Design of Spur Gears Under Fatigue Load Parameters

[1]Pritam Jadhav, [2]Hanumant Borate
[1]*Lecturer, Mechanical Engineering Department, SVPMS ITE Malegaon BK, Baramati, India*
[2]*Professor at VPKBIET Vidyanagari, Baramati, India*

***Abstract*-This paper reviews the study on mainly design of spur gear under the fatigue loading on gear tooth profiles by using different design parameters. The bending strength and surface strength of the gear tooth are consider to be the mainly contributor in failure of gear tooth. It mainly the designs of spur gear Fatigue approach by using with the help of modified Lewis Equation and Buckingham's equation respectively. By this study we prefer theoretical design of spur gear by considering all fatigue design parameters. For analysis of spur gear stresses it can be calculated by using Ansys workbench and get the comparative results with theoretical values for confirmation of manufacturing of specified spur gear geometrical parameters.**

***Keywords*- AGMA, Lewis, Buckingham's equation**

I. INTRODUCTION

Gears are specifically used in power and motion transmission work under different loading and speed parameters. In modern engineering power transmission elements are generally found. It is possible that gears will predominate as the most effective means of transmitting power in future machines due to their high degree of reliability and compactness. In advance, most of the heavy industries such as shipbuilding to industries such as automobile manufacture and office automation tools will require a refined application of gear technology [1].

A pair of gear teeth in action is generally subjected to two types of cyclic stresses: which is one is bending stresses inducing bending fatigue and second is contact stress causing contact fatigue. Both these types of stresses may not attain their maximum values at the same point of contact. However, combined action of both of them is the reason of failure of gear tooth leading to fracture at the root of a tooth under bending fatigue and surface failure, like pitting or flaking due to contact fatigue. In addition there may be surface damage associated seizure of surfaces due to poor lubrication and overloading. However the fracture failure at the root due to bending stress and pitting and flaking of the surfaces due to contact stress cannot be fully avoided. These types of failures can be minimized by careful analysis of the problem during the design stage and creating proper tooth surface profile with proper manufacturing methods. In spite of all the cares, these stresses are sometimes very high either due to overloading or wear of surfaces with use and need proper investigation to accurately predict them under stabilized working conditioned so that unforeseen failure of gear tooth can be minimized [3].

Numerous fatigue-critical parts could be found in ground vehicles, and time-varying loads have always challenged automotive designers. Fatigue design and life assessment of these components are essentially influenced by the material used and manufacturing processes chosen. Explore the design parameters and optimize the strength parameters with respect to manufacturing processes is main aspect to the industry. This study was aimed at developing general procedures for fatigue analysis and optimization of safety-critical automotive components like gears with manufacturing considerations. This was obtained by, making specimen and component tests were performed, and finite element stress analysis and optimization evaluations of similar components are produced to achieve the objectives [4].

Finite element models of the components were analysed, using linear and nonlinear stress analyses. The nominal stress and local stress and strain approaches were employed to assess durability of the components. Experimental and analytical stress and fatigue life results were compared to evaluate the validity of the analytical approaches. The strength and shortages of the applied models and alternative analyses were also investigated. It was concluded that the local life prediction approaches in combination with either nonlinear finite element analysis results, or linear finite element analysis results corrected for local plasticity, yielded satisfactory predictions [5].

II. DESIGN OF SPUR GEAR BY AGMA PROCEDURE

2.1 Theoretical Design of Gears

The key step in gear design has been the determination of the allowable tooth bending stress. Prototyping of gears is expensive and time consuming, so an error in the initial choice of the tooth bending stress can be costly.

For any given material, the allowable stress is dependent on a number of factors, including:

- Total lifetime cycle.
- Intermittent or continuous duty.
- Environment – temperature, humidity, solvents, chemicals, etc.
- Change in diameter and center to center distance with temperature and humidity.
- Pitch line velocity.
- Diametric pitch (size of teeth) and tooth form.
- Accuracy of tooth form, helix angle, pitches diameter, etc.
- Mating gear material including surface finish and hardness.
- Type of lubrication (frictional heat).

Gear analysis can be performed using analytical methods which required a number of assumptions and simplifications which aim at getting the maximum stress values only but gear analyses are multidisciplinary including calculations related to the tooth stresses and to failures like wear. Various formulae are derived to account for various factors which have influence on gear rating and many instances these are derived in empirical manner. One is the Hertz equation, which can be used to calculate the Hertz contact stresses and other is the Lewis formula, which can be used to calculate the bending stress.

2.2 Bending Strength of Gear Tooth

The analysis of bending stresses in gear tooth was done by Wilfred Lewis in 1892 and even today, the Lewis equation is considered as the basic equation in the design of gears. In the Lewis analysis, the gear tooth is treated as a cantilever beam. The tangential component (Pt) causes the bending moment about the base of the tooth. Following assumptions have been made in Lewis equation.

1. The effect of radial component (Pr), which induces compressive stresses, is neglected.
2. It is assumed that the tangential component (Pt) is uniformly distributed over the face width of the gear. This is possible when the gears are rigid and accurately machined.
3. The effect of stress concentration is neglected.
4. It is assumed that at any time, only one pair of teeth is in contact and takes the total load.

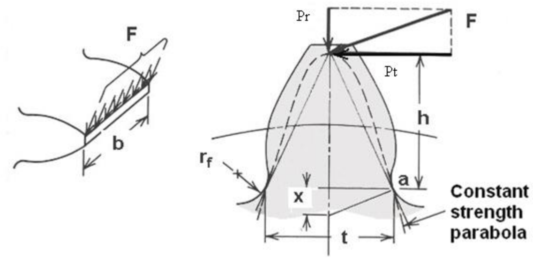

Fig.1 Gear Tooth as Cantilever Beam

It is observed that the cross section of the tooth varies from the free end to the fixed end. Therefore, a parabola is constructed within the tooth profile and shown by dotted line. The advantage of parabolic outline is that it is a beam of uniform strength. For this beam, the stress at any cross section is uniform or same as shown in figure 1.

The bending stresses are given by,

$$\sigma_b = \frac{M_b \times y}{I} = \frac{(P_t \times h)\left(\frac{t}{2}\right)}{\left[\left(\frac{1}{12}\right) \times b \times t^3\right]} \quad (1)$$

Rearranging the terms

$$P_t = b \times \sigma_b \times \left(\frac{t^2}{6h}\right) \quad (2)$$

Multiplying the numerator and denominator of the right hand side by m,

$$P_t = m \times b \times \sigma_b \times \left(\frac{t^2}{6hm}\right) \quad (3)$$

Defining a factor Y,

The equation is rewritten as,

$$P_t = m \times b \times \sigma_b \times Y$$

$$\text{where } Y = \left(\frac{t^2}{6hm}\right) \quad (4)$$

In the above equation, Y is called the Lewis form factor. Equation 4 gives the relationship between the tangential force (P_t) and the corresponding bending stress σ_b. when the tangential force is increased, the stress also increases. When the stress reaches the permissible magnitude of bending stresses, the corresponding force (P_t) is called the beam strength. Therefore the beam strength (S_b) is the maximum value of the tangential force that the tooth can transmit without bending failure. Replacing Pt by S_b above equation can be written as

$$S_b = m \times b \times \sigma_b \times Y \qquad (5)$$

Where

S_b = Beam strength of gear tooth (N)

σ_b =Permissible bending stress (N/mm^2)

In order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth.

Therefore

$$S_b \geq P_{eff}$$

Calculating effective load on gear Tooth (P_{eff})

$$P_{eff} = \frac{C_s \times P_t}{C_v} \qquad (6)$$

Where

C_s= Service factor, Pt. =Tangential Component of resultant force,

C_v= Velocity factor

$$C_s = \frac{starting\ torque}{Rated\ torque}$$

$$C_s = \frac{(M_t)Max}{M_t} \qquad (7)$$

For ordinary and commercially cut gears made with form cutters and with v< 10m/s,

$$C_v = \frac{3}{3+v} \qquad (8)$$

2.3 Permissible Bending Stresses

Since the teeth are subjected to fluctuating stresses, endurance limit stress (S_e) is the criterion of design. Therefore maximum bending stress is equal to the endurance limit stress of the gear teeth. The endurance limit stress of the gear tooth depends upon the following factors

1. Surface finish of the gear tooth
2. Size of the gear tooth
3. Reliability used in design
4. Stress concentration in the gear tooth
5. Gears rotating in one direction or both directions

In practice it is difficult to get above mentioned data for each and every case of gear design. Earle Buckingham suggested that the endurance limit stress of gear tooth is approximately one third of the ultimate tensile strength of the material.

$$\sigma_b = S_e = \left(\frac{1}{3}\right) S_{ut} \qquad (9)$$

2.3.1 Estimation of module based on beam strength

In order to avoid failure of gear tooth due to bending $S_b > P_{eff}$

Introducing factor of safety

$$S_b = P_{eff}(FS) \qquad (10)$$

The recommended factor of safety is from 1.5 to 2.

Finally from above all equation formula for gear module is derived as

$$m = \left\{ \frac{60 \times 10^6}{\Pi} \left[\frac{Kw \times C_s \times F.S.}{Z_p \times n_1 \times \frac{S_{ut}}{3} \times \left(\frac{b}{m}\right) \times Y \times C_v} \right] \right\}^{\frac{1}{3}} \qquad (11)$$

2.4 Drawbacks of Lewis equation

1. The tooth load in practice is not static. It is dynamic and is influenced by pitch line velocity.
2. The whole load is carried by single tooth is not correct. Normally load is shared by teeth since contact ratio is near to 1.5.
3. The greatest force exerted at the tip of the tooth is not true as the load is shared by teeth. It is exerted much below the tip when single pair contact occurs.
4. The stress concentration effect at the fillet is not considered.

III. DESIGN OF GEAR BY USING BUKINGHAMS EQUATION

As per the application based requirements the firstly we calculate the module of gear as it fulfill the all design consideration values and check it for that material also. While design of gear we must know about input data as its power required, pinion and gear speed. By considering all input data the following design steps are followed. Power= W =HP= kW; n_1= n_2 = rpm; 20^0 full depth involutes spur gear. Speed ratio= i = n_1 / n_2

In order to keep the size of gears small and avoid interference also, now, angular velocity of spur gear pair, $\omega = \frac{2\Pi n}{60} = rad/\sec$ so,

Torque transmitted by pair,

$$T = \frac{1000 \times w}{\omega} \text{ Nmm}$$

AGMA equation for tooth bending stress,

Where $b = mz$,

$$\sigma_1 = \frac{2T}{bzm^2 J} \times K_v \times K_m \times K_o \qquad (12)$$

These values are obtained from the table.

$$K_v = \left(\frac{78 + (200v)^2}{78}\right)^{0.5} = 1.15$$

TABLE1 OVERLOAD FACTOR K_O

	Driven Machinery		
Driven Machinery	Source of power	Uniform	Moderate Shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

TABLE2 LOAD DISTRIBUTIONS FACTOR *km*

	Face width (mm)			
Characteristics of Support	0 – 50	150	225	400 up
Accurate mountings, small bearing	clearances, minimum deflection, precision	Gears	1.3	1.4
Less rigid mountings, less accurate gears,	contact across the full face	1.6	1.7	1.8
Accuracy and mounting such that less than	full-face contact exists	Over 2.2	Over 2.2	Over 2.2

Now endurance stress of pair,

$$\sigma_e = \sigma_{e'} \times k_L \times k_v \times k_s \times k_R \times k_T \times k_f \times k_m \quad (13)$$

For pinion $\sigma_{e'} = 0.5\sigma_{ut} = Mpa$

TABLE 3 RELIABILITY FACTOR k_R

Reliability factor R	0.50	0.90	0.95	0.99	0.999
Factor Kr	1.00	0.89	0.868	0.814	0.753

Factor of safety on bending of 1.5 assumed,

$$[\sigma] = \sigma_e / S \quad (14)$$

Now from tooth bending fatigue considerations, Tooth bending stress should be less than design strength for that gear, i.e.

$$\sigma_1 \leq \sigma \quad (15)$$

From above relation we get the value of designed gear module .From module we have decided all geometric dimensional parameters of the spur gear.

Tangential load on gear tooth becomes,

$$F_t = \frac{T}{r} \text{ .... } (16)$$

So tooth checked by surface durability consideration, Hertzs Contact stress equation from AGMA,

$$\sigma_H = C_p \sqrt{\frac{F_t \times K_v \times K_o \times K_m}{bdI}} \text{ .... } (17)$$

Elastic coefficient of material, $C_p = Mpa$

$$I = \frac{\sin\theta \times \cos\theta}{2} \times \frac{i}{i+1} \text{ ...where i=1}$$

Calculating surface fatigue strength,

$$\sigma_{sf} = \sigma_{sf'} \times k_L \times k_R \times k_T \text{ .... } (18)$$

$$\sigma_{sf'} = (2.8 \times Bhn) - 69 = Mpa$$

Assuming a factor of safety s = 1.1

$$[\sigma_H] = \sigma_{sf} / S = Mpa$$

So, Surface contact stress $\sigma_H = Mpa <$ Surface fatigue strength $[\sigma_H] = Mpa$

The design is safe and surface fatigue failure will not occur. So new terminology of spur gear pair.

If design is not safe then Increase the surface hardness of the material and also increase the module to next value, So again check for new terminology of spur gear pair as per above process.

IV ANALYTICAL DESIGN OF TOOTH STRENGTH OF GEARS

Earle Buckingham’s equation for finding incremental dynamic or fatigue load on gear teeth under the desired motion in engaged pair,

$$F_i = \frac{21v \times (Ceb + F_t)}{21v + \sqrt{Ceb + F_t}} \text{ .... } (19)$$

Where,

F_i= incremental dynamic load

V= pitch line velocity

C= deformation factor

e = sum of error between two meshing teeth

b= face width

Ft= tangential force due to rated torque

So, Total dynamic load

$$F_d = (F_t + F_i) \text{ .... } (20)..$$

Analytical Tooth Strength of gear is as

$$F_{td} = \sigma \times b \times Y \times m \text{ .... } (21)$$

As per design consideration, Design of gear is safe When,

$$F_{td} \geq (F_d)$$

So, check the safe design for different materials gears.

V.CONCLUSION

From above review paper we get the design procedure aspects and parameters while fatigue loading on the gear tooth. By using modified AGMA bending stress equations will get the analytical design of gear tooth by considering all fatigue considerations in design of gears. From Buckingham's equations we will calculate the loading capacity of gear while considering its fatigue failure due to the contact surface stress and bending strength parameters in dynamic conditions.

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