

# Fundamentals of Quantum Computing: A Comprehensive Study from Schrödinger Wave Equation to Quantum States

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**Abstract** - The subject of quantum computing is a combination of classical information theory, computer science, and quantum physics. This paper aims to mathematically derive several potential Quantum States from Schrödinger Wave Equation and representation of these states in Bloch Sphere using corresponding Bloch Vector. The authors summarize in this paper not just the core concepts of quantum computing like Superposition, Entanglement, and Decoherence, but also the need for Quantum Computers, and its applications in different fields.

**Index Terms** — Algorithm, Bloch Sphere, Bloch Vector, Bracket, Copenhagen Interpretation, Density Matrix, Dirac Notation, Eigenstates, Entanglement, Hilbert Space, Quantum Decoherence, Quantum Gates, Quantum Tunneling, Qubits, Superposition, Z-Measurement.

## I. INTRODUCTION

With continuous growth of research and development, quantum computing may eventually be millions of times faster than the computing power of the processors that we all use today within our computers, smartphones, and other electronic computing devices. This new model of computing has moved from the theoretical concept to reality, with quantum computers has begun to be used for scientific and commercial use. The difference between classical and quantum computing is that the classical computing is limited to processing bits (or, binary bits), which are like small switches that can be either on or off, represented by just one of the two states at any given time, a one (1) or a zero (0), whereas quantum computing possesses qubits (or, quantum bits). These are nothing but an induced property of a subatomic particle that can be represented as a one, zero, both or any value in between, most interestingly, all at the same time. This phenomenon is known as superposition. Thus,

superposition allows us to perform calculation on multiple states at the same time. In classical computers, if we have a bit string of length 3 bits, we can compute only one out of 8 combinations of bits. Whereas, in quantum computers, every qubit can be in superposition of 0 or 1, thus we can be in superposition of all the 8 combinations at the same time. As a result, quantum algorithms are exponentially faster than any classical algorithms that are known to us. However, once we read or measure the superposition state of a qubit, it collapses to one of its states, i.e., either 0 or 1, which is known as wave-function collapse. That is why it is not much easy to design quantum algorithms, but we use interference effect that allows us to work with all the states at the same time and we can obtain one output state at the end. In quantum mechanics, the concept of particle existing in multiple states until it is measured at a point in time is called the Copenhagen Interpretation, proposed by Niels Bohr and Werner Heisenberg in the 1920's.

## II. NEED FOR QUANTUM COMPUTERS

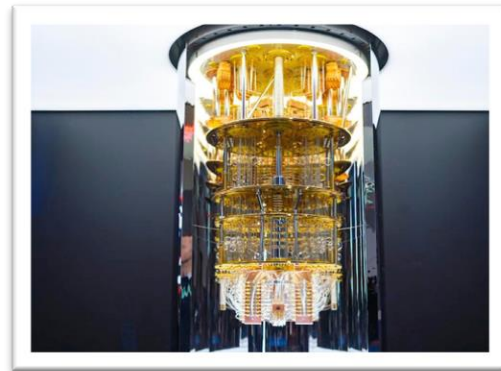
Classical processors (CPU microchips) are made up of transistors. A transistor is a physical object, thus, being purely physical it is governed by the laws of physics. That means there must be a physical limit to how small a transistor can be. Today's transistors are being manufactured using silicon, whose atomic size is about 0.2 nanometers, while transistors are about 70 silicon atoms wide. At some point of time, it might happen that chips and their transistors will be converging on the scale of atoms which will likely trigger quantum phenomenon. Electrons will not be flowing predictably anymore, instead, it will behave unpredictably and may pass through solid surfaces or even teleport mysteriously to another part of the chip, called quantum tunneling. In other words, one day, it

won't be possible to miniaturize the transistors further. However, it is believed that classical computers will exist along with any new emerging computing platforms.

### III. LITERATURE REVIEW

In early 1900s, Quantum mechanics emerged as a branch of physics to explain nature on the scale of atoms and led to advances in transistors, lasers, magnetic resonance imaging, etc. In the 1970s and early 1980s, computer scientists started to explore the possibilities of creating a computing device based on quantum mechanics. Since 1920s, physicists have recognized the unique nature of the universe of subatomic particles, but the computer scientists took another 50 years to start their research on using quantum effects in computing. In 1982, Richard Feynman [1] was one of the few physicists to make an effort to conceptualize a new type of computational device that might be developed using the quantum mechanics principles. In 1985, David Deutsch proposed an algorithm named as Deutsch-Jozsa algorithm, illustrates that quantum computers have major advantages over classical computers where the fundamental principles of quantum physics can be used to exploit computation that is not inherent in classical computers. Peter Shor, an American Mathematician, introduced another quantum algorithm for finding the prime factors of Integers, in the year 1994. His algorithm is popularly known as Shor's Algorithm. Some studies have been done on the advantages of using this technology in various emerging technologies. Quantum Computing uses quantum-mechanical phenomenon, which we will be explaining further in this paper. Quantum Cloud Computation – this is a combination of Quantum Computing and Cloud Computing. Quantum Computers are meant for superfast computation and Cloud Computing provides the power of computing as a service, thus combining both will allow anyone in this world to experience the power of Quantum Computing via Cloud [2]. Another important field is Cryptography where Quantum Computing can bring a massive revolution. Charles H. Bennet and Gilles Brassard developed the concept of quantum cryptography in 1984. Bennet and Brassard show that an encryption key can be generated depending on the number or photons that reaches the receiver and the way they are being received [3]. Quantum Computing

in Blockchain – it is a different type of database, more secured than traditional or classical database and these are not governed by any central authority. The best quality of Blockchain is its security. It uses cryptography, to restrict unauthorized access. Cryptography uses critical mathematics that is difficult for human as well as classical computers to solve, while Quantum Computers have the potential to solve complex math problems in order to break the security. Quantum Computers can also be implemented in the area of Machine Learning. Learning tasks can be classified into three categories – supervised, unsupervised and reinforcement learning. Quantum Computing can be used to accelerate various supervised learning strategies such as Curve fitting or Regression, using HHL algorithm for solving linear systems of equations. There is another popular algorithm, known as Grover's algorithm, along with a special oracle function can be used for Quantum Cluster Analysis, which is an unsupervised learning task [4].



Quantum Computer - IBM's Q System One [5]

### IV. BACKGROUND

Electron plays a vital role in Classical Computers. These particles orbit around an atom. The electrical current is generated due to the movement of electrons, which is the necessary ingredient of a transistor to carry out its job. As electrons move around a digital circuit, there is a certainty about its location, speed, and time. For example, if a series of transistors are connected, we can identify when, where, and how fast electrons will arrive at a specific location. It's a predictable and measurable phenomenon. The classical computing is based on what is termed as classical mechanics. However, quantum computers use a completely different

approach. So far by 20th century, a series of discoveries are performed by several scientists that resulted questioning about the predictability of the behavior of particles. While researching on the smallest discrete unit of a physical property, called quantum, gave rise to the questions about the certainty of a particle's behavior and location at a given time. Various scientists concluded that this behavior may be subject to probability. The science that deals with the mathematical description of the motion and interaction of subatomic particles, known as Quantum mechanics.

*A. Superposition: The core of quantum computing*

The vulnerability of a qubit to have multiple states simultaneously is called superposition. It's the physics of quantum mechanics that allows us to perform calculation on multiple states at the same time. For example, in classical computers, if we have a bit string of 3 bits, we can have/compute only one out of 8 combinations of bits. Whereas, in quantum computers, every qubit can be in superposition of 0 or 1, thus we can be in superposition of all the 8 combinations at the same time. As a result, quantum algorithms are exponentially faster than any classical algorithms we know. But once we read/measure the superposition state of a qubit, it collapses to one of its states, i.e., although qubits can be in superposition, but when measured, it gives only once answer.

*A classical computing model computes all possible solutions in sequence to obtain the correct answer.*

*A quantum computer guesses many answers in parallel, thus finding the solution in no time at all.*

Qubits are the most essential thing that makes quantum computing work. Qubits are created by changing the state of certain atoms and other quantum-scale particles, like electrons, nucleus or even photons. To change an electron into a qubit, laser beams, electromagnetic fields, radio waves, and some other techniques, are generally used. These qubits can be tied together to create quantum gates, they are equivalent to logic gates. Although quantum computing is already achieving remarkable results, it has some big challenges to solve. Among them is managing qubits. Any vibration or changes in temperature immediately make them unstable, this can hamper computation or cause errors in the result. We need to take care of these current limitations to make quantum computing reliable and use with confidence.

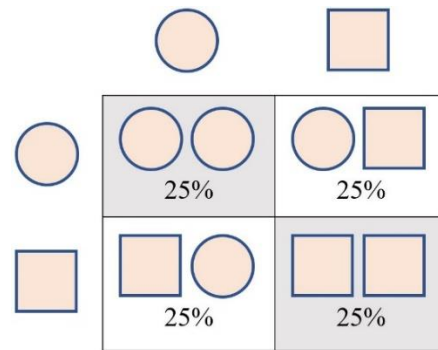
*B. Entanglement: The Unusual characteristics of quantum mechanics*

Entanglement is when two particles that have no physical connection respond to each other as if they know about each other. In quantum entanglement, the particles can be at any distance from each other (they might exist on the opposite sides of the Universe). When one particle is observed, the other has the identical but opposite measurement. They are connected without communicating. There is no exact answer to this, but some believe, “*Quantum entanglement is perhaps what is holding the universe together*”. Qubits are in a state of superposition, until they are being observed, i.e., each qubit spins until it is measured, and then it is assumed to be in an up or down position, that is nothing but a zero or a one. If two qubits are in an entangled state, it is obvious that, if one of them is measured, then immediately other will be known. Thus, in contrast to classical computing, where each bit is processed in a sequential manner, qubits are capable of producing enormous computational outputs in parallel. Albert Einstein famously described entanglement as –

*“Spooky action at a distance.”*

Let us take a simple example to understand entanglement.

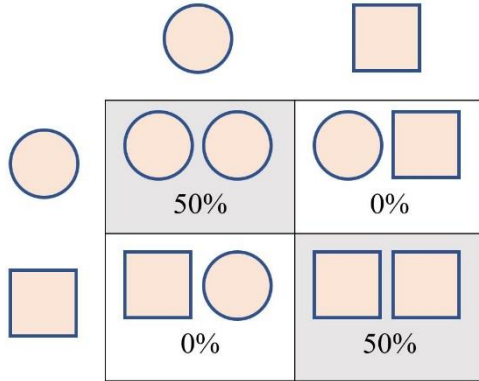
A bakery produces cake in two shapes – circular and square (which we can consider as their possible states). If we pair them, then we can get cakes in four possible combinations.



Normal States

The above four cases are known as “Independent” as knowledge of one state does not give information about the other. Say, if the first cake is circular then the other can be either circular or square, same happens with the square cakes too. Thus, the probability getting any one of the above four cases becomes ¼. But if the two states are “entangled”, then

the scenario would be something as shown in the table below –



Entangled States

In this case, whenever the first cake is circular, then the second one is also circular, and when the first cake is square, then the second one is also square. This is the phenomenon of Entanglement, when one state is known, we get to know the other.

C. Quantum Decoherence

Decoherence is the description of a thermodynamically irreversible exchange of information with the environment. When the environment is barely measurable, decoherence quickly renders most quantum systems indistinguishable from the classical ensemble of possible (classical) states [6].

Qubits are being created using specific atomic particles, or even photons, which are light waves. For example, an electron of phosphorus atom. To prepare a qubit, the atoms are first placed in a superconducting magnet. This forces the electron into a spin-down position. Under normal temperature, the electron's spin direction is not stable, as heat produces energy, forcing the electron to get excited and move. To create stability, we need to set up an environment within a supercooled refrigerator. To make the electron spin up, a pulse of microwaves is fired at it. A superposition can be created by stopping the microwaves at a point anywhere between spin-down and spin-up. It is now a stable, coherent qubit. It is to be noted that, there will be an impact on the qubit's state, if any changes to light, sound, vibration, temperature, and other external factors.

V. MATHEMATICAL PRELIMINARIES IN CONNECTION TO QUANTUM COMPUTATION

In Quantum Mechanics, everything can be described by a wave-function, generally denoted using the Greek

letter 'Ψ'. The wave-function (Ψ) is a complex-valued function and from its absolute square, we can obtain the probability of a measurement outcome. We have a very popular equation for wave-function, known as Schrödinger equation and the solutions to this equation are the possible things that the system can perform.

$$\hat{H}|\Psi\rangle = i\hbar \frac{\delta}{\delta t}|\Psi\rangle$$

Schrödinger Wave Equation

The Schrödinger equation has a very interesting property, i.e., if we have two solutions to the equation, then the sum of those two solutions with arbitrary pre-factors is also a solution (superposition). In other words, if  $|\Psi_1\rangle$  is a solution and  $|\Psi_2\rangle$  is the other, then  $a|\Psi_1\rangle + b|\Psi_2\rangle$  is also a solution.

Let us create two superpositions, that are a sum and difference of the two original solutions,  $\Psi_1$  and  $\Psi_2$ . Then we have two new solutions, let them be  $\Psi_3$  and  $\Psi_4$ . Interestingly, now we can write the original solution  $\Psi_1$  and  $\Psi_2$  as a superposition of  $\Psi_3$  and  $\Psi_4$ . For example, we have obtained two solutions,  $\Psi_1$  and  $\Psi_2$ . Let us now create two superpositions which will be the sum and difference of the original solution, say  $\Psi_3$  and  $\Psi_4$ .

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}} (|\Psi_1\rangle + |\Psi_2\rangle)$$

$$|\Psi_4\rangle = \frac{1}{\sqrt{2}} (|\Psi_1\rangle - |\Psi_2\rangle)$$

Interestingly, we can now say that the original solutions as the superposition of  $\Psi_3$  and  $\Psi_4$ .

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}} (|\Psi_3\rangle + |\Psi_4\rangle)$$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}} (|\Psi_3\rangle - |\Psi_4\rangle)$$

As it is already mentioned that wave-function is the one that is being used in Quantum Mechanics to describe everything. Now, the wave-function in quantum mechanics is a vector, although it's not a vector in the space that we see around us, but a vector in an abstract mathematical thing called a Hilbert-space. One of the most important differences between the wave-function and vector is that the coefficients in quantum mechanics are complex numbers instead of real numbers, so they in general have a non-zero imaginary part.

In quantum mechanics, we use the following kind of brackets to represent wave-function, instead of regular vector notations,

$$|\Psi\rangle = \begin{pmatrix} a1 \\ a2 \\ a3 \end{pmatrix}$$

This notation helps to keep track of whether a vector is a row vector or a column vector. The above one is a column vector. If we have a row-vector, we need to draw the bracket on the other side, shown as follows,

$$\langle \Psi | = (a1^* \ a2^* \ a3^*)$$

Note that, if we want to convert a row vector to a column vector, we must take the complex conjugate of the coefficients.

$$\begin{aligned} \text{If} \quad & a = X + iY \\ \text{Then} \quad & a^* = X - iY \end{aligned}$$

Here,  $a^*$  is the complex conjugate of complex number  $a$ . The following notation was proposed by Paul Dirac and is called the bra-ket notation or Dirac notation. The left side (representing row vector) is the “bra” and the right side (representing column vector), is the “ket”.

$$\langle \text{bra} | \text{ket} \rangle$$

**A. Dirac Notation in Detail:**

- ket:  $|\Psi\rangle = \begin{pmatrix} a1 \\ a2 \end{pmatrix}$
- bra: The “bra” has complex conjugates on the coefficients.

$$\langle \Phi | = |\Phi\rangle^* = \begin{pmatrix} b1 \\ b2 \end{pmatrix}^* = (b1^* \ b2^*)$$

- bra-ket: This notation can be suitably used to write a scalar product as follows,

$$\langle \Phi | \Psi \rangle = a1b1^* + a2b2^*$$

The scalar product is the sum over the products of the coefficients.

- ket-bra: Here, we can multiply one of the bra or ket with itself, to obtain ket-bra. In the following, notice that what we have got is not a single number. It’s in a matrix form, each element of which is a product of coefficients of the vectors. This is known as the “density matrix”.

$$|\Psi\rangle\langle \Phi | = \begin{pmatrix} a1b1^* & a1b2^* \\ a2b1^* & a2b2^* \end{pmatrix}$$

- We define the eigenstates of qubits as –
  - $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ , which are orthogonal:  $\langle 0|1\rangle = (1 \ 0) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1.0 + 0.1 = 0$ .
- Another orthonormal bases can be built from the computational bases, such as  $|+\rangle$  and  $|-\rangle$ 
  - $|+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $|-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
- All quantum states are normalized –
  - $\langle \Psi | \Psi \rangle = 1$ , e.g.,  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ 

$$= \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

**B. Measurements:**

- We use orthogonal bases to describe and measure quantum states (also known as projective measurements).
- If we have a measurement onto the bases  $\{|0\rangle, |1\rangle\}$  (orthogonal states), the state will collapse into either state  $|0\rangle$  or  $|1\rangle$ , as those are the eigenstates of sigma Z operator ( $\sigma_z$ ). This measurement is known as as Z-measurement.
- There are infinitely many different bases, of them few popular ones are –

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

$$|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

- The above also correspond to the eigenstates of sigma  $\sigma_x$  and  $\sigma_y$  respectively.

**C. Born Rule:** The probability that a state  $|\Psi\rangle$  collapses during a projective measurement onto the bases  $\{|x\rangle, |x^+\rangle\}$  to the state  $|x\rangle$ , is given by –

$$P(x) = |\langle x | \Psi \rangle|^2, \text{ where } \sum P(i) = 1$$

Let us take some examples:

Example 1:  $|\Psi\rangle = \frac{1}{\sqrt{3}}(|0\rangle + \sqrt{2}|1\rangle)$  is measurement in the bases  $\{|0\rangle, |1\rangle\}$ :

$$\begin{aligned} P(0) &= |\langle 0 | \frac{1}{\sqrt{3}}(|0\rangle + \sqrt{2}|1\rangle) \rangle|^2 \\ &= |\frac{1}{\sqrt{3}}\langle 0|0\rangle + \frac{\sqrt{2}}{\sqrt{3}}\langle 0|1\rangle|^2 \\ &= \frac{1}{3} \end{aligned}$$

[Since  $\langle 0|0\rangle = 1$  (normalized state);  $\langle 0|1\rangle = 0$  (as they are orthogonal)]

$$\begin{aligned} P(1) &= |\langle 1 | \frac{1}{\sqrt{3}}(|0\rangle + \sqrt{2}|1\rangle) \rangle|^2 \\ &= |\frac{1}{\sqrt{3}}\langle 1|0\rangle + \frac{\sqrt{2}}{\sqrt{3}}\langle 1|1\rangle|^2 \\ &= \frac{2}{3} \end{aligned}$$

[Since  $\langle 1|1\rangle = 1$  (normalized state);  $\langle 1|0\rangle = 0$  (as they are orthogonal)]

Example 2:  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$  is measurement in the bases  $\{|+\rangle, |-\rangle\}$ :

$$\begin{aligned} P(+)&= |\langle + | \Psi \rangle|^2 \\ &= |\frac{1}{\sqrt{2}}\langle 0|0\rangle + \frac{1}{\sqrt{2}}\langle 0|1\rangle - \frac{1}{\sqrt{2}}\langle 1|0\rangle - \frac{1}{\sqrt{2}}\langle 1|1\rangle|^2 \\ &= \frac{1}{4} |\langle 0|0\rangle - \langle 0|1\rangle + \langle 1|0\rangle - \langle 1|1\rangle|^2 \\ &= 0 \end{aligned}$$

The above result is expected, as  $\langle + | \Psi \rangle = \langle + | - \rangle = 0$  [Since,  $\langle + | - \rangle$  is orthogonal.]

VI. EXPERIMENT

Bloch Sphere: Two qubit states  $|0\rangle$  &  $|1\rangle$  are represented by positive and negative Z-axis respectively. State  $|0\rangle$  denotes upward spin of electron, while state  $|1\rangle$  denotes downward spin of electron. Any point  $|\Psi\rangle$  on this sphere is represented by equation, thus generic state of a qubit is represented by a linear combination of the two eigen kets:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where the coefficients are complex numbers related to each other by,  $|\alpha|^2 + |\beta|^2 = 1$ , ( $\alpha^2$  represents the probability of electron having upward spin and  $\beta^2$  represents the probability of electron having downward spin).

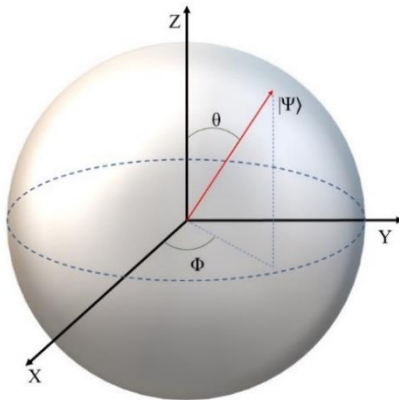


Fig 1: Bloch Sphere

This state can be parametrized by angles  $\theta$  and  $\phi$ , such as  $\alpha = \cos \theta/2$  and  $\beta = \exp(i\phi) \sin \theta/2$ :

$$|\Psi\rangle = \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow$$

Where, Azimuthal angle  $\phi \in [0, 2\pi]$  describes the relative phase and Polar angle  $\theta \in [0, \pi]$  determines the probability to measure  $|0\rangle$  or  $|1\rangle$ ,  $p(0) = \cos^2 \theta/2$  and  $p(1) = \sin^2 \theta/2$ . The above expression allows us to get a geometric visualization of the qubit states as a point on the surface of a unit radius sphere, called Bloch sphere. It is named after the physicist Felix Bloch. [All normalized pure states can be illustrated on the surface of the Bloch sphere.]

To get the value of  $e^{i\phi}$ , we need to apply Euler's Formula, that gives a fundamental relationship between exponential and trigonometric function. For a complex number,

$$e^{i\phi} = \cos \phi + i \sin \phi \quad \text{-Euler's Formula}$$

The coordinates of such a state are given by the Bloch Vector,  $\hat{r} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ . It is a unit vector used to represent points on a Bloch Sphere.

Important Points: The most important points on Bloch sphere are shown on the table below,

$\Theta$	$\phi$	$ \Psi\rangle$	Bloch Vector ( $\hat{r}$ )
0	-	$ 0\rangle$	(0 0 1)
$\pi$	0 or $2\pi$	$ 1\rangle$	(0 0 -1)
$\pi/2$	0	$ +\rangle$	(1 0 0)
$\pi/2$	$\pi$	$ -\rangle$	(-1 0 0)
$\pi/2$	$\pi/2$	$ +i\rangle$	(0 1 0)
$\pi/2$	$3\pi/2$	$ -i\rangle$	(0 -1 0)

Table 1: Important Points on Bloch Sphere

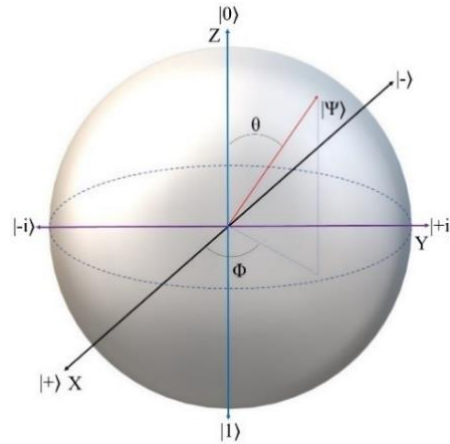


Fig 2: Different Quantum States

Note/- The angles on the Bloch Sphere are twice as large as in Hilbert Space, e.g.,  $|0\rangle$  &  $|1\rangle$  are orthogonal, but on the Bloch Sphere their angle is  $180^\circ$ . For a general state  $|\Psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$ , here  $\theta$  is the angle on the Bloch sphere, while  $\theta/2$  is the actual angle in Hilbert Space!

Observations Table:

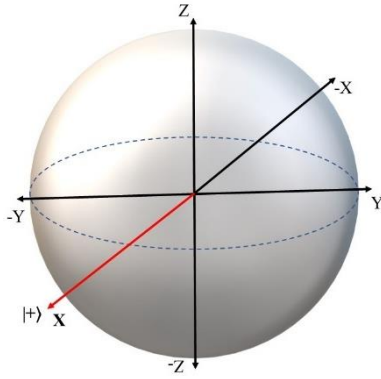
$ \Psi\rangle$	Observation
$ 0\rangle$	North pole on the Z axis
$ 1\rangle$	South pole on the Z axis
$ +\rangle$	Equator line on the X axis
$ -\rangle$	Equator line on the X axis
$ +i\rangle$	Equator line on the Y axis
$ -i\rangle$	Equator line on the Y axis

Table 2: Observations based on the position of different points on the Bloch Sphere

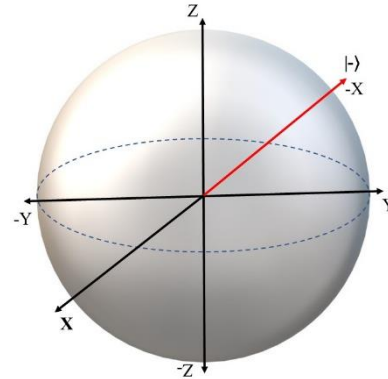
VII. REPRESENTATION OF IMPORTANT QUANTUM STATES IN BLOCH SPHERE

The following are the representation of all the points in respective Bloch Sphere that are derived and shown in the Table 1.

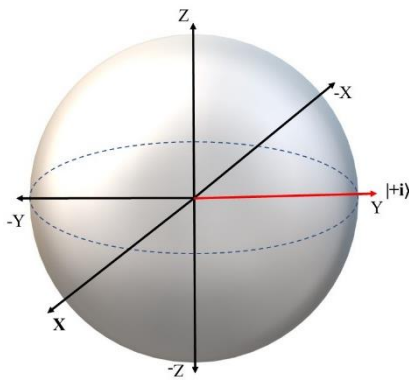




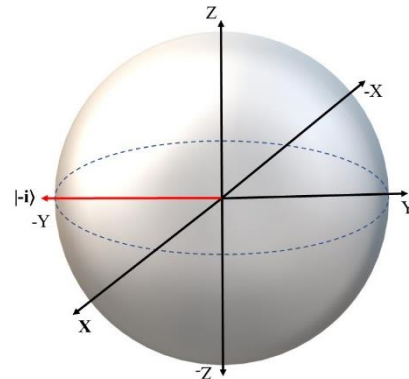
Bloch Sphere representing the Quantum State  $|+\rangle$   
 Bloch Vector:  $\hat{r} = (1 \ 0 \ 0)$ , where  $\theta = \pi/2$  &  $\phi = 0$   
 $|\Psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$



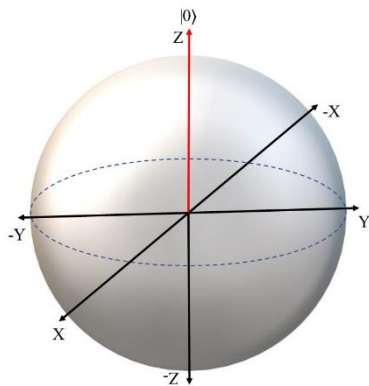
Bloch Sphere representing the Quantum State  $|-\rangle$   
 Bloch Vector:  $\hat{r} = (-1 \ 0 \ 0)$ , where  $\theta = \pi/2$  &  $\phi = \pi$   
 $|\Psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$



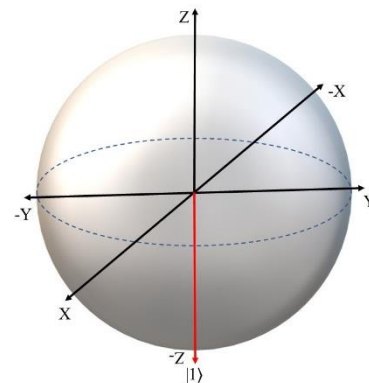
Bloch Sphere representing the Quantum State  $|+i\rangle$   
 Bloch Vector:  $\hat{r} = (0 \ 1 \ 0)$ , where  $\theta = \pi/2$  &  $\phi = \pi/2$   
 $|\Psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}i|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$   
 $= \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle)$



Bloch Sphere representing the Quantum State  $| -i\rangle$   
 Bloch Vector:  $\hat{r} = (0 \ -1 \ 0)$   
 Where  $\theta = \pi/2$  &  $\phi = 3\pi/2$   
 $|\Psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}i|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) = \frac{1}{\sqrt{2}}(|\uparrow\rangle - i|\downarrow\rangle)$



Bloch Sphere representing the Quantum State  $|0\rangle$   
 Bloch Vector:  $\hat{r} = (0 \ 0 \ 1)$ , where  $\theta = 0$  &  $|\Psi\rangle = |0\rangle = \uparrow$



Bloch Sphere representing the Quantum State  $|1\rangle$   
 Bloch Vector:  $\hat{r} = (0 \ 0 \ -1)$ , where  $\theta = \pi$  &  $\phi = 0$  or  $2\pi$   
 $|\Psi\rangle = |1\rangle = \downarrow$

VIII. SOME MORE STATES ON BLOCH SPHERE

With different combinations of  $\theta$  and  $\phi$ , where  $\theta \in [0, \pi]$  and  $\phi \in [0, 2\pi]$ , we are computing the values of Bloch Vector ( $\hat{r}$ ) and  $|\Psi\rangle$ , using the following formulas:

$$|\Psi\rangle = \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow$$

$$\text{where } e^{i\phi} = \cos \phi + i \sin \phi$$

$$\text{Bloch Vector } (\hat{r}) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

1. For  $\theta = \pi/2$  &  $\phi = \pi/6$

$$\begin{aligned} \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\ &= (\sin \pi/2 \cos \pi/6, \sin \pi/2 \sin \pi/6, \cos \pi/2) \\ &= \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right) \end{aligned}$$

$$\begin{aligned} e^{i\phi} &= \cos \phi + i \sin \phi \\ &= \cos \pi/6 + i \sin \pi/6 \\ &= \frac{\sqrt{3}}{2} + i \frac{1}{2} \end{aligned}$$

$$\begin{aligned} |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\ &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i\pi/6} \sin \frac{\pi/2}{2} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \left(\frac{\sqrt{3}}{2} + i \frac{1}{2}\right) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \frac{1}{2\sqrt{2}} (\sqrt{3} + i) |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} [ |0\rangle \uparrow + \frac{1}{2} (\sqrt{3} + i) |1\rangle \downarrow ] \end{aligned}$$

2. For  $\theta = \pi/2$  &  $\phi = \pi/4$

$$\begin{aligned} \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\ &= (\sin \pi/2 \cos \pi/4, \sin \pi/2 \sin \pi/4, \cos \pi/2) \\ &= \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \end{aligned}$$

$$\begin{aligned} e^{i\phi} &= \cos \phi + i \sin \phi \\ &= \cos \pi/4 + i \sin \pi/4 \\ &= \frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\ &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i\pi/4} \sin \frac{\pi/2}{2} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}\right) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \frac{1}{2} (1 + i) |1\rangle \downarrow \end{aligned}$$

3. For  $\theta = \pi/2$  &  $\phi = \pi/3$

$$\begin{aligned} \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\ &= (\sin \pi/2 \cos \pi/3, \sin \pi/2 \sin \pi/3, \cos \pi/2) \\ &= \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right) \end{aligned}$$

$$\begin{aligned} e^{i\phi} &= \cos \phi + i \sin \phi \\ &= \cos \pi/3 + i \sin \pi/3 \\ &= \frac{1}{2} + i \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\ &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i\pi/3} \sin \frac{\pi/2}{2} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \left(\frac{1}{2} + i \frac{\sqrt{3}}{2}\right) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \frac{1}{2} (1 + i\sqrt{3}) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} [ |0\rangle \uparrow + \frac{1}{2} (1 + i\sqrt{3}) |1\rangle \downarrow ] \end{aligned}$$

4. For  $\theta = \pi/2$  &  $\phi = 2\pi/3$

$$\begin{aligned} \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\ &= (\sin \pi/2 \cos 2\pi/3, \sin \pi/2 \sin 2\pi/3, \cos \pi/2) \\ &= \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right) \end{aligned}$$

$$\begin{aligned} e^{i\phi} &= \cos \phi + i \sin \phi \\ &= \cos 2\pi/3 + i \sin 2\pi/3 \\ &= -\frac{1}{2} + i \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\ &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i2\pi/3} \sin \frac{\pi/2}{2} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2}\right) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} |0\rangle \uparrow - \frac{1}{2} (1 - i\sqrt{3}) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} [ |0\rangle \uparrow - \frac{1}{2} (1 - i\sqrt{3}) |1\rangle \downarrow ] \end{aligned}$$

5. For  $\theta = \pi/2$  &  $\phi = 3\pi/4$

$$\begin{aligned} \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\ &= (\sin \pi/2 \cos 3\pi/4, \sin \pi/2 \sin 3\pi/4, \cos \pi/2) \\ &= \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \end{aligned}$$

$$\begin{aligned} e^{i\phi} &= \cos \phi + i \sin \phi \\ &= \cos 3\pi/4 + i \sin 3\pi/4 \\ &= -\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\ &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i3\pi/4} \sin \frac{\pi/2}{2} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \left(-\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}\right) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\ &= \frac{1}{\sqrt{2}} |0\rangle \uparrow - \frac{1}{2} (1 - i) |1\rangle \downarrow \end{aligned}$$

6. For  $\theta = \pi/2$  &  $\phi = 5\pi/6$

$$\begin{aligned} \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\ &= (\sin \pi/2 \cos 5\pi/6, \sin \pi/2 \sin 5\pi/6, \cos \pi/2) \\ &= \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right) \end{aligned}$$

$$\begin{aligned} e^{i\phi} &= \cos \phi + i \sin \phi \\ &= \cos 5\pi/6 + i \sin 5\pi/6 \\ &= -\frac{\sqrt{3}}{2} + i \frac{1}{2} \end{aligned}$$



$$\begin{aligned}
 |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\
 &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i5\pi/6} \sin \frac{\pi/2}{2} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow - \frac{1}{2\sqrt{2}} (\sqrt{3} - i) |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} [ |0\rangle \uparrow - \frac{1}{2} (\sqrt{3} - i) |1\rangle \downarrow ]
 \end{aligned}$$

7. For  $\theta = \pi/2$  &  $\phi = 7\pi/6$

$$\begin{aligned}
 \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\
 &= (\sin \pi/2 \cos 7\pi/6, \sin \pi/2 \sin 7\pi/6, \cos \pi/2) \\
 &= \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0\right)
 \end{aligned}$$

$$\begin{aligned}
 e^{i\phi} &= \cos \phi + i \sin \phi \\
 &= \cos 7\pi/6 + i \sin 7\pi/6 \\
 &= -\frac{\sqrt{3}}{2} - i\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\
 &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i7\pi/6} \sin \frac{\pi/2}{2} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \left(-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow - \frac{1}{2\sqrt{2}} (\sqrt{3} + i) |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} [ |0\rangle \uparrow - \frac{1}{2} (\sqrt{3} + i) |1\rangle \downarrow ]
 \end{aligned}$$

8. For  $\theta = \pi/2$  &  $\phi = 5\pi/4$

$$\begin{aligned}
 \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\
 &= (\sin \pi/2 \cos 5\pi/4, \sin \pi/2 \sin 5\pi/4, \cos \pi/2) \\
 &= \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)
 \end{aligned}$$

$$\begin{aligned}
 e^{i\phi} &= \cos \phi + i \sin \phi \\
 &= \cos 5\pi/4 + i \sin 5\pi/4 \\
 &= -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\
 &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i5\pi/4} \sin \frac{\pi/2}{2} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \left(-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow - \frac{1}{2} (1 + i) |1\rangle \downarrow
 \end{aligned}$$

9. For  $\theta = \pi/2$  &  $\phi = 4\pi/3$

$$\begin{aligned}
 \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\
 &= (\sin \pi/2 \cos 4\pi/3, \sin \pi/2 \sin 4\pi/3, \cos \pi/2) \\
 &= \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right)
 \end{aligned}$$

$$\begin{aligned}
 e^{i\phi} &= \cos \phi + i \sin \phi \\
 &= \cos 4\pi/3 + i \sin 4\pi/3 \\
 &= -\frac{1}{2} - i\frac{\sqrt{3}}{2}
 \end{aligned}$$

$$|\Psi\rangle = \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow$$

10. For  $\theta = \pi/2$  &  $\phi = 5\pi/3$

$$\begin{aligned}
 \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\
 &= (\sin \pi/2 \cos 5\pi/3, \sin \pi/2 \sin 5\pi/3, \cos \pi/2) \\
 &= \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right)
 \end{aligned}$$

$$\begin{aligned}
 e^{i\phi} &= \cos \phi + i \sin \phi \\
 &= \cos 5\pi/3 + i \sin 5\pi/3 \\
 &= \frac{1}{2} - i\frac{\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\
 &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i5\pi/3} \sin \frac{\pi/2}{2} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \frac{1}{2} (1 - i\sqrt{3}) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} [ |0\rangle \uparrow + \frac{1}{2} (1 - i\sqrt{3}) |1\rangle \downarrow ]
 \end{aligned}$$

11. For  $\theta = \pi/2$  &  $\phi = 7\pi/4$

$$\begin{aligned}
 \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\
 &= (\sin \pi/2 \cos 7\pi/4, \sin \pi/2 \sin 7\pi/4, \cos \pi/2) \\
 &= \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)
 \end{aligned}$$

$$\begin{aligned}
 e^{i\phi} &= \cos \phi + i \sin \phi \\
 &= \cos 7\pi/4 + i \sin 7\pi/4 \\
 &= \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\
 &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i7\pi/4} \sin \frac{\pi/2}{2} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \frac{1}{2} (1 - i) |1\rangle \downarrow
 \end{aligned}$$

12. For  $\theta = \pi/2$  &  $\phi = 11\pi/6$

$$\begin{aligned}
 \hat{r} &= (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \\
 &= (\sin \pi/2 \cos 11\pi/6, \sin \pi/2 \sin 11\pi/6, \cos \pi/2) \\
 &= \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0\right)
 \end{aligned}$$

$$\begin{aligned}
 e^{i\phi} &= \cos \phi + i \sin \phi \\
 &= \cos 11\pi/6 + i \sin 11\pi/6 \\
 &= \frac{\sqrt{3}}{2} - i\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 |\Psi\rangle &= \cos \frac{\theta}{2} |0\rangle \uparrow + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \downarrow \\
 &= \cos \frac{\pi/2}{2} |0\rangle \uparrow + e^{i11\pi/6} \sin \frac{\pi/2}{2} |1\rangle \downarrow
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + (\frac{\sqrt{3}}{2} - i \frac{1}{2}) \frac{1}{\sqrt{2}} |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} |0\rangle \uparrow + \frac{1}{2\sqrt{2}} (\sqrt{3} - i) |1\rangle \downarrow \\
 &= \frac{1}{\sqrt{2}} [ |0\rangle \uparrow + \frac{1}{2} (\sqrt{3} - i) |1\rangle \downarrow ]
 \end{aligned}$$

Summary Table of the above Calculations

$\theta$	$\phi$	$ \psi\rangle$	$\hat{r}$
$\pi/2$	$\pi/6$	$\frac{1}{\sqrt{2}}[ 0\rangle\uparrow + \frac{1}{2}(\sqrt{3}+i) 1\rangle\downarrow]$	$(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0)$
$\pi/2$	$\pi/4$	$\frac{1}{\sqrt{2}} 0\rangle\uparrow + \frac{1}{2}(1+i) 1\rangle\downarrow]$	$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$
$\pi/2$	$\pi/3$	$\frac{1}{\sqrt{2}}[ 0\rangle\uparrow + \frac{1}{2}(1+i\sqrt{3}) 1\rangle\downarrow]$	$(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$
$\pi/2$	$2\pi/3$	$\frac{1}{\sqrt{2}}[ 0\rangle\uparrow - \frac{1}{2}(1-i\sqrt{3}) 1\rangle\downarrow]$	$(\frac{-1}{2}, \frac{\sqrt{3}}{2}, 0)$
$\pi/2$	$3\pi/4$	$\frac{1}{\sqrt{2}} 0\rangle\uparrow - \frac{1}{2}(1-i) 1\rangle\downarrow]$	$(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$
$\pi/2$	$5\pi/6$	$\frac{1}{\sqrt{2}}[ 0\rangle\uparrow - \frac{1}{2}(\sqrt{3}-i) 1\rangle\downarrow]$	$(\frac{-\sqrt{3}}{2}, \frac{1}{2}, 0)$
$\pi/2$	$7\pi/6$	$\frac{1}{\sqrt{2}}[ 0\rangle\uparrow - \frac{1}{2}(\sqrt{3}+i) 1\rangle\downarrow]$	$(\frac{-\sqrt{3}}{2}, \frac{-1}{2}, 0)$
$\pi/2$	$5\pi/4$	$\frac{1}{\sqrt{2}} 0\rangle\uparrow - \frac{1}{2}(1+i) 1\rangle\downarrow]$	$(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0)$
$\pi/2$	$4\pi/3$	$\frac{1}{\sqrt{2}}[ 0\rangle\uparrow - \frac{1}{2}(1+i\sqrt{3}) 1\rangle\downarrow]$	$(\frac{-1}{2}, \frac{-\sqrt{3}}{2}, 0)$
$\pi/2$	$5\pi/3$	$\frac{1}{\sqrt{2}}[ 0\rangle\uparrow + \frac{1}{2}(1-i\sqrt{3}) 1\rangle\downarrow]$	$(\frac{1}{2}, \frac{-\sqrt{3}}{2}, 0)$
$\pi/2$	$7\pi/4$	$\frac{1}{\sqrt{2}} 0\rangle\uparrow + \frac{1}{2}(1-i) 1\rangle\downarrow]$	$(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0)$
$\pi/2$	$11\pi/6$	$\frac{1}{\sqrt{2}}[ 0\rangle\uparrow + \frac{1}{2}(\sqrt{3}-i) 1\rangle\downarrow]$	$(\frac{\sqrt{3}}{2}, \frac{-1}{2}, 0)$

### IX. LATEST TRENDS IN QUANTUM COMPUTING

Quantum Computing is an emerging trend. The giant players who have already step into the field of Quantum Computers includes – IBM, D Wave, Microsoft, Google.

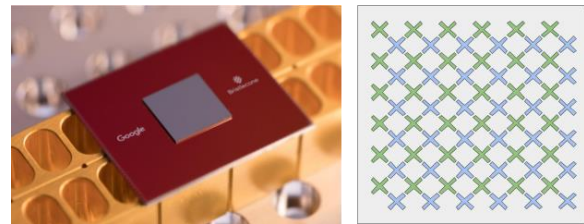
IBM: Beside continuous research and experiments, IBM provide public access to their quantum computers via their cloud platform called Qiskit (an open-source quantum software development kit) and IBM Q Experience (a virtual interface for coding). The number of qubits in these machines’ ranges from 5 to 32, but IBM has already developed machines with higher number of qubits which are not publicly accessible. The latest IBM processor is Eagle, which is a 127-qubit quantum processor. IBM has also announced that they will be launching their 433 qubit Osprey Processor and increase the speed from 1.4K CLOPs to 10K CLOPs by the end of 2022.

D Wave: The company was founded in 1999, claim’s to be the World’s first commercial Quantum Computer

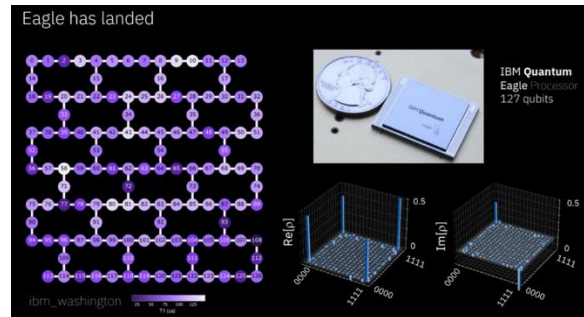
supplier. They develop both hardware and software and has been granted over 200 US patents.

Google: Google is nowhere behind in the research and development of Quantum Computers. Their latest processor named Bristlecone, is a 72-qubit quantum processor. Like IBM, Google also provide two open-source platforms – Cirq and OpenFermion. The company’s focus is on quantum powering artificial intelligence.

Microsoft: They are also working on this new computing platform for over a decade, both on hardware and software. They have built their own quantum programming language and named it as Q#.



Bristlecone by Google (left) and a cartoon of the device – Each “X” represents a qubit, with nearest neighbor connectivity (right) [7]



IBM’s Eagle – Quantum Processing Unit [8]

Quantum Computing in India:

Quantum Computer Simulation Toolkit (QSim): One of the first initiative by Government of India (Ministry of Electronics and Information Technology or MeitY) to overcome common challenges in research and development of Quantum Computing in India. Major organizations like IISc, IIT Roorkee and C-DAC, plays an important role in development of this project. QSim provides an integrated GUI based workbench for simulation of different quantum circuits and programs. This platform is publicly available for students, researchers as well as the enthusiasts who

want to explore the field of Quantum Computing. QSim provides the following key features:

- Simulate: Simulation of quantum circuits dynamically with custom parameters.
- Noise: Realistic simulator considering the different effects of Quantum noise.
- User Interface: Intuitive UI / UX helps users to conceptualize and simulate their own quantum programs.
- Secured: Secure user management system along with options to save our quantum programs or circuits.
- Code Editor: Code editor to create quantum circuits using advance Python language.
- This platform also provides a pack of learning materials with associated solved examples, that can be referred by the programmers for help or guidance.

QRDLab: It is an industry first initiative in Kolkata, India, primarily focusses on high-end quantum research, education, and consultation. QRDLab is open to all independent researchers as well as academic institutions to speed up quantum research and development in India. Research activities of QRDLab in knowledge partnership with University of Calcutta's includes collaborative contribution to the field of Quantum Computing with universities and academic institutions across globe, along with that, they provide quantum ecosystem for researchers, developers, and industry representatives to solve real-world hard problems that remained unsolved by the classical computers.

Qulabs: Qulabs Software (India) Private Limited, incorporated on 22 February 2018, progressing with a vision to introduce the world's first Quantum Internet and as a result create a completely secure internet and communication at immense speed and security. Its registered office is in Hyderabad, Telangana, India. This private company is working on the process to deliver reliable, high quality, cost-effective, efficient, scalable, and secure quantum communication solutions globally. Qulabs plan to set up their first "Quantum Optics Lab" in India at its Kolkata office. There are two optical labs spread over an area of 600 square feet.

Taqbit Labs: A Bengaluru based Indian startup, founded by Animesh Aaryan, Sugata Sarkar, and Sreeram Sreerishna in 2018, keeps a vision to commercialize globally the Quantum technologies for the safety & security of cyber systems. They provide Quantum based encryption which is truly random and claims that it can never be hacked.

Quanta Computacao: It was founded in 2018, a Chennai based startup, primarily focusses on developing quantum cryptographic tools which will be able to provide quantum proof data security, mainly useful for banking sectors to protect transactions. They are also working in Quantum Machine Learning and Artificial Intelligence. Quanta Computacao is developing Alchemy, a Quantum Virtual Simulator, which can be used to compile and run different quantum capable software tools.

## X. FUTURE SCOPE

The era of Quantum Computing has just started. A lot more development is yet to be performed in this field. This new computing technology will require –

- Manufacturers
- Software Developers
- Service Providers
- Consultants or Assistance Services

Software that is meant to run on classical computers, works with binary bits, it cannot run on a system that operates with qubits. Different Quantum Computers varies in different aspects including the number of qubits used and require different algorithms. Yet the future of quantum computer is very much promising.

## XI. CONCLUSION

This study demonstrated the origin of different Quantum States from the famous Schrödinger Wave Equation and the representations of some obtained states into the Bloch Sphere using the corresponding Bloch Vector. One can easily obtain a different quantum state with alternative values of  $\theta$  and  $\phi$  by putting them into the mentioned formulas of  $|\Psi\rangle$  and  $\hat{r}$ .

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