:

Write (3) as

$$\alpha^2 + 56\beta^2 = X^2 = X^2 * 1$$

(6)

Assume

$$X = a^2 + 56b^2 \quad (7)$$

Write 1 on the R.H.S. of (6) as

$$1 = \frac{(5 + i\sqrt{56})(5 - i\sqrt{56})}{81}$$

(8)

Substituting (7) and (8) in (6) and employing the method of factorization,consider

$$\alpha + i\sqrt{56}\beta = \frac{(5 + i\sqrt{56})(a + i\sqrt{56}b)^2}{9}$$

(9)

Equating the real and imaginary parts in (9) & replacing a by 3A ,b by 3B,we have

$$X = 9(A^2 + 56B^2),$$
$$\alpha = 5(A^2 - 56B^2) - 112AB,$$
$$\beta = (A^2 - 56B^2) + 10AB$$

In view of (2),the corresponding integer solutions to (1) are given by

$$x = 18(A^2 + 56B^2),$$
$$y = 12(A^2 - 56B^2) - 42AB,$$
$$z = -6(A^2 - 56B^2) - 384AB$$

Note 1:

In addition to (8),the integer 1 is also represented as below:

$$1 = \frac{(13 + i\sqrt{56})(13 - i\sqrt{56})}{225},$$
$$1 = \frac{(5 + i3\sqrt{56})(5 - i3\sqrt{56})}{529},$$
$$1 = \frac{(1 + i2\sqrt{56})(1 - i2\sqrt{56})}{225}$$

The above process leads to three more integer solutions to (1).

Illustration 4:

Write (3) as

$$X^2 - 56\beta^2 = \alpha^2 = \alpha^2 * 1$$

(10)

Assume

$$\alpha = a^2 - 56b^2$$

(11)

Write 1 on the R.H.S. of (10) as

$$1 = (15 + 2\sqrt{56})(15 - 2\sqrt{56})$$

(12)

Substituting (11) and (12) in (10) and employing the method of factorization,consider

$$X + \sqrt{56}\beta = (15 + 2\sqrt{56})(a + \sqrt{56}b)^2$$

(13)

Equating the rational &irrational parts in (13) and from (11) ,(2) the corresponding integer solutions to (1) are given by

$$x = 30(a^2 + 56b^2) + 448ab,$$
$$y = 15a^2 + 13 * 56b^2 + 210ab,$$
$$z = -30a^2 - 34 * 56b^2 - 480ab$$

Note 2:

In addition to (12),the integer 1 is also represented as below:

$$1 = \frac{(23 + 3\sqrt{56})(23 - 3\sqrt{56})}{25},$$
$$1 = \frac{(15 + \sqrt{56})(15 - \sqrt{56})}{169},$$
$$1 = \frac{(9 + \sqrt{56})(9 - \sqrt{56})}{25}$$

The above process leads to three more integer solutions to (1).

Remark:

In addition to (2),one may consider the following linear transformations:

(i): $x = 2\alpha + 16\beta$, $y = \alpha + 15\beta, z = 2w$

(ii): $x = 2\alpha + 14\beta$, $z = 2\alpha + 30\beta$

Applying the analysis presented above, one may obtain some more integer solutions to (1).

CONCLUSION

In this paper, an attempt has been made to obtain non-zero distinct integer solutions to the cone represented by the ternary quadratic equation $15x^2 - 32y^2 = 7z^2$. It is well known that quadratic equation with three unknowns are rich in variety. To conclude, one may search for integer solutions to other choices of cone.

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