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Applications of Karry-Kalim Adnan Transformation (KKAT) in Growth and Decay problems

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Abstract- Growth decay problems are solved by many researchers by using various methods and various integral transforms. In this paper we use, the integral transform known as Karry-Kalim Adnan transformation (KKAT) to solve the problems of growth and decay.

Key words: Growth and decay problems, Integral transforms, Karry-Kalim Adnan Transform.

I.INTRODUCTION

Recently, integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform in November 2021. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [3] used Kushare transform for obtaining the solution of the problems on Newton's law of Cooling.

In April 2022 D. P. Patil, et al [4] solved the problems on growth and decay by using Kushare transform. D.P. Patil [5] also used Sawi transform in Bessel functions. Further, Patil [6] evaluate improper integrals by using Sawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [7]. Dinkar Patil [8] used Sawi transform and its convolution theorem for solving wave equation. Using Mahgoub transform, parabolic boundary value problems are solved by D.P. Patil [9].

D.P. Patil [10] used double Laplace and double Sumudu transforms to obtain the solution of wave equation. Further Dr. Patil [11] also obtained dualities between double integral transforms. Kandalkar, Gatkal and Patil [12] solved the system of differential equations using Kushare transform. D. P. Patil [13] solved boundary value problems of the system of ordinary differential equations by using Aboodh and Mahgoub transforms. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.

Laplac, Sumudu, Aboodh, Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [15]. D. P. Patil et al [16] solved Volterra Integral equations of first kind by using Emad-Sara transform. Futher Patil with Tile and Shinde [17] used Anuj transform and solved Volterra integral equations for first kind. Rathi sisters and D. P. Patil [18] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [19] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil [20] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil[21] obtained the solution of parabolic boundary value problems by using double general integral transform. Dinkar Patil used Emad- Falih transform for solving problems based on Newton's law of cooling [22]. D. P. Patil et al [23] used Soham transform to obtain the solution of Newton's law of cooling. Dinkar Patil et al [24] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY transform for Newton's law of cooling [25]. D. P. Patil et al [26] used Emad-Falih transform for general solution of telegraph equation. Dinkar Patil et al [27] introduced double kushare transform. Recently, D. P. Patil et al [28] solved population growth and decay problems by using Emad Sara transform. Alenzi transform is used in population growth and decay problems by patil et al [29]. Thete, et al [30] used Emad Falih transform for handling growth and decay problems. Nikam, Patil et al [31] used, Kushare transform of error functions in evaluating improper integrals. Wagh sisters and Patil used Kushare [32] and Soham [33] transform in chemical Sciences. Malpani, Shinde and Patil [34] used Convolution theorem for Kushare transform and applications in convolution type Volterra integral equations of first kind. Raundal and Patil [35] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, Derle and Patil [36] developed generalized double rangaig integral transform. Patil et al [37] used Kharrat Toma transform for solving growth and decay problems. Kushare transform is used for solving Volterra Integro-Differential equations of first kind by Shinde, et al [38]. Kandekar et al [39] used new general integral equation to solve Abel's integral equations.

II.PRELIMINARIES

In this section we state some important and required definitions, properties and formulae.

Definition of KKAT[40]: A transformation defined for function of exponential order from set S.

S = {f(x): ∃P; a_1 ,a_2> 0|f(x)|<P.$e^{|x|/a_i}$ },x∈ (= $1)^i$.[0,∞) …….(1)

Where constant P is a finite number and $a_{1,}a_2$ may be finite or infinite.

KKAT is represented by operator K (.) and is defined by

K(f(x))=$\frac{1}{\beta}\int_0^\infty f(\alpha x)\, e^{-(\beta).x}$dx ,x≥0; α,β∈[$a_{1,}a_2$]

Where α and β are constants and α, β≠0

K{f(x)} can also be written as

K{f(x)} = $\frac{1}{\alpha\beta}\int_0^\infty f(x)e^{-\left(\frac{\beta}{\alpha}\right).x}dx$=F($\left(\frac{\beta}{\alpha}\right)$ ……(2)

Also

f(x)=$K^{-1}\left\{F\left(\frac{\beta}{\alpha}\right)\right\}$ …………..(3)

Some useful formulae of KKAT [40]:

Sr.No.	f(x)	F(β/α)
1.	1	$1/\beta^2$
2.	x^n,n∈N	n!.α^n/β^{n+2}
3.	$e^{\lambda x}$	1/β(β-$\lambda\alpha$)
4.	f '(x)	(β/α)K{f(x)}-f(0)/$\alpha\beta$
5.	f "(x)	(β^2/α^2).K{f(x)}-f '(0)/$\alpha\beta$-f(0)/α^2
6.	$e^{\lambda x}$f(x)	(β-$\lambda\alpha/\beta$)F(α,β-$\lambda\alpha$)

Inverse of KKAT [40]:

If F(β/α) be the KKAT of f(x) then f(x) is called the inverse of KKAT of F(β/α).

The inverse of KKAT is expressed in equation (3)

$K^{-1}\left\{F\left(\frac{\beta}{\alpha}\right)\right\}$ = f(x).

Applications of KKAT on the Integral function [40]:

If $g(x) = \int_0^x f(z)$.dz ,then K{$\int_0^x f(z).dz$} = (α/β) F(β/α).

III.APPLICATIONS OF KKAT

In this section we use KKAT to solve some problems of growth dcay type.

Application 1: one model used in medicine is that the rate of growth of tumor is proportional to the size of the tumor. Write a differential equation satisfied by S. the size of tumor in mm as a function of time t, the tumor is 5mm across at time t=0. Find the solution in addition if the tumor is 8mm across at time t=3, find particular solution.

⇒ This problem is governed by differential equation,

$\frac{dS}{dt} = \lambda S$ …….(1)

Where λ is Constant.

By applying KKAT to equation (1)

K$\left\{\frac{ds}{dt}\right\}$ = K λ {S(t)}

∴$\frac{\beta}{\alpha}$ K { S(t) } $-\frac{S(0)}{\alpha\beta}$ = λK {s(t)}

$\frac{\beta}{\alpha}$ K { S(t) } - λK {s(t)}$\frac{\beta}{\alpha}$ = $\frac{S(0)}{\alpha\beta}$

K { S(t) } $\left(\frac{\beta}{\alpha} - \lambda\right)$ = $\frac{S(0)}{\alpha\beta}$

Now, for t = 0 , S(0) = 5,

K { S(t) } $\left(\frac{\beta}{\alpha} - \lambda\right)$ = $\frac{5}{\alpha\beta}$

K { S(t) } = $\frac{5}{\alpha\beta}.\left(\frac{\alpha}{\beta-\alpha\lambda}\right)$

K { S(t) } = $\left(\frac{5}{\beta(\beta-\alpha\lambda)}\right)$

∴By applying inverse of KKAT , we get ,

{ S(t) } = $K^{-1}\left[\left(\frac{5}{\beta(\beta-\alpha\lambda)}\right)\right]$

S(t) = $5e^{\lambda t}$ ....(2)

Now the condition is that, the tumor is 8mm across at t = 3

Therefore the situation is, from eq (2), $8 = 5e^{3\lambda}$

Taking log on both sides, we get,

$\ln(8) = \ln 5 . \ln e^{3\lambda}$

$$\therefore \ln\left(\frac{8}{5}\right) = 3\lambda$$

$$\therefore \lambda = \frac{1}{3}\ln\left(\frac{8}{5}\right)$$

≈ 0.1567

$$S = 5e^{3(0.1567)}$$

$$S = 5e^{0.4701} = 5(1.600154201)$$

S = 8.00077

Application 2: Hydrocodone bitartite is used as cough suppressant after the drug is fully absorbed the quantity of drug in body decreases at a rate proportional to the amount left in th body. The half-life of hydrocodone bitartite in body is 3.8 hours. And the usual oral dose is 10 mg. Use half-life to find constant of proportionality C. How much of the 10 mg dose is still in the body after 12 hours.

⇒ The problem is governed by the differential equation ,

$$\frac{dQ}{dt} = -CQ \qquad ....(1)$$

Where C is proportionality constant.

Applying KKAT to equation (1) ,

$$K\left\{\frac{dQ}{dt}\right\} = -C\, K\{Q(t)\}$$

$$\frac{\beta}{\alpha} K\{Q(t)\} - \frac{Q(0)}{\alpha\beta} = -C\, K\{Q(t)\}$$

$$\frac{\beta}{\alpha} K\{Q(t)\} + C\, K\{Q(t)\} = \frac{Q(0)}{\alpha\beta}$$

$$K\{Q(t)\} \left[\frac{\beta}{\alpha} + c\right] = \frac{Q(0)}{\alpha\beta}$$

$$K\{Q(t)\} = \frac{Q(0)}{\alpha\beta} . \frac{\alpha}{\beta+\alpha c}$$

$$K\{Q(t)\} = \frac{Q(0)}{\beta(\beta+\alpha c)}$$

Now for t = 0 , Q(0) = Q_0

$$K\{Q(t)\} = \frac{Q_0}{\beta(\beta+\alpha c)}$$

By applying inverse of KKAT ,

$$Q\{T\} = Q_0 . K^{-1} \frac{1}{\beta(\beta+\alpha c)}$$

$$Q\{T\} = Q_0 . e^{-ct} \qquad ....(2)$$

Now, Here t = 3.8 , Q = $\frac{1}{2}Q_0$

So equation (2) becomes,

$$\frac{1}{2}Q_0 = Q_0 . e^{-3.8\,c}$$

$$\frac{1}{2} = e^{-3.8\,c}$$

Therefore taking log on both sides, we get

$$C = \ln\frac{1}{2} . \left(\frac{-1}{3.8}\right) \approx 0.182$$

For $Q_0 = 10$,

$$Q(12) = 10\, e^{-0.182\times 12}$$

$$Q(12) \cong 1.126 \text{ mg}$$

∴ 1.126 mg dose is still in the body after 12 hours.

Appliction 3: The population of the city grows at the rate of proportional to the number of people presently living in the city. If after three years the population is 20,000. Estimate the number of people initially in city.

⇒ This problem is governed by the differential equation

$$\frac{dN}{dt} = PN \qquad ....(1)$$

Where P is constant.

Applying KKAT to equation (1), $K\left(\frac{dN}{dt}\right) = K(PN)$

$$\frac{\beta}{\alpha} . K\{N(t)\} - \frac{N(0)}{\alpha\beta} = P\{N(t)\}$$

$$\frac{\beta}{\alpha} . K\{N(t)\} + P\{N(t)\} = \frac{N(0)}{\alpha\beta}$$

Since t = 0 then N(0) = N_0 ,

$$K\{N(t)\} = N_0\left[\frac{1}{\beta(\beta-\alpha\beta)}\right]$$

$$K\{N(t)\}\left(\frac{\beta}{\alpha} - P\right) = \frac{N_0}{\alpha\beta}$$

$$K\{N(t)\} = \frac{N_0}{\beta(\beta-\alpha P)}$$

$$K\{N(t)\} = N_0\left[\frac{1}{\beta(\beta-\alpha P)}\right] \quad (2)$$

Now applying inverse of KKAT to equation(2)

$$\{N(t)\} = N_0 K^{-1}\left[\frac{1}{\beta(\beta-\alpha P)}\right]$$

$$\{N(t)\} = N_0 e^{Pt} \qquad ....(3)$$

Now at t = 2 , N = $2N_0$ we have $2 = e^{2P}$

Now applying log on both sides,

$2P = \log_e(2) \Rightarrow P = 0.3465$

Now for t = 3, N = 20000

Now put in equation (3),

$20000 = N_0 e^{3\times 0.3465}$

$20000 = N_0(2.8278)$

$$N_0 = 7072.6359 \cong 7072$$

∴ 7072 People were initially in the city.

Application 4 :A radioactive substance is known to decay at a rate proportional to the amount present suppose that initially there is 100 miligrams of the radioactive substance present and after two hours it is observed that the substance has lost 10 percent of its original mass. Find the half-life of radioactive substance.

⇒ Differential equation governed by this problem is,

$$\frac{dN}{dt} = -PN \qquad \ldots\ldots\ldots\ldots(1)$$

Applying KKAT transform to equation (1)

$$K\left(\frac{dN}{dt}\right) = -P\,K\{N(t)\}$$

$$\frac{\beta}{\alpha}.\,K\{N(t)\} - \frac{N(0)}{\alpha\beta} = -P\,K\{N(t)\}$$

$$\frac{\beta}{\alpha}.\,K\{N(t)\} + P\,K\{N(t)\} = \frac{N(0)}{\alpha\beta}$$

$$K\{N(t)\}\,.\left(\frac{\beta}{\alpha} + P\right) = \frac{N(0)}{\alpha\beta}$$

$$K\{N(t)\} = \frac{N(0)}{\beta(\beta+\alpha P)}$$

Since now t = 0 , N = N_0 = 100 , we have $K\{N(t)\} = \frac{100}{\beta(\beta+\alpha P)}$

Applying inverse of KKAT, we get, $\{N(t)\} = K^{-1}\left[\frac{100}{\beta(\beta+\alpha P)}\right]$

$$N(t) = 100\,e^{-Pt} \qquad \ldots\ldots\ldots(2)$$

Now at t = 2 , Radioactive substance has lost 10 percent of its original mass 100 mg.

So, N = 100 - 10 = 90.

∴ 90 = 100 e^{-2P}

2P = $\log_e(0.9)$⇒ P = 0.0526

We required half time of radioactive substance when N = $\frac{N_0}{2}$ = 50

Put in equation (2),

$$50 = 100\,e^{-0.0526\,t}$$

$$\therefore\ (-0.0526\,t) = \log_e(0.5)$$

$$\therefore\ -0.0526\,t = -0.6931$$

$$\therefore\ t = 13.1768 \text{ hrs.}$$

∴ Half life of radioactive substance is 13.178 hrs.

IV. CONCLUSION

By using Raj transform we can easily solve the mathematical models in biochemistry, health sciences and environmental sciences, containing ordinary differential equations.

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